# Decay of Collective Nuclear States Excited in Muon Capture*

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Excitation of giant-dipole and related collective states has been found to give predominant contributions to muon capture. The present work obtains polarizations of the neutrons emitted in the subsequent decay of these states, for capture in some $0^+$ closed-shell nuclei (their angular distributions are shown to be isotropic). The polarization of the neutrons is due to the polarization of the captured muon, and is obtained both according to type (longitudinal and transverse) and according to magnitude (it depends on the configuration mixing of the dipole states). The generalized Goldhaber-Teller model is used for the calculation of the excitation, and Boeker's form of the Wigner $R$-matrix theory for the decay, together with standard Racah methods. In an Appendix, the longitudinal polarization of the directly emitted neutrons of highest energy following muon capture in nuclei, and its dependence on the weak coupling constants, are discussed briefly.

## I. INTRODUCTION

THE muon-capture reaction has often been said to be unique insofar as it represents a tool for probing nuclear structure besides permitting a study of the weak interaction proper. Very little use has been made so far, however, of its former aspect, all nuclear-structure-dependent phenomena having been interpreted as means for determining coupling constants, such as the induced pseudoscalar.[1–4] With the recent realization of the dominant role (80–90%) that giant-dipole states play in muon capture,[5–7] this process may turn out to constitute a useful probe into the properties of the collective states, complementing the photonuclear and $(N,\gamma)$ giant-dipole-excitation studies by the large axial-vector (Gamow-Teller) transition probability it features over and above the vector (Fermi) matrix element which it shares with the electromagnetic reactions. (Inelastic electron scattering can also excite the collective states through a magnetic transition containing $\boldsymbol{\sigma}$.) Spectra of neutrons emitted after muon capture have been worked out by Balashov *et al.*[6] on the basis of the Elliott-Flowers[8,9] single-particle-hole model of the giant-dipole resonant states in $0^+$ nuclei. In the present work, we study angular distributions and especially polarizations of the neutrons emitted in the decay of the excited collective states. The decay is described using Wigner's $R$-matrix theory in a form suitable for particle-hole states as developed by Boeker and Jonker.[10,11] Since the accuracy[11] of this method, as far as widths are concerned, seems to be somewhat less than the accuracy of the excitation calculations (which are able to predict, e.g., the photonuclear transitions rather well[8,9,12–15]), we employed the Goldhaber-Teller[16,17] model of collective states, extended to include spin-isotopic-spin modes of collective vibrations[18] (which contribute to the axial-vector matrix element) for a description of the giant-dipole excitation in muon capture; this is expected to be of an accuracy comparable to that obtained in the decay process. The configurations occurring in the latter phase were of course properly linked to the collective states in the excitation. It was found that the neutron emission should be isotropic, but that there should be sizeable neutron polarizations caused by the captured muon being polarized, both longitudinal and transverse, the magnitudes of which vary with emission angle (whereas neutrons emitted directly after muon capture carry predominantly longitudinal polarization only[19,20]) and depend strongly on the mixing of configurations in the decaying states (i.e., the small components), so that a study of these neutron polarizations could be another way of verifying the small admixtures, besides, e.g., measurements of the magnitudes of photonuclear absorption peaks. The theoretical methods used here apply to muon capture in closed-shell $0^+$ nuclei, and numerical examples were worked out for $C^{12}$, $O^{16}$, and $Ca^{40}$; in the case of $Si^{28}$, for which

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1 R. C. Cohen, S. Devons, and A. D. Kanaris, Nucl. Phys. 57, 255 (1964).
2 T. Ericson, J. C. Sens, and H. P. C. Rood, Nuovo Cimento 34, 51 (1964).
3 H. P. C. Rood, thesis, University of Groningen, 1964 (unpublished).
4 D. A. Jenkins, University of California Radiation Laboratory Report No. UCRL-11531, 1964 (unpublished).
5 J. Barlow, J. C. Sens, P. J. Duke, and M. A. R. Kemp, Phys. Letters 9, 84 (1964).
6 V. V. Balashov, V. B. Beliaev, R. A. Eramjian, and N. M. Kabachnik, Phys. Letters 9, 168 (1964).
7 L. L. Foldy and J. D. Walecka, Nuovo Cimento 34, 1026 (1964).
8 J. P. Elliott and B. M. Flowers, Proc. Roy. Soc. (London) A242, 57 (1957).
9 G. E. Brown, L. Castillejo, and J. A. Evans, Nucl. Phys. 22, 1 (1961).
10 E. Boeker and C. C. Jonker, Phys. Letters 6, 80 (1963).
11 E. Boeker, Thesis, University of Amsterdam, 1963 (unpublished).
12 N. Vinh-Mau and G. E. Brown, Nucl. Phys. 29, 89 (1962).
13 V. V. Balashov, V. G. Shevchenko, and N. P. Yudin, Nucl. Phys. 27, 323 (1961).
14 E. Hayward, Rev. Mod. Phys. 35, 324 (1963).
15 E. Hayward, Lecture Notes on Photonuclear Reactions (Scottish Universities' Summer Schook, 1964) (unpublished).
16 M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948).
17 J. Goldemberg, Y. Torizuka, W. C. Barber, and J. D. Walecka, Nucl. Phys. 43, 242 (1963).
18 H. Überall, Phys. Rev. 137, B502 (1965).
19 M. Cini and R. Gatto, Nuovo Cimento 11, 253 (1959).
20 L. Wolfenstein, Nuovo Cimento 7, 706 (1958).

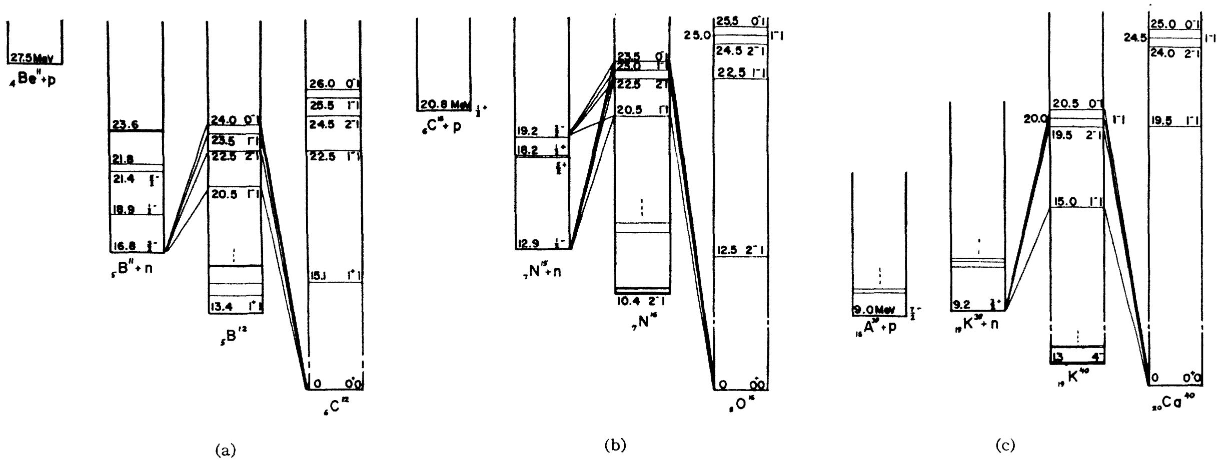

FIG. 1. (a) Nuclear level scheme for muon capture in $C^{12}$ and subsequent nucleon emission. (b) Nuclear level scheme for muon capture in $O^{16}$ and subsequent nucleon emission. (c) Nuclear level scheme for muon capture in $Ca^{40}$ and subsequent nucleon emission.

configurations have also been obtained,[21,22] these seem to be too violently mixed to permit a proper identification of the collective modes. Charge independence has been assumed throughout.

## II. EVALUATION

We consider muon capture in closed-shell $0^+$ nuclei, and in particular the following capture reactions and their subsequent decays:

$$\mu^- + {}_6C^{12} \rightarrow {}_5B^{12}{}_{\text{g.dip.}} + \nu\,, \tag{1a}$$

$$ {}_5B^{12}{}_{\text{g.dip.}} \rightarrow n + {}_5B^{11(*)}\,; \tag{2a}$$

$$\mu^- + {}_8O^{16} \rightarrow {}_7N^{16}{}_{\text{g.dip.}} + \nu\,, \tag{1b}$$

$$ {}_7N^{16}{}_{\text{g.dip.}} \rightarrow n + {}_7N^{15(*)}\,; \tag{2b}$$

$$\mu^- + {}_{20}Ca^{40} \rightarrow {}_{19}K^{40}{}_{\text{g.dip.}} + \nu\,, \tag{1c}$$

$$ {}_{19}K^{40}{}_{\text{g.dip.}} \rightarrow n + {}_{19}K^{39(*)}\,, \tag{2c}$$

where "g.dip." stands for "giant dipole." Calling $\Delta$ the excitation energy of the final $T_3=-1$ nucleus measured from the $T_3=0, J=0^+$ ground state of the initial nucleus, energy conservation requires (neglecting the nuclear recoil):

$$\nu = \mu - \Delta\,, \tag{3}$$

where $\mu$ = muon mass, $\nu$ = neutrino momentum. Of course, out of the 15 components of the giant-dipole Wigner supermultiplet,[7,23] only those with $T=1$ come into play here. In the supermultiplet theory, all these states are degenerate; experimentally, they are split up, and this is also the case in the existing particle-hole models of the giant-dipole states. These are

(a) for $C^{12}$, the models of Gillet[24] and of Lewis and Walecka[25];

(b) for $O^{16}$, the models of Gillet,[24] of Brown *et al.*,[9] and of Lewis[26];

(c) for $Ca^{40}$, the models of Gillet,[24] of Brown *et al.*,[9] and of Balashov *et al.*[13]

Of all these, Gillet's models seem to be the most complete, and we shall in the following make predominant use of them. It turns out that in this model, after the possible particle-hole configurations have been mixed by a two-body interaction, one configuration still remains dominant in general in each state, but it is only *one state* $J=1^-$, $T=1$ that stands out among the others in dipole strength; for the axial-vector transition probability, there is likewise a predominant one with $J=1^-$, $T=1$, which has two companions with $J=0^-$ and $J=2^-$ of the same configuration. For $O^{16}$, e.g., the dominant vector state is[7] $(1p\frac{3}{2})^{-1}(1d\frac{5}{2})$, the dominant axial vector $(1p\frac{3}{2})^{-1}(1d\frac{3}{2})$. The model[26] gives their energies at 23.9 MeV and at 27.3 ($0^-$), 26.6 ($1^-$), and at 24.5 MeV ($2^-$), respectively. We prefer, however, to use later on a compromise between theoretical and experimental level energies, which can then be roughly taken as[14] 22.5 MeV and 25.5, 25.0, and 24.5 MeV, respectively (note that the "axial-vector" levels also have a certain amount of dipole strength). The situations are depicted in Figs. 1 (a)–(c), in which these $T=1$ collective levels have been drawn in the $0^+$ and the neighboring nuclei ($T_3=-1$), properly shifted by their Coulomb energy, together with some of their decay schemes (including the energy situation for

[21] L. N. Bolen and J. M. Eisenberg, Phys. Letters **9**, 52 (1964).
[22] J. B. Seaborn and J. M. Eisenberg, unpublished report.
[23] E. P. Wigner, Phys. Rev. **51**, 106 (1937).
[24] V. Gillet, thesis, University of Paris, 1962 (unpublished); see also Ref. 11.
[25] F. H. Lewis and J. D. Walecka, Phys. Rev. **133**, B849 (1964).
[26] F. H. Lewis, Phys. Rev. **134**, B331 (1964); see also Ref. 7.

possible proton emission). The experimental energies were mostly taken from the figures of Ref. 14; the sometimes considerable experimental widths of the levels were not taken into account, and they would be somewhat different in any case for the levels in the $T_3=-1$ nuclei reached by muon capture.

These states of dominant "vector" and "axial-vector" character can now easily be identified with the corresponding collective states of the generalized Goldhaber-Teller model (see Ref. 18 for a more detailed description of this model), which will be used to calculate the excitation probability of the states. Since we calculate mostly polarizations, total rates are of little importance, so mainly the geometrical transformation properties of the transition amplitudes predicted by the model will be needed.

The combined transition probability $\lambda$ for excitation and subsequent decay of the collective states can be written, calling $J, M_J$ the spin of the final nucleus and $\Omega, \sigma$ direction and spin coordinate of the emitted neutron, and $J_0, M$ the spin of the intermediate resonant state (which later we consider noninterfering with its neighbors, and identifiable by observing the corresponding peak in the spectrum of emitted neutrons[27]):

$$\lambda=\sum_{MM'}\omega_{J_0MM'}\rho_{J_0MM'}\,, \tag{4}$$

where $\omega_{J_0MM'}$ represents the muon capture probability, and $\rho_{J_0MM'}$ the decay probability of the collective states (which are polarized owing to the muon being polarized):

$$\rho_{J_0MM'}=\sum_{M_J}\langle JM_J,\Omega\sigma|\mathcal{T}|J_0M\rangle^*\langle JM_J,\Omega\sigma'|\mathcal{T}|J_0M'\rangle\,, \tag{5}$$

with $\mathcal{T}$ the transition operator. By the standard Racah methods[28,29] Eq. (5) can be expressed as

$$\rho_{J_0MM'}=\sum_{\bar{J}\bar{M}}(-1)^{M'}(J_0M,J_0-M'|\bar{J}\bar{M})F(\bar{J}\bar{M})\,, \tag{6a}$$

$$F(\bar{J}\bar{M})=\hat{J}_0{}^2\sum_{\bar{M}'}\sum_{jlj'l'}\tau_{\bar{J}\bar{M}'}(jlj'l')(-1)^{\bar{J}+J-j}\times W(J_0J_0jj';\bar{J}J)\mathfrak{D}_{\bar{M}\bar{M}'}{}^{\bar{J}}(R)\times\langle J\|jl\|J_0\rangle^*\langle J\|j'l'\|J_0\rangle\,, \tag{6b}$$

where $\hat{L}$ means $(2L+1)^{1/2}$, $\mathfrak{D}$ is the rotation matrix corresponding to a rotation $R$ that brings the $z$ axis into the direction of the outgoing (neutron) radiation of total and orbital angular momentum $j$ and $l$, respectively; the radiation parameter $\tau_{\bar{J}\bar{M}'}$ is given by

$$\tau_{\bar{J}\bar{M}'}(jlj'l')=\sum_{\mu\mu'}(-1)^{j'-\mu'}(j\mu,j'-\mu'|\bar{J}\bar{M}')\times\langle 0\sigma|jl\mu\rangle^*\langle 0\sigma'|j'l'\mu'\rangle\,, \tag{7}$$

and the $\langle J\|jl\|J_0\rangle$ are reduced matrix elements, used in the sense of Refs. 28, 29, and defined in more detail in Eqs. (24)–(26) below.

The capture probability for polarized muons is given using the Primakoff Hamiltonian (see Ref. 18):

$$H=2^{-1/2}(\phi_n{}^\dagger u_\nu{}^\dagger(1+\gamma_5)[G_F+G_G\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}^N+G_P\hat{\nu}\cdot\boldsymbol{\sigma}^N]u_\mu\phi_P)\,, \tag{8}$$

where the $G_i$ are the effective coupling constants (nuclear-velocity terms are neglected). We obtain

$$\omega_{J_0}=(8\pi^3a_\mu{}^3)^{-1}\nu^2\int d\Omega_\nu\{G_F{}^2|\mathfrak{M}|^2+G_G{}^2|\boldsymbol{\mathfrak{M}}|^2-(2G_GG_P-G_P{}^2)|\hat{\nu}\cdot\boldsymbol{\mathfrak{M}}|^2+P\mathbf{n}\cdot[2G_FG_G\,\mathrm{Re}\mathfrak{M}^*\boldsymbol{\mathfrak{M}}+iG_G{}^2\boldsymbol{\mathfrak{M}}^*\times\boldsymbol{\mathfrak{M}}-2G_FG_P\hat{\nu}\hat{\nu}\cdot\mathrm{Re}\mathfrak{M}^*\boldsymbol{\mathfrak{M}}+2G_GG_P\,\mathrm{Im}(\hat{\nu}\cdot\boldsymbol{\mathfrak{M}}\hat{\nu}\times\boldsymbol{\mathfrak{M}})]\}\,, \tag{9}$$

with $a_\mu=137/(Z\mu)$, $\hat{\nu}=\mathbf{v}/\nu$, $P$ and $\mathbf{n}$ the degree of polarization and direction of the muon spin at the moment of its capture ($P$ being of order 15–20% experimentally), and the matrix elements

$$\mathfrak{M}=(\Phi_{Z-1}{}^{*\dagger}\sum_{i=1}^{A}e^{-i\nu\cdot\mathbf{r}_i}\tau_-{}^{(i)}\varphi_\mu(\mathbf{r}_i)\Phi_Z)\,, \tag{10}$$

$$\boldsymbol{\mathfrak{M}}=(\Phi_{Z-1}{}^{*\dagger}\sum_{i=1}^{A}e^{-i\nu\cdot\mathbf{r}_i}\boldsymbol{\sigma}^{(i)}\tau_-{}^{(i)}\varphi_\mu(\mathbf{r}_i)\Phi_Z)\,, \tag{11}$$

[where the bound-muon wave function will be assumed as $\varphi_\mu\cong\exp(-r/a_\mu)$], which can be taken from Ref. 18:

$$\boldsymbol{\mathfrak{M}}_M=CY_{1M}{}^*(\hat{\nu})\,, \tag{12a}$$

$$C=-i\nu F'(\nu)(4\pi/3Am\Delta)^{1/2}\,, \tag{12b}$$

where $A$ = atomic weight, $m$ = nucleon mass, and the form factor

$$F'(\nu)=Z(M_0/M_0')^3(1+\nu^2/M_0'^2)^{-2}\,, \tag{13}$$

with $M_0=0.725mA^{-1/3}$, $M_0'=M_0+Z\mu/137$. In the case of the axial-vector matrix element, in which the oscil-

[27] Experiments of Hagge for muon capture in $Ca^{40}$ and heavier nuclei seem to indicate quite clearly the peaks in the neutron spectrum corresponding to dipole–ground-state transitions, although the experimental resolution will still have to be improved. In this work, however, neutron emission was solely interpreted in terms of the evaporation model. See D. E. Hagge, University of California Radiation Laboratory Report No. UCRL-10516, 1963 (unpublished).

[28] S. Devons and L. J. B. Goldfarb, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. XLII, p. 362.

[29] L. J. B. Goldfarb, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, p. 159.

lator transition $0^+ \rightarrow 1^-$ (magnetic quantum number $m_1$) has to be coupled vectorially with the nuclear total spin angular momentum $S=1$ (magnetic quantum number $m_s$) to a total spin $J_0=0^-$, $1^-$, or $2^-$, care has to be taken that the tensors, corresponding to final states, transform cogradiently[30] under rotations, whereas spherical components of $\mathfrak{M}$ (which we index by $\mu$) are preferably taken as contragradient. In the uncoupled expression of Ref. 18,

$$\mathfrak{M}_{\mu,m_s m_1}=(-1)^{\mu}\delta_{\mu,-m_s}\mathfrak{M}_{m_1}, \qquad (14)$$

$\mu$ can be made contragradient by use of Wigner's metric tensor.[31] We then have

$$\mathfrak{M}_{\mu,M}=-C\sum_m (1\mu,1m|J_0M)Y_{1m}^*(\hat{p}). \qquad (15)$$

Remembering that the vector $(1^-)$ and axial vector $(J_0^-)$ (for each value $J_0$) transitions are to be considered separately, the $F-G$ interference terms in Eq. (9) do not appear, and numbering the remaining terms I$\cdots$V in the order in which they appear in (9), we obtain for the corresponding terms in $\lambda$, using some Racah algebra, and introducing the factor

$$D=\nu^2(8\pi^3a_\mu^3)^{-1}|C|^2: \qquad (16)$$

$$\lambda_{\rm I}=DG_F^2(-1)^{J_0}\hat{J}_0F(00), \qquad (17.{\rm I})$$

$$\lambda_{\rm II}=DG_G^2(-1)^{J_0}\hat{J}_0F(00), \qquad (17.{\rm II})$$

$$\lambda_{\rm III}=-3D(2G_GG_P-G_P^2)\hat{J}_0^{-1}(10,10|J_00)^2F(00), \qquad (17.{\rm III})$$

$$\lambda_{\rm IV}=\sqrt{2}PDG_G^2(-1)^{J_0}\hat{J}_0^2W(1J_011;J_01)F(10), \qquad (17.{\rm IV})$$

$$\lambda_{\rm V}=-6\sqrt{2}PDG_GG_P(10,10|J_00)^2W(1J_011;J_01)F(10), \qquad (17.{\rm V})$$

where the last two quantities were calculated setting $\mathbf{n}\|\mathbf{z}$.

The radiation parameters to be used in $F(\bar{J}\bar{M})$ can be derived for spin-$\frac{1}{2}$ particles[28,29] and consist of a part independent of their polarization,

$$8\pi\tau_{\bar{J}\bar{M}'}^{(i)}=\tfrac{1}{2}\hat{j}\hat{j}'(j\tfrac{1}{2},j'-\tfrac{1}{2}|\bar{J}0)(-1)^{j+\bar{J}-(1/2)}\times[1+(-1)^{\bar{J}}]\delta_{\bar{M}'0}, \qquad (18a)$$

a longitudinally polarized part

$$8\pi\tau_{\bar{J}\bar{M}'}^{(l)}=\tfrac{1}{2}\hat{p}\cdot\mathbf{s}\hat{j}\hat{j}'(-1)^{j-(1/2)}(j\tfrac{1}{2},j'-\tfrac{1}{2}|\bar{J}0)\times[1-(-1)^{\bar{J}}]\delta_{\bar{M}'0}, \qquad (18b)$$

and two transversally polarized parts,

$$8\pi\tau_{\bar{J}\bar{M}'}^{(\pm)}=-\tfrac{1}{2}(s_x\mp is_y)\hat{j}\hat{j}'(j\tfrac{1}{2},j'\tfrac{1}{2}|\bar{J}1)(-1)^{j-j'+l}\times(-1)^{(1/2)(1\mp1)\bar{J}}\delta_{\bar{M}',\pm1}, \qquad (18c)$$

where it has been anticipated that $l+l'$ is always even in the configurations considered in the following. Here, $\hat{p}=\mathbf{p}/p$ where $\mathbf{p}$ is the momentum of the outgoing neutron, and $\mathbf{s}$ is its polarization vector.

From Eq. (17), it follows that all neutron angular distributions are isotropic, since $F(00)$ implies that only $\mathfrak{D}_{00}^0$ enters, and $\tau_{1\bar{M}'}^{(i)}=0$. This follows from parity conservation,[32] which permits even powers of $\mathbf{n}\cdot\hat{p}$ to appear, but only one power of $\mathbf{n}$ is present in Eq. (9). $F(00)$ only can be present in $\lambda_{\rm I}\cdots\lambda_{\rm III}$ which contain no reference directions. The situation is somewhat similar to beta decay of polarized nuclei before the advent of parity violation.[33]

As no $G_F^2$ term appears multiplied by $\mathbf{n}$ in Eq. (9), it is clear that the vector states cannot emit polarized neutrons, using the results $\tau_{00}^{(l)}=\tau_{00}^{(\pm)}=0$ (or: no pseudovectors can be formed without $\mathbf{n}$); likewise, the neutrons from the $0^-$ axial-vector states are unpolarized because $W(1011;01)=0$. The total transition probabilities are found as (before summing over the neutron spin directions)

$$\lambda=D\{A\Sigma_1+PB[\mathbf{s}\cdot\hat{p}\hat{p}\cdot\mathbf{n}\Sigma_2+\mathbf{s}\cdot(\hat{p}\times(\mathbf{n}\times\hat{p}))\Sigma_3]\}, \qquad (19)$$

with

$$A=G_F^2\hat{J}_0^2$$

or

$$G_G^2\hat{J}_0^2-3(2G_GG_P-G_P^2)(10,10|J_00)^2, \qquad (20a)$$

$$B=\hat{J}_0^2W(1J_011;J_01)\times[G_G^2\hat{J}_0^2-6G_GG_P(10,10|J_00)^2], \qquad (20b)$$

and

$$\Sigma_1=\sum_{jll'}\langle J\|jl\|J_0\rangle^*\langle J\|jl'\|J_0\rangle, \qquad (21a)$$

$$\Sigma_2=6^{1/2}\sum_{jlj'l'}\hat{j}(j\tfrac{1}{2},10|j'\tfrac{1}{2})W(J_01Jj';J_0j)\times\langle J\|jl\|J_0\rangle^*\langle J\|j'l'\|J_0\rangle, \qquad (21b)$$

$$\Sigma_3=\sum_{jlj'l'}\hat{j}\hat{j}'(-1)^{j-j'+l}(j\tfrac{1}{2},j'\tfrac{1}{2}|11)W(J_01Jj';J_0j)\times\langle J\|jl\|J_0\rangle^*\langle J\|j'l'\|J_0\rangle. \qquad (21c)$$

Equation (19) can be put into the form $\frac{1}{2}(1+\mathbf{s}\cdot\boldsymbol{\zeta})$, from which the neutron spin vector is obtained as

$$\boldsymbol{\zeta}=P(B/A)[\hat{p}\mathbf{n}\cdot\hat{p}(\Sigma_2/\Sigma_1)+\hat{p}\times(\mathbf{n}\times\hat{p})(\Sigma_3/\Sigma_1)]; \qquad (22)$$

this contains both a longitudinal component and a transverse one in the $\mathbf{p},\mathbf{n}$ plane. The polarization is dependent on the emission angle and will be purely transverse for $\mathbf{p}\perp\mathbf{n}$, and purely longitudinal for $\mathbf{p}\|\mathbf{n}$. It is practically independent of the coupling, since neglecting the small square of the pseudoscalar, we have

$$B/A\cong R\equiv\hat{J}_0^2W(1J_011;J_01). \qquad (23)$$

It remains to evaluate the $\Sigma_i$. The reduced matrix

[30] U. Fano and G. Racah, *Irreducible Tensorial Sets* (Academic Press Inc., New York, 1959); see also Ref. 11.

[31] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957), p. 46.

[32] Note that our integration over $d\Omega_\nu$ removes all parity-violating terms from the muon-capture reaction; for this reason, the only observable in this process besides the rate is a polarization of $J_0$ along the muon spin direction.

[33] H. A. Tolhoek and S. R. deGroot, Physica 17, 81 (1951).

elements contained therein can be obtained using the $R$-matrix methods of Boeker.[11] They can be written as[11,34]

$$\langle J\|jl\|J_0\rangle = g_{jl}e^{-i\phi_l-(1/2)il\pi}; \tag{24}$$

the total width $\Gamma$ entering in

$$g_{jl}=\pm(\Gamma_{jl}/\Gamma)^{1/2} \tag{25}$$

is of no importance for polarization calculations, and neither is the sign which is only over-all for a given level.[11] The partial widths are related to the reduced width by

$$\Gamma_{jl}=(2P_l/a)\gamma_{jl}^2. \tag{26}$$

The penetration factors $P_l$ and phases $\phi_l$ for neutrons can be found in tables.[35] For the channel radius we use the formula

$$a=(1.20A^{1/3}+0.80)\text{ F}, \tag{27}$$

but for $C^{12}$, shall also use $a'=1.4A^{1/3}$ and $a''=1.2A^{1/3}$ F in order to test the dependence of our results upon this quantity. Finally, the reduced widths are given by the $R$-matrix expression[11] (for the $T_3=-1$ state decay)

$$\gamma_{jl}=(3\hbar^2/2ma)^{1/2}\sum_{n_A}(-1)^{n_A-1}X_{Aa}, \tag{28}$$

where $X_{Aa}$ are the coefficients of the mixed configurations [9,13,24–26] of particles $A$ (principal quantum number $n_A$) and holes $a$; only those configurations enter that correspond to the hole state of the daughter nucleus, and to the $j$, $l$ of the outgoing channel.

## III. RESULTS AND DISCUSSION

### 1. $C^{12}$ Capture

For $C^{12}$ capture, only the highest energy transitions, from the $0^-$, $1^-$, and $2^-$ axial-vector states of ${}_5B^{12}$ to the ground state of ${}_5B^{11}$, are considered; they should be identified experimentally as the group of three peaks in the neutron spectrum around 6–7 MeV [see Fig. 2(a)], rising up from a small[6] tail of evaporated or directly emitted neutrons that reaches to higher energies. In this figure, the positions of neutron groups corresponding to transitions to excited $B^{11}$ states are indicated also; this lower part of the spectrum will furthermore contain contributions from muon capture to levels below[36] the collective states of $B^{12}$, which will make proper identification extremely difficult.[37] The relative heights of the three states in Fig. 2(a), 1.3:11.9:16.0, were obtained using Gillet's[24] wave functions and Eqs. (19) and (27). (Note that the phases of some hole states in Gillet's tabulation need to be corrected. This was pointed out, e.g., by Boeker[11]).

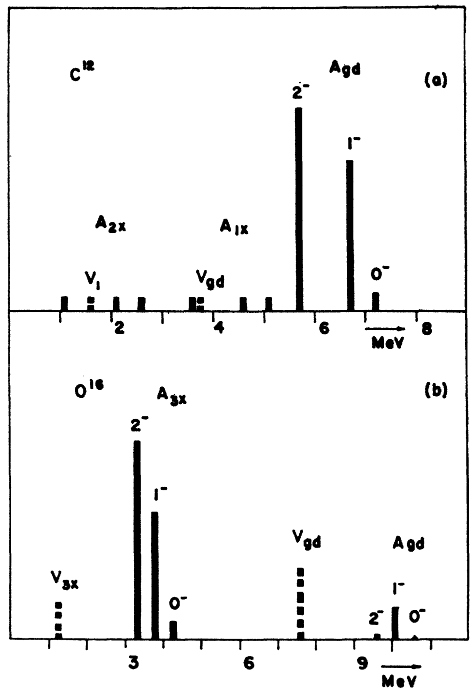


FIG. 2. Energies of neutron groups emitted after muon capture in $C^{12}$ and $O^{16}$, corresponding to transitions from the $1^-$ "vector" ($V$) and $0^-$, $1^-$, $2^-$ "axial-vector" ($A$) collective states to the ground (gd), first excited (1x), etc. states of the final nucleus (see Fig. 1). The relative magnitudes of some of the transitions are indicated.

The $B^{11}$ ground state can be taken $(1p_{3/2})^{-1}$; it will be fed by large components of the $(1p_{3/2})^{-1}(1d_{3/2})$ axial-vector state of $B^{12}$, and thus the results for the neutron polarization may be considered as relatively reliable. To be consistent, it might not really be necessary to retain contributions of small components in those decays of collective states where a large component participates, since they have not been kept in the excitation calculation either. Their effect is essential, however, when this is not the case; but we believe that even in this situation, use of the Goldhaber-Teller model (being lowest order) for the excitation is sufficiently accurate. Table I shows the coefficients in Eq.

TABLE I. Degree of polarization of neutrons emitted after muon capture in $C^{12}$, using Gillet's (Ref. 24) [in parentheses: Lewis' and Walecka's (Ref. 25)] wave functions.

| | $1^-$ state | | | $2^-$ state | | |
|---|---|---|---|---|---|---|
| | $a=3.55$ F | $a'=3.20$ F | $a''=2.75$ F | $a$ | $a'$ | $a''$ |
| $R(\Sigma_2/\Sigma_1)$ | 0.16 | 0.15 | 0.14 | −0.35 | −0.36 | −0.37 |
| | (0.23) | (0.22) | (0.20) | | | |
| $R(\Sigma_3/\Sigma_1)$ | −0.09 | −0.09 | −0.08 | 0.29 | 0.29 | 0.29 |
| | (−0.04) | (−0.04) | (−0.03) | | | |

[34] See also H. E. Gove, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, p. 259.

[35] J. E. Monahan, L. C. Biedenharn, and J. D. Schiffer, Argonne National Laboratory Report No. ANL-5846, 1958 (unpublished).

[36] For capture rates to low excited states, see M. Ruel and J. G. Brennan, Phys. Rev. **129**, 866 (1963).

[37] Compare the corresponding situation for the $(\gamma,n)$ reaction in $O^{16}$: P. F. Yergin, R. H. Augustson, N. N. Kaushal, H. A. Medicus, W. R. Moyer, and E. J. Winhold, Phys. Rev. Letters **12**, 733 (1964).

(22), calculated on the basis of the wave functions of Gillet[24] (values in parentheses: using the wave function of Lewis and Walecka[25]). We see that some polarizations are sizeable, and depend little on the value of *a* and not excessively on different wave functions.

### 2. $O^{16}$ Capture

For $O^{16}$ capture, transitions to the negative-parity ground and third excited states of $N^{15}$ were considered, and their positions and relative magnitudes (from top: 0.02:2.53:0.32:5.65:1.36:9.87:15.68) are shown in Fig. 2(b). No transitions to the positive-parity first and second excited levels can occur in the single-particle, single-hole dipole-state model, although some such transitions seem to have been observed following photonuclear excitation.[37] The $N^{15}$ ground state is $(1p_{1/2})^{-1}$ and will thus be fed only by small admixed components of the $(1p_{3/2})^{-1}(1d_{3/2,5/2})$ collective states. The calculated neutron polarizations may therefore not be too reliable, but on the other hand, as mentioned earlier, this sensitivity to small components may be used to probe into their admixture. The third excited state transitions are also displayed in Fig. 2(b) since they may possibly be identified over the background of ground-state transitions from noncollective states by their large probability—we use the assignment $(1p_{3/2})^{-1}$ for the third excited state (although excited states are less well described by the Mayer-Jensen shell model than the ground states), so that it will be fed by large components of the collective states—and by the gap corresponding to the large gap between the ground state and the excited states in $N^{15}$. Table II presents values of the polarization parameters using Gillet's wave functions (in parentheses: Lewis' wave functions[26]).

TABLE II. Degree of polarization of neutrons emitted after muon capture in $O^{16}$, using Gillet's (Ref. 24) [in parentheses: Lewis' (Ref. 26)] wave functions.

| | $1^-$ state | | $2^-$ state | |
|---|---|---|---|---|
| | $N^{15}$ ground | 3rd excited | $N^{15}$ ground | 3rd excited |
| $R(\Sigma_2/\Sigma_1)$ | 0.18 | 0.04 | 0.00 | −0.32 |
| | (0.18) | (0.01) | (−0.05) | (−0.35) |
| $R(\Sigma_3/\Sigma_1)$ | 0.34 | −0.16 | −0.09 | −0.32 |
| | (0.34) | (−0.15) | (−0.37) | (−0.30) |

### 3. $Ca^{40}$ Capture

For $Ca^{40}$ capture, only $1^-$ configurations are available. These are much more mixed than in $C^{12}$ or $O^{16}$, and different calculations give considerably different coefficients. The $\frac{3}{2}^+$ $K^{39}$ ground state, taken as $(1d_{3/2})^{-1}$, is not fed by large components of the $(1d_{5/2})^{-1}(1f_{5/2,7/2})$ collective states. This and the generally large configuration mixing, which makes the identification of the collective states ambiguous, will render the calculated neutron polarizations, shown in Table III, not very reliable. The wave functions used were Gillet's (in parentheses: Balashov's[13]).

TABLE III. Degree of polarization of neutrons emitted after muon capture in $Ca^{40}$ ($1^-$ axial-vector state only), using Gillet's (Ref. 24) [in parentheses: Balashov's (Ref. 13)] wave functions.

| | |
|---|---|
| $R(\Sigma_2/\Sigma_1)$ | 0.10 |
| | (0.13) |
| $R(\Sigma_3/\Sigma_1)$ | −0.30 |
| | (−0.28) |

In closing, we would like to point out the following fact: For muon capture in $Ca^{40}$ (and partly also in $O^{16}$), energetics would permit a decay of the collective states $_{19}K^{40}$ (or $_7N^{16}$) to the ground state (or excited states) of $_{18}Ar^{39}$ ($_6C^{15}$) by *proton* emission; see Fig. 1(c), 1(b). The $Ar^{39}$ ground state, $\frac{7}{2}^-$, should be a $(1f_{7/2})$ state with two extra holes. If, therefore, emitted protons are observed with an energy corresponding to the ground-state transition, this would constitute an argument for the presence of 2-particle, 2-hole states in the giant-dipole configuration, and it may turn out to represent a better check than that of Yergin *et al.*[37] who looked for photoneutron transitions to the excited $\frac{1}{2}^+$, $\frac{5}{2}^+$ doublet in $O^{15}$, because this is a maximum-energy transition which requires no ambiguous procedures of separating out the background from other transitions of comparable size in the same energy region.

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## APPENDIX

As mentioned earlier, the neutrons emitted directly in the muon-capture process in complex nuclei will be relatively few, but their spectrum may reach beyond 20 MeV. These have been discussed before,[19,38–41] sometimes on the basis of a Fermi-gas model.[19,40,41] Their longitudinal polarizations, being a consequence of parity violation in the weak process, are not multiplied by the degree of muon polarization at capture, $P$, and therefore they may (and do) reach close to 100%. Their measurement can give information on the weak coupling constants. In particular, it may provide an independent check on the size of the pseudoscalar

[38] L. D. Blokhintsev, Zh. Eksperim. i Teor Fiz. **36**, 258 (1959) [English transl.: Soviet Phys.—JETP **9**, 175 (1959)].
[39] M. K. Akimova, L. D. Blokhintsev, and E. I. Dolinsky, Nucl. Phys. **23**, 309 (1961).
[40] H. Überall, Nuovo Cimento **6**, 533 (1957).
[41] R. Klein, T. Neal, and L. Wolfenstein, Phys. Rev. **138**, B86 (1965).

coupling $g_P$. The ratio $\xi=g_P/g_A$ is predicted as $\xi\cong7$ using dispersion relations (see, e.g., Ref. 3). Experimental results seem to lead consistently to higher values of $\xi$. If one assumes universal Fermi interaction (UFI) and the conserved-vector-current hypothesis (CVC), with no terms violating $G$ invariance:

(1) from the muon-capture rate in hydrogen (see, e.g., Ref. 3):

$$4\lesssim\xi\lesssim14; \tag{29}$$

(2) from capture in $He^3$ (see, e.g., Ref. 42):

$$8\lesssim\xi\lesssim16 \quad (\text{or } 25\lesssim\xi\lesssim33); \tag{30}$$

(3) from radiative capture[3,43] in $Ca^{40}$:

$$10\lesssim\xi\lesssim16; \tag{31}$$

(4) from capture[1–4] in $O^{16}$;

(a) $0^+\rightarrow0^-$ transition:

$$7\lesssim\xi\lesssim18; \tag{32}$$

(b) $0^+\rightarrow2^-$ transition: even a $\xi$ as high as

$$\xi>20 \tag{33}$$

still disagrees with the experimental results.

The same is the case for the extremely large asymmetries $A=\beta P\alpha$ of neutrons with energies $\gtrsim20$ MeV for capture in $Ca^{40}$, measured[44] as $A=-0.235\pm0.040$, while the polarization of the $s$-state muon was measured in the same experiment as $P=0.190\pm0.015$. This requires not only that the neutron-energy-dependent factor $\beta\leq1$ describing the influence of nuclear structure on the direct emission be $\beta\cong1$ in this energy region, but also that $\alpha=-1.00\pm0.05$. The latter result cannot be obtained with the Primakoff Hamiltonian, which predicts for capture in hydrogen without hyperfine effects (a result changed by no more than 10% in nuclei due to nuclear structure effects[39]):

$$\alpha_H=\frac{G_F^2-2G_G^2+(G_G-G_P)^2}{G_F^2+2G_G^2+(G_G-G_P)^2}, \tag{34}$$

as this can be made $=-1$ only if[45] both $G_F=0$ and $G_G-G_P=0$; with the value of $G_F$ given by UFI, it is $-0.35$ for $\xi=7$, and reaches the extremum of $-0.57$ for $\xi=24$ (where $G_P=G_G$, taking $g_A/g_V=-1.20$ and $\nu\cong80$ MeV. Thus, the large pseudoscalar at least helps to go towards the experimental result.

For the longitudinal polarization of neutrons directly emitted from nuclei, the Primakoff Hamiltonian predicts[39]

$$P_n^{||}\cong\beta P_H^{||} \tag{35}$$

(within 1%; we also may now set $\beta=1$), where

$$P_H^{||}=\frac{-2G_G^2+2G_F(G_G-G_P)}{G_F^2+2G_G^2+(G_G-G_P)^2}; \tag{36}$$

UFI gives for this $-1.00$ at $\xi=7$, $-0.78$ at $\xi=24$. (Again, for $G_F=G_G-G_P=0$, we obtain $P_H^{||}=-1$.) Therefore, the longitudinal polarization of the highest energy neutrons is a reasonably sensitive indicator of large pseudoscalar couplings.

The Primakoff Hamiltonian takes account of only some relativistic terms of first order in the nucleon velocity. Wolfenstein *et al.*[41] have shown that the contribution of the additional first-order terms can easily be included for the highest energy neutrons on the basis of a Fermi-gas model by replacing Primakoff's effective coupling constants

$$\begin{aligned} G_F&=g_V(1+\epsilon),\\ G_G&=g_A-g_V(1+\kappa)\epsilon,\\ G_P&=\epsilon(-g_V-\kappa g_V+g_P-g_A), \end{aligned} \tag{37}$$

where $\epsilon=\nu/2m$, $\kappa=\mu_p-\mu_n=3.71$, by

$$\begin{aligned} G_F&\rightarrow G_F'=g_V(1+\lambda),\\ G_G&\rightarrow G_G,\\ G_P&\rightarrow G_P'=\epsilon(-g_V-\kappa g_V+g_P)-\lambda g_A, \end{aligned} \tag{38}$$

where $\lambda=(2k_F+\nu)/2m=0.32$, $k_F=$ Fermi momentum $\cong270$ MeV/$c$ (which gives $\nu\cong70$ MeV corresponding to the highest energy neutrons). In this case, one finds $\alpha=-0.06$ for $\xi=7$, $-0.30$ for $\xi=24$. Considering $G_F'$ as given, one can again make $\alpha$ extreme $=-0.37$ for $G_G-G_P=0$, which corresponds to the even larger pseudoscalar $\xi=36$.

The effect of all these relativistic terms on the longitudinal polarizations of the highest energy neutrons is not as extreme as on the asymmetry: We find $P^{||}=-1.00$, $-0.89$, and $-0.68$ at $\xi=7$, 24, and 36, respectively. It is seen, therefore, that even after all the relativistic terms have been included in an approximate fashion, the longitudinal polarization of the neutrons of highest energy directly emitted after muon capture in nuclei can give us new information on the magnitude of the induced pseudoscalar coupling.

[42] A. I. Mukhin, in *Proceedings of the Twelfth International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[43] M. Conversi, R. Diebold, and L. diLella, Phys. Rev. **136**, B1077 (1964).

[44] V. S. Evseev, V. S. Roganov, F. A. Chernogorova, Chang Run-Hwa, and M. Szymczak, Phys. Letters **6**, 332 (1963).

[45] $G_F=0$ could be obtained by assuming a scalar coupling with $g_S=-g_V$. However, $G_F=G_G-G_P=0$ would reduce the theoretical rate of muon capture in $He^3$, which is the primary source of information on $G_F$, to ~75% of the experimental rate.