Properties of Weak Amplitudes in Broken $\tilde{U}(12)$

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Weak nonleptonic amplitudes are discussed in the $\tilde{U}(12)$ scheme on the assumption of octet dominance. The contributions to hyperon-decay S waves of spurions of representations higher than **143** of the current ⊗ current product are discussed in detail. In addition to Gell-Mann's relations for hyperon S waves we find that $\Sigma^+ \to n+\pi^+$ is forbidden in S waves apart from minor kinematical corrections. Possible modifications from a kinetic-energy spurion are also examined. Some remarks concerning consequences of CP invariance and concerning $K_{2\pi}$ decays are included.

INTRODUCTION

A RELATIVISTIC generalization of the SU_6 symmetry of strong interactions has recently been discussed by Bég and Pais,[1] by Delbourgo, Salam, and Strathdee[2] and by Sakita and Wali.[3] We derive here some properties of the weak nonleptonic amplitudes in relativistic $SU(6)$ theory. We follow the approach by Delbourgo, Salam and Strathdee,[2] who make use of the symmetry group $\tilde{U}(12)$. We discuss in detail the contributions of spurions of the representations higher than **143** of the current ⊗ current product to the hyperon nonleptonic S waves. On the assumptions of CP invariance and octet enhancement[4] we find in addition to Gell-Mann's relation for hyperon S waves,[5] that $\Sigma^+ \to n+\pi^+$ in S waves is also forbidden with higher spurions, apart from minor kinematical corrections. We also look for possible modifications from the introduction of a kinetic-energy spurion, but find that such modifications are essentially negligible. In connection with the problem of the ratio $(K^+ \to 2\pi)/(K^0 \to 2\pi)$, we indicate a possible suppression mechanism for $K^0 \to 2\pi$. We find, too, that some results following from CP invariance are also obtainable from approximate kinematical arguments.

1. THE ASSUMPTION OF 143 DOMINANCE

To introduce the discussion, we first derive the amplitudes for hyperon decay on the assumption of dominance of the representation **143** for the spurion.[6] From the baryon tensor Ψ_{ABC}, the meson tensor $M_A{}^B$, and the spurion $S_A{}^B$ we can form in $\tilde{U}(12)$ the invariants:

$$\bar{\Psi}^{ABC}\Psi_{ABC}M_G{}^F S_F{}^G, \tag{1}$$

$$\bar{\Psi}^{ABC}\Psi_{ABD}M_C{}^F S_F{}^D, \tag{2}$$

$$\bar{\Psi}^{ABC}\Psi_{ABD}M_F{}^D S_C{}^F, \tag{3}$$

$$\bar{\Psi}^{ABC}\Psi_{ADE}M_B{}^D S_C{}^E. \tag{4}$$

The tensors $\bar{\Psi}$, Ψ, and M are decomposed in $SU(3)\otimes\tilde{U}(4)$, and are assumed to satisfy Bargmann-Wigner equations.[7] The parity-violating (conserving) spurion is a suitable pseudoscalar (scalar) component of **143**. No explicit assumptions are made as to the behavior under CP.

The invariants (1) and (3) do not contribute to hyperon nonleptonic decays. The invariant (2) contributes to both S and P waves.

The invariant (4) contributes to the S wave with terms of the order μ/m, where μ is the meson mass and m the baryon mass. One is strongly tempted to neglect such contributions: and, reassuringly, by neglecting them one obtains the beautiful prediction $S(\Sigma_+{}^+)=0$ for the S wave of $\Sigma^+ \to n+\pi^+$.

Furthermore one can show that in the frame of the lower symmetry[8] $\tilde{U}_T(8)\otimes\tilde{U}_X(4)\otimes W_Y$, where $\tilde{U}_T(8)$ acts

[1] M. A. B. Bég and A. Pais, Phys. Rev. Letters 14, 267 (1965).
[2] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) 284, 146 (1965).
[3] B. Sakita and K. C. Wali, Phys. Rev. Letters 14, 404 (1965).
[4] R. F. Dashen, S. C. Frautschi, M. Gell-Mann, and Y. Hara in *Proceedings of the Twelfth Annual International Conference on High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).
[5] M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964).
[6] The relations following from such an assumption are also derived by R. Oehme, Phys. Letters 15, 284 (1965); D. Horn, M. Kugler, H. J. Lipkin, S. Meshkov, J. C. Carter, and J. J. Coyne, Phys. Rev. Letters 14, 717 (1965) and by M. P. Khanna (to be published). The relations obtained by K. Kawarabayashi and R. White, Phys. Rev. Letters 14, 527 (1965) disagree with ours and with those of the above authors.
[7] V. Bargmann and E. Wigner, Proc. Natl. Acad. Sci. U.S. 34, 211 (1948).
[8] A subgroup $SU_T(4)\otimes SU_X(2)\otimes W_Y$ of SU_6 was introduced by F. Gürsey, A Pais and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

on the non-strange quarks, $\tilde{U}_X(4)$ acts on the strange quark, and W_Y is the hypercharge gauge group (to allow for the possibility of physical kinematical situations), these S-wave contributions have the property of vanishing in the limit where the pion is emitted at rest. With the assumption that the emitted S wave can be reasonably approximated with its value at zero pion momentum one can neglect such contributions.

Independently of such speculations, one can always eliminate the invariant (4) for S waves by simply assuming that CP invariance holds in these decays.

The contribution to the P wave from the invariant (4) is of the same order as that from the invariant (2).

In addition to the $\Delta T=\frac{1}{2}$ relations one obtains for S waves the relations:

$$S(\Sigma_+^+)=0\,, \tag{5}$$

$$S(\Xi_-^-)=S(\Lambda_-^0)\,, \tag{6}$$

$$S(\Sigma_0^+)=-(1/\sqrt{3})S(\Lambda_-^0)\,, \tag{7}$$

and for P waves:

$$\sqrt{2}P(\Sigma_+^+)=3P(\Sigma_0^+)-(1/\sqrt{3})P(\Lambda_-^0)\,, \tag{8}$$

$$P(\Sigma_0^+)-(1/\sqrt{3})P(\Xi_-^-)=(3/5\sqrt{2})P(\Sigma_+^+)\,. \tag{9}$$

The S-wave relations (5), (6), and (7) were derived in SU_6 from the assumption of behavior of the parity-violating spurion as a component of **35** and from CP invariance.[9] They were found to agree fairly well with the data.[9]

Comparison of Eqs. (8) and (9) with the choice of empirical solutions for the amplitudes, reported in Table I of the first paper of Ref. (9), gives: Eq. (8), $(5.8\pm0.20)=(3.9\pm0.75)$ or $=(9.8\pm1.25)$, respectively, for the choice (ii) or (i) of Σ_0^+; Eq. (9), $(1.7\pm0.6)=(2.5\pm0.9)$ or (4.4 ± 1.0), again corresponding to choice (ii) or (i), respectively. The choice (ii) seems preferable. This choice is known to fit better the Gell-Mann relation for S waves, which is independent of the assumption of **143** dominance. The agreement with the data of the P-wave relation (8) and (9) is however much less satisfactory than for S waves. Equation (8) would compare better with the data if instead of the choices (i) or (ii) for Σ_0^+ one were to take $P(\Sigma_0^+)$ from the $\Delta T=\frac{1}{2}$ relation $\sqrt{2}\Sigma_0^+=\Sigma_+^+-\Sigma_-^-$.

One may speculate about possible relations connecting S and P waves arising from a definite form of the spurion. If, for instance, the spurion consists [in $\tilde{U}(4)$] of a $\delta_\alpha{}^\beta$ (parity-conserving) added to $(\gamma_5)_\alpha{}^\beta$ (parity-violating) with equal coefficients, one obtains the further relation $P(\Lambda_-^0)-\frac{1}{6}(\sqrt{6})P(\Sigma_+^+)=S(\Lambda_-^0)(2m-\mu)/\mu$. This relation is not consistent with the data.

[9] G. Altarelli, F. Buccella and R. Gatto, Phys. Letters **14**, 70 (1965); K. Kawarabayashi, Phys. Rev. Letters **14**, 86 (1965) and *ibid.* **14**, 169 (1965); P. Babu, Phys. Rev. Letters **14**, 166 (1965); some of the relations are also given by S. P. Rosen and S. Pakvasa, Phys. Rev. Letters **13**, 773 (1964) and by M. Suzuki, Phys. Letters **14**, 64 (1965).

To account for some of the symmetry-breaking effects we consider the restriction to the subgroup $\tilde{U}_T(8)\otimes\tilde{U}_X(4)\otimes W_Y$.[8] The pions belong in $\tilde{U}_T(8)\otimes\tilde{U}_X(4)\otimes W_Y$ to a representation **(63,1)**; the K,K^* to **(8*,4)**; the $\bar{K},\bar{K}^*$ to **(8,4*)**; the Σ,Λ and Y_1^* to **(36,4)**; N,N^* to **(120,1)**; Ξ,Ξ^* to **(8,10)**; Ω^- to (1,20). [Inside the parenthesis the first number denotes the $\tilde{U}_T(8)$ representation, the second the $\tilde{U}_X(4)$ representation.] The spurion will be chosen in **(8*,4)** and **(8,4*)**. The restriction to the subgroup leaves Eqs. (5), (7), and (8) unchanged. One thus expects these equations to be more accurate than Eqs. (6) and (9).

Furthermore, whereas the assumption of an imaginary pion momentum was implicit in the derivation in full symmetry (with all baryon masses equal), this is no longer the case for the restricted symmetry.

Our last remark concerns the decays $K\to 2\pi$. Again in the frame of the lower symmetry $\tilde{U}_T(8)\otimes\tilde{U}_X(4)\otimes W_Y$ one finds that the contribution to such S-wave amplitudes from any parity-violating spurion of the current $\otimes$ current product vanishes when the pions are emitted with zero c.m. momentum. For a small range of the decay interaction such peculiar behavior of the emitted S wave would suggest a partial suppression of these decay modes even in the nonsymmetric limit, and independently of CP invariance. This result may facilitate the resolution of the old problem of the $(K^+\to 2\pi)/(K_1^0\to 2\pi)$ ratio.

2. SPURIONS OF THE HIGHER REPRESENTATIONS

We shall now calculate the contributions to the hyperon-decay S waves from the spurions of the higher representations of the symmetric current$\otimes$current product.

With a spurion $S_{[CD]}{}^{[AB]}$ of **4212** (the bracket $[\cdots]$ denotes antisymmetrization) one can only form one coupling,

$$a\bar{\Psi}^{ABC}\Psi_{ABD}M_S{}^RS_{[RC]}{}^{[SD]}\,, \tag{10}$$

where a is a constant.

A spurion $S_{\{CD\}}{}^{\{AB\}}$ of **5940** (the bracket $\{\cdots\}$ denotes symmetrization) gives rise to four independent couplings,

$$b_1\bar{\Psi}^{ABC}\Psi_{ABD}M_S{}^RS_{\{RC\}}{}^{\{SD\}}\,, \tag{11}$$

$$b_2\bar{\Psi}^{ABC}\Psi_{ADE}M_B{}^RS_{\{RC\}}{}^{\{DE\}}\,, \tag{12}$$

$$b_3\bar{\Psi}^{ABC}\Psi_{ADE}M_R{}^DS_{\{BC\}}{}^{\{RE\}}\,, \tag{13}$$

$$b_4\bar{\Psi}^{ABC}\Psi_{DEF}M_A{}^DS_{\{BC\}}{}^{\{EF\}}\,, \tag{14}$$

where $b_1,\cdots,b_4$ are constants.

The addition of the invariants (10) and (11) to the invariants $(1),\cdots,(4)$, formed from the spurion of **143**, leaves the relations among the amplitudes unchanged.

We shall now examine in detail the invariants (12), (13), and (14).

We make the assumption of "octet dominance." The representation **5940** contains the following octets:

(**8**,**84**), (**8**,**45**), (**8**,**45***), three times (**8**,**15**), and (**8**,**20''**) (the first number inside the brackets denotes the SU_3 representation, the second number the $\tilde{U}_4$ representation).

Only (**8**,**15**) and (**8**,**20''**) contain pseudoscalars. We have a total of four octet pseudoscalars which we call $A_{CD}{}^{AB}$, $B_{CD}{}^{AB}$, $C_{CD}{}^{AB}$, and $D_{CD}{}^{AB}$:

$$A_{CD}{}^{AB}=[(C^{-1}\gamma_5)^{\alpha\beta}C_{\gamma\delta}-(C^{-1})^{\alpha\beta}(\gamma_5 C)_{\gamma\delta}]\times\epsilon^{abe}\epsilon_{cdf}(T_6)_e{}^f, \quad (15)$$

$$B_{CD}{}^{AB}=\epsilon_{\mu\nu\rho\lambda}(C^{-1}\sigma_{\mu\nu})^{\alpha\beta}(\sigma_{\rho\lambda}C)_{\gamma\delta}\delta_{\{c}{}^{\{a}(T_6)_{d\}}{}^{b\}}, \quad (16)$$

$$C_{CD}{}^{AB}=(C^{-1}\gamma_\mu)^{\alpha\beta}(i\gamma_\mu\gamma_5 C)_{\gamma\delta}\delta_{\{c}{}^{\{a}(T_6)_{d\}}{}^{b\}}, \quad (17)$$

$$D_{CD}{}^{AB}=(C^{-1}i\gamma_\mu\gamma_5)^{\alpha\beta}(\gamma_\mu C)_{\gamma\delta}\delta_{[c}{}^{[a}(T_6)_{d]}{}^{b]}. \quad (18)$$

In Eqs. (15)–(18), the Latin indices refer to SU_3 and the Greek indices α, β, $\cdots$ refer to $\tilde{U}_4$, C is the charge conjugation matrix, and T_6 is the appropriate unitary spin matrix.

Assuming CP-invariance to be approximately valid, the requirement of negative charge conjugation for the parity violating spurion and the current⊗current assumption leave us only with two possible spurions: A and C-D.

Furthermore, for both spurions, CP invariance requires that the couplings (12) and (13) be added with equal coefficient.

Let us first consider the contributions from the sum (12)+(13). The baryon term $\bar{\Psi}^{ABC}\Psi_{ADE}=J_{DE}{}^{BC}$ must be saturated to the tensor obtained from the product of M and S. The product of M and A gives a term ($\hat{q}=q^\mu\gamma_\mu$)

$$\left\{-\tfrac{1}{2}\left[C^{-1}\left(1-\frac{\hat{q}}{\mu}\right)\gamma_5\right]^{\delta\epsilon}(\gamma_5 C)_{\beta\gamma}+\tfrac{1}{2}(C^{-1}\gamma_5)^{\delta\epsilon}\left[\left(1-\frac{\hat{q}}{\mu}\right)\gamma_5 C\right]_{\beta\gamma}\right\}(T_\pi)_{[b}{}^{[d}(T_6)_{c]}{}^{e]} \quad (19)$$

plus terms which have octet behavior under SU_3, plus terms of the kind $(C^{-1})^{\delta\epsilon}$ $(\cdots)$, which vanish when saturated to the baryons. In $\tilde{U}(4)$ the expression (19) is of the form $R_{\beta\gamma}{}^{\delta\epsilon}(q)-R'_{\beta\gamma}{}^{\delta\epsilon}(q)$, where R and R' are related by $R_{\beta\gamma}{}^{\delta\epsilon}(q)=(C^{-1})^{\delta\delta'}(C^{-1})^{\epsilon\epsilon'}C_{\beta\beta'}C_{\gamma\gamma'}R'_{\delta'\epsilon'}{}^{\beta'\gamma'}(q)$. A similar analysis of the product of M and C-D again gives terms of such a general form, plus terms with octet behavior in SU_3, plus terms which vanish after saturating to the baryons. In the limit of equal masses of the octet baryons the current $J_{DE}{}^{BC}$ decomposes into $\tilde{U}(4)$ terms $J_{\delta\epsilon}{}^{\beta\gamma}(p,p')$ which satisfy $(C^{-1})^{\beta\beta'}(C^{-1})^{\gamma\gamma'}\times C_{\delta\delta'}C_{\epsilon\epsilon'}J_{\beta'\gamma'}{}^{\delta'\epsilon'}(-p',-p)=J_{\delta\epsilon}{}^{\beta\gamma}(p,p')$. Each product $J_{\delta\epsilon}{}^{\beta\gamma}(p,p')[R_{\beta\gamma}{}^{\delta\epsilon}(q)-R'_{\beta\gamma}{}^{\delta\epsilon}(q)]$ must therefore vanish in this limit.

Finally, let us consider the coupling (14). In the equal-mass limit for the octet baryons the amplitude from (18) contains a factor

$$\frac{1}{m^2}\left(1+\frac{2m}{\mu}\right)(2(p\cdot p')-2m^2).$$

One can then reasonably assume that the coupling (19) is negligible in comparison with the other terms.

We thus find that the addition of the higher parity-violating spurions of the current⊗current product leaves the following two results unchanged:

(1) The absence of $\Sigma^+\rightarrow n+\pi^+$ in S wave, $S(\Sigma_+{}^+)=0$.

(2) The Gell-Mann relation for S-waves[5] which in fact only follows from the current⊗current hypothesis, from CP, and from octet dominance.

Minor corrections to $S(\Sigma_+{}^+)=0$ may be caused by the coupling (18).

3. THE KINETIC-ENERGY SPURION

We have also considered an additional breaking of $\tilde{U}(12)$ from kinetic-energy spurions $T_A{}^B$ of the kind

$$\hat{q}_\alpha{}^\beta\delta_a{}^b \quad \text{and} \quad \hat{P}_\alpha{}^\beta\delta_a{}^b$$

where, as before, $q=p-p'$ and $P=p+p'$. The weak spurion S will here be assumed to belong to **143**. For both parity-violating and parity-conserving amplitudes we can conclude that the introduction of the kinetic-energy spurion does not modify the relations obtained in the absence of such a breaking.

In fact, couplings in which $T_A{}^B$ is saturated to the baryons are directly reduced to unperturbed couplings by eliminating $T_A{}^B$ through the equations of motion. Couplings where $T_A{}^B$ is saturated to the meson will conserve the SU_3 character of the unperturbed couplings and therefore will not change the relations among the decay amplitudes. Those couplings where $T_A{}^B$ is saturated to the spurion are reduced to unperturbed couplings by use of the equations of motion. Finally the couplings where $T_A{}^B$, the meson tensor, and the weak spurion are saturated together still contribute SU_3 terms of the kind $3D+2F$ for P waves and F for S waves, leaving the relations unchanged.

It thus appears that the introduction of the kinetic-energy spurions does not modify the conclusions of the original analysis, and, in particular, it brings no remedy to the unsatisfactory P-wave results.