PHYSICAL REVIEW VOLUME 139, NUMBER 5B 6 SEPTEMBER 1965

Reciprocal Bootstrap Possibilities for Baryons

L. A. P. BALÁZS, VIRENDRA SINGH, AND B. M. UDGAONKAR
Tata Institute of Fundamental Research, Bombay, India
(Received 2 April 1965)

In πN scattering, using an N/D static model with linear approximation to the D function, the reciprocal bootstrap between the nucleon and the $(\frac{3}{2}, \frac{3}{2})$ resonance is shown to hold for any l, with an $I=\frac{1}{2}$, $j=l-\frac{1}{2}$ and an $I=\frac{3}{2}$, $j=l+\frac{1}{2}$ isobar supporting each other. It is also shown that this is in fact the only pair of multiplets capable of reciprocally bootstrapping. The following question is then raised: Given the existence of the pion, what is the simplest possible set of baryon multiplets which can support itself? It is found that the aforementioned pair is in fact the simplest set. All these results are then generalized to $SU(3)$, where this pair becomes a $j=l-\frac{1}{2}$ octet and a $j=l+\frac{1}{2}$ decimet.

1. INTRODUCTION

THE first bootstrap scheme for baryons was proposed by Chew[1] who suggested that the nucleon (N) and the $(\frac{3}{2},\frac{3}{2})$ isobar (Δ) are capable of supporting each other. The model he used was the static limit of the N/D method for πN scattering in which only baryon-exchange forces are considered and the D function is approximated by a straight line. In this approximation the baryons are considered so heavy compared to the mesons that their recoil can be neglected. Using the same approach, a similar scheme has also been shown to hold in the corresponding $SU(3)$ model, with the N and Δ generalized to the $\frac{1}{2}^+$ octet and the $(\frac{3}{2})^+$ decimet, respectively.[2,3] We find that this is in fact the case for any l in the static approximation. There exists a reciprocal bootstrap between the $I=\frac{1}{2}$, $j=l-\frac{1}{2}$ and $I=\frac{3}{2}$, $j=l+\frac{1}{2}$ isobars.[4] In the d wave this would explain the physically observed πN $D_{3/2}$ resonance.

In the simplest version of the above model, only ratios of coupling constants can be calculated. This, however, is sufficient for checking whether a given set of particles is cc nsistent with the model. If, for instance, the coupling for some particle turns out to be small or negative, we can certainly rule out the existence of this particle within our scheme. This means that we have a particularly simple approach for studying the question of whether the physically observed particles are the only possible set within a bootstrap approach, or at least whether they are in some sense the simplest.

The first such question we study is whether other pairs of multiplets can support each other in πN scattering. We find that the $I=\frac{1}{2}$, $j=l-\frac{1}{2}$ and $I=\frac{3}{2}$, $j=l+\frac{1}{2}$ isobars are the only consistent pair. We then ask the question: Given the existence of the pion, what is the simplest set of baryons which can support itself in a nonstrange $SU(2)$ model? For instance, could there be a world having only an isosinglet baryon? We find that the aforementioned case is also the simplest from this point of view. Finally, we generalize this $SU(2)$ model to the corresponding $SU(3)$ case, where similar conclusions are reached for the $j=l-\frac{1}{2}$ octet and the $j=l+\frac{1}{2}$ decimet.

2. THE STATIC MODEL

We shall begin by reviewing the static N/D model with linear D. A meson-baryon state is specified by its spin J, isotopic spin I, orbital angular momentum l, and total energy W. For a given l we use the amplitude

$$g_{IJ}(\omega)=e^{i\delta}\sin\delta/q^{2l+1}, \tag{1}$$

where δ=phase shift, $q^2=\omega^2-1$, $\omega=W-M$, and M=baryon mass, with the meson mass taken to be unity. The forces can be obtained through the crossing relation

$$g_{IJ}(\omega)=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}g_{I'J'}(-\omega), \tag{2}$$

where α and β are the crossing matrices for isotopic spin and spin, respectively. Of course, in the static limit, the g's on the right-hand side of Eq. (2) all have the same l as the g on the left-hand side.

If some isobar (a bound state or resonance) occurs in the (I,J) state, the corresponding amplitude has a pole $\gamma_{IJ}/(\omega_{IJ}-\omega)$. From Eq. (2), the force (Born term) in the lth wave then has the form

$$B_{IJ}(\omega)=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}(\gamma_{I'J'}/(\omega_{I'J'}+\omega)). \tag{3}$$

In the sum we take $\gamma_{IJ}=0$ whenever there is no particle in the (I,J) state. If we use $B_{IJ}(\omega)$ as the input to an N/D calculation, we obtain

$$g_{IJ}(\omega)=N_{IJ}(\omega)/D_{IJ}(\omega), \tag{4}$$

$$N_{IJ}(\omega)=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}(\gamma_{I'J'}/(\omega_{I'J'}+\omega))D(-\omega_{I'J'}), \tag{5}$$

$$D_{IJ}(\omega)=1-\frac{\omega-\omega_0}{\pi}\int_1^{\Lambda}d\omega'\frac{(\omega'^2-1)^{(\frac{1}{2}+l)}N_{IJ}(\omega')}{(\omega'-\omega_0)(\omega'-\omega-i\epsilon)}, \tag{6}$$

where ω_0 is some subtraction point and Λ is a cutoff which parametrizes high-energy effects. Equations (4)–(6) are constructed so as to satisfy elastic unitarity

---

[1] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).
[2] R. Dashen, Phys. Letters **11**, 89 (1964).
[3] Y. Hara, Phys. Rev. **135**, B1079 (1964).
[4] The arbitrary l case has also been discussed by P. Carruthers, Phys. Rev. Letters **10**, 538, 540 (1963) and Phys. Rev. **133**, B497 (1964), who did not find such a reciprocal bootstrap. He, however, did not use a static model.

and at the same time give the correct force singularities coming from the B_{IJ} contribution to g_{IJ}.

If we have an isobar in the (I,J) state we can approximate D by a straight line

$$D_{IJ}(\omega)=(\omega_{IJ}-\omega)/(\omega_{IJ}-\omega_0). \tag{7}$$

Then

$$\gamma_{IJ}=-N_{IJ}(\omega_{IJ})/D_{IJ}'(\omega_{IJ})=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}\gamma_{I'J'}. \tag{8}$$

It is convenient to introduce[5]

$$F_{IJ}=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}\gamma_{I'J'} \tag{9}$$

even if the (I,J) state does not have any particle in it. As discussed in Ref. 5, F_{IJ} provides a measure of the force in the (I,J) state. For instance, we would expect a low-lying particle to exist in a state (I,J) only if F_{IJ} is positive and large, at least with reasonably smooth high-energy behavior in Eqs. (5) and (6). Of course, if we had a particle in a state for which F_{IJ} is negative, we would have had $\gamma_{IJ}<0$ from Eq. (8); this is clearly impossible.

3. THE $SU(2)$ CASE

Chew's reciprocal bootstrap[1] follows directly from Eq. (8) if we assume the existence of a $(\frac{1}{2},\frac{1}{2})$ and $(\frac{3}{2},\frac{3}{2})$ isobar, i.e., the N and Δ. Here

$$\alpha=\begin{pmatrix}-\frac{1}{3} & \frac{4}{3}\\ \frac{2}{3} & \frac{1}{3}\end{pmatrix} \tag{10}$$

for $I=\frac{1}{2}$, $\frac{3}{2}$ and $\beta=\alpha$. In this case, the two equations (8) are identical, and give

$$\gamma_{\frac{1}{2}\frac{1}{2}}=2\gamma_{\frac{3}{2}\frac{3}{2}}, \tag{11}$$

a result which agrees with experiment. If this is substituted into Eq. (9), we obtain

$$\begin{aligned}F_{\frac{3}{2}\frac{3}{2}}&=\gamma_{\frac{3}{2}\frac{3}{2}},\\ F_{\frac{1}{2}\frac{3}{2}}&=F_{\frac{3}{2}\frac{1}{2}}=0,\\ F_{\frac{1}{2}\frac{1}{2}}&=2\gamma_{\frac{3}{2}\frac{3}{2}}.\end{aligned} \tag{12}$$

Thus it is consistent to assume only the N and Δ.

Suppose we now generalize the above result to any $l>0$. We have

$$\beta=\frac{1}{2l+1}\begin{pmatrix}-1 & 2l+2\\ 2l & 1\end{pmatrix}, \tag{13}$$

but with the same α as in Eq. (10). The most natural generalization of Chew's reciprocal bootstrap is to assume that a $(\frac{1}{2}, l-\frac{1}{2})$ and a $(\frac{3}{2}, l+\frac{1}{2})$ isobar bootstrap each other. In this case the Eqs. (8) become

$$\gamma_{\frac{1}{2},l-\frac{1}{2}}=(4(l+1)/(3l+1))\gamma_{\frac{3}{2},l+\frac{1}{2}} \tag{14}$$

and

$$\gamma_{\frac{3}{2},l+\frac{1}{2}}=(2l/(3l+1))\gamma_{\frac{1}{2},l-\frac{1}{2}}. \tag{15}$$

These two equations agree exactly for $l=1$, as we have already seen. For all $l>1$, they agree to quite a good approximation, e.g., for $l=2$ we get $\gamma_{\frac{1}{2},\frac{3}{2}}/\gamma_{\frac{3}{2},\frac{5}{2}}\approx 1.71$ from Eq. (14) and equal to 1.75 from Eq. (15). If we now use Eq. (9) we find

$$F_{\frac{1}{2},l-\frac{1}{2}}=\frac{1}{3}\frac{1}{2l+1}\gamma_{\frac{1}{2},l-\frac{1}{2}}+\frac{4}{3}\frac{2l+2}{2l+1}\gamma_{\frac{3}{2},l+\frac{1}{2}}, \tag{16}$$

$$F_{\frac{3}{2},l+\frac{1}{2}}=\frac{2}{3}\frac{2l}{2l+1}\gamma_{\frac{1}{2},l-\frac{1}{2}}+\frac{1}{3}\frac{1}{2l+1}\gamma_{\frac{3}{2},l+\frac{1}{2}}, \tag{17}$$

$$F_{\frac{1}{2},l+\frac{1}{2}}=-\frac{1}{3}\frac{2l}{2l+1}\gamma_{\frac{1}{2},l-\frac{1}{2}}+\frac{4}{3}\frac{1}{2l+1}\gamma_{\frac{3}{2},l+\frac{1}{2}}, \tag{18}$$

$$F_{\frac{3}{2},l-\frac{1}{2}}=-\frac{2}{3}\frac{1}{2l+1}\gamma_{\frac{1}{2},l-\frac{1}{2}}+\frac{1}{3}\frac{2l+2}{2l+1}\gamma_{\frac{3}{2},l+\frac{1}{2}}. \tag{19}$$

If we substitute either Eq. (14) or Eq. (15) into Eqs. (16)–(19), we find that $F_{\frac{1}{2},l+\frac{1}{2}}$ and $F_{\frac{3}{2},l-\frac{1}{2}}$ are sufficiently small compared with $F_{\frac{1}{2},l-\frac{1}{2}}$ and $F_{\frac{3}{2},l+\frac{1}{2}}$ to make it consistent to have only the $(\frac{1}{2}, l-\frac{1}{2})$ and $(\frac{3}{2}, l+\frac{1}{2})$ isobars support each other. In higher waves these isobars are simply the Regge recurrences of the $(\frac{1}{2},\frac{1}{2})-(\frac{3}{2},\frac{3}{2})$ and $(\frac{1}{2},\frac{3}{2})-(\frac{3}{2},\frac{5}{2})$ pairs. In the s wave, the $j=l-\frac{1}{2}$ states are absent and the Eqs. (8) when combined with (10) and (13) can be easily seen to be inconsistent; there would thus be no s-wave isobars in our model.

We now turn to the question of whether the above pair is the only one capable of supporting itself. Now the only other possible pairs are $(\frac{1}{2}, l-\frac{1}{2})$-$(\frac{1}{2}, l+\frac{1}{2})$, $(\frac{1}{2}, l-\frac{1}{2})$-$(\frac{3}{2}, l-\frac{1}{2})$, $(\frac{1}{2}, l+\frac{1}{2})$-$(\frac{3}{2}, l-\frac{1}{2})$, $(\frac{1}{2}, l+\frac{1}{2})$-$(\frac{3}{2}, l+\frac{1}{2})$, $(\frac{3}{2}, l-\frac{1}{2})$-$(\frac{3}{2}, l+\frac{1}{2})$. In the first and second cases, Eq. (8) leads to negative ratios of γ's; these cases are thus automatically excluded. In the remaining cases the two ratios of γ's that one gets by looking at Eq. (8) for the two members of the pair are grossly inconsistent with each other. We thus conclude that the $(\frac{1}{2}, l-\frac{1}{2})$ and $(\frac{3}{2}, l+\frac{1}{2})$ pair is the only one capable of reciprocally bootstrapping.

So far we have been restricting ourselves only to pairs of particles in the πN system (this also trivially includes the case of one multiplet by itself). If we go beyond pairs, then, as pointed out in Ref. 5, it is possible to have other consistent solutions. For instance, in the p-wave case, we can have such a solution with $(\frac{1}{2},\frac{1}{2})$-$(\frac{1}{2},\frac{3}{2})$-$(\frac{3}{2},\frac{1}{2})$-$(\frac{3}{2},\frac{3}{2})$; Chew's solution is only a particularly simple case of this more general solution. Of course, if we had a better calculational technique, we might have found that this more general solution is inconsistent. Lacking this, we shall fall back on the *criterion of simplicity*.[6] We shall pick out the Chew solution in p-wave πN scattering because it is the simplest.

[5] E. S. Abers, L. A. P. Balázs, and Y. Hara, Phys. Rev. **136**, B1382 (1964).

[6] W. Oakham (unpublished).

Suppose we now assume the existence of only the pion with $I=1$ and try to see what baryons are demanded by a static $SU(2)$ model. We shall start with the simplest possibility, and then go to more and more complicated sets until we find one which is consistent for our model. We can then use our criterion of simplicity to select out this solution.

The simplest possibility is to start with the scattering of a pion with an isoscalar spin-$\frac{1}{2}$ baryon. The spin crossing matrix is again given by Eq. (13) while the isospin crossing matrix is just unity. Using Eq. (8) we see that although we can have a $(1,\frac{1}{2})$ baryon supporting itself in the s wave and a reciprocal bootstrap between a $(1, l-\frac{1}{2})$ and $(1, l+\frac{1}{2})$ baryon for higher waves, we cannot get out the original $(0,\frac{1}{2})$ particle for the simple reason that the external isospins 1 and 0 cannot be combined to give an $I=0$ composite. We might get it out if we consider $\pi-(1,\frac{1}{2})$ scattering in addition to $\pi-(0,\frac{1}{2})$ scattering. But before considering this more complicated problem which involves a scattering particle with a higher I, we must try the simpler case of $\pi-(\frac{1}{2},\frac{1}{2})$ scattering. If this is capable of giving a self-supporting system without the consideration of more complicated problems, it would satisfy our criterion of simplicity and we would not have to consider any more complicated problem. But this is simply the πN problem which, as we have already seen, does lead to a closed self-supporting system. Thus the physically interesting case is also the simplest from our point of view.

4. THE $SU(3)$ CHEW RECIPROCAL BOOTSTRAP FOR ANY l

In this section we shall follow the same procedure with all the above particles generalized to $SU(3)$ multiplets. Let us first consider 0^- octet$-\frac{1}{2}^+$ octet scattering, which is the generalization of πN scattering. Here we can use exactly the same techniques as in the $SU(2)$ case except for the octet state, which has to be treated as a two-channel problem since it occurs twice in the direct product reduction

$$8\otimes 8=1\oplus 8_s\oplus 8_a\oplus 10\oplus \overline{10}\oplus 27 \tag{20}$$

of the meson-baryon states. Since the N and Δ become the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decimet, respectively, this difficulty has to be overcome if we want to generalize Chew's reciprocal bootstrap to the $SU(3)$ case.

There have been several attempts to treat the two-channel nature of the problem in a simple way.[2,3,7] Of these the method of Gerstein and Mahanthappa[7] appears to be a pure ansatz. Dashen's method[2] does not reduce to the correct result in the limit in which the octet and decimet masses become equal, as we shall see. Hara's method[3] appears to be correct but is somewhat complicated to use in practice. We have therefore decided to simply assume that the masses of the exchanged octet and decimet are degenerate. This means that the positions of the pseudopoles in Eq. (3) are taken to coincide, which makes it possible to reduce the problem to a one-channel problem, since we can diagonalize the input with an energy-independent matrix. This assumption should be a reasonable approximation, since the pseudopoles are distant from the physical region. Moreover Dashen and Frautschi[8] have noticed in their N/D perturbation approach that the breaking of the mass degeneracy of exchanged particles usually leads to fairly small effects.

In practice, instead of actually diagonalizing the input we shall find it convenient to diagonalize the output first. Since, in this new representation, the input must also be diagonal, we therefore have to impose the condition that the off-diagonal terms be zero. This procedure is completely equivalent to first diagonalizing the input and getting out a diagonal output. Now the output in the usual 8_s-8_a representation has the form

$$\frac{R}{\omega_B-\omega}=\begin{pmatrix} 20\alpha^2/9 & \frac{4}{3}(\sqrt{5})\alpha(1-\alpha) \\ \frac{4}{3}(\sqrt{5})\alpha(1-\alpha) & 4(1-\alpha)^2 \end{pmatrix}\frac{\gamma_8}{\omega_B-\omega}, \tag{21}$$

where ω_B is the position of the octet pole, $\gamma_8=3f^2$ where f^2 is the usual pseudovector coupling with the value $f^2=0.08$, and $\alpha/(1-\alpha)$ is the D to F ratio.[9] Equation (21) can be diagonalized with the matrix

$$U=a^{1/2}\begin{pmatrix} \alpha & 3(1-\alpha)/\sqrt{5} \\ -3(1-\alpha)/\sqrt{5} & \alpha \end{pmatrix}, \tag{22}$$

where $a=[\alpha^2+(9/5)(1-\alpha)^2]^{-1}$, to give

$$\frac{R_d}{\omega_B-\omega}=\begin{pmatrix} 20/9a & 0 \\ 0 & 0 \end{pmatrix}\frac{\gamma_8}{\omega_B-\omega}. \tag{23}$$

Now with 8 and 10 exchange with equal masses for the 8 and 10 the input as given by Eq. (3) has the form

$$\frac{R^{\times}}{\omega_B+\omega}=\begin{pmatrix} (2/9)[3(1-\alpha)^2+\alpha^2]\gamma_8+\frac{2}{3}\gamma_{10} & \frac{1}{3}(\sqrt{5})\gamma_{10} \\ \frac{1}{3}(\sqrt{5})\gamma_{10} & -(2/9)[3(1-\alpha)^2-(5/3)\alpha^2]\gamma_8 \end{pmatrix}\times\frac{1}{\omega_B+\omega}, \tag{24}$$

[7] I. S. Gerstein and K. T. Mahanthappa, Nuovo Cimento 32, 239 (1964). In addition, attempts have been made using relativistic treatments by R. E. Cutkosky, Ann. Phys. (N. Y.) 23, 415 (1963); A. W. Martin and K. C. Wali, Nuovo Cimento 31, 1324 (1964); and by P. Carruthers (see Ref. 4); these are, however, too complicated for our purpose, particularly since it has not been demonstrated that such treatments lead to any better results than the static model.

[8] R. Dashen and S. C. Frautschi, Phys. Rev. 137, B1318 (1965).

[9] We shall also use $\alpha/(1-\alpha)$ to denote the D to F ratio for the coupling of $(8, l-\frac{1}{2})$ baryons to mesons in the general l case. The corresponding ratio for $(8, l+\frac{1}{2})$ baryons will be denoted by $\beta/(1-\beta)$.

where $\omega=-\omega_B$ is the assumed common position of the exchange pseudopole. Here we have used the $SU(3)$ crossing matrix[10] instead of $\alpha_{II'}$. In our new diagonalized representation $R^\times$ becomes

$$R_d^\times=a\begin{bmatrix} \frac{2}{9}\alpha^2[6(1-\alpha)^2+\alpha^2+3\Gamma]+2\Gamma\alpha(1-\alpha)-\frac{6}{5}(1-\alpha)^4 & -\frac{2}{3\sqrt{5}}\alpha(1-\alpha)[6(1-\alpha)^2-\frac{2}{3}\alpha^2+3\Gamma]+\frac{\sqrt{5}}{3}\alpha^2\Gamma-\frac{3(1-\alpha)^2\Gamma}{\sqrt{5}} \\ -\frac{2}{3\sqrt{5}}\alpha(1-\alpha)[6(1-\alpha)^2-\frac{2}{3}\alpha^2+3\Gamma]+\frac{\sqrt{5}}{3}\alpha^2\Gamma-\frac{3(1-\alpha)^2\Gamma}{\sqrt{5}} & \frac{2}{5}(1-\alpha)^2[3(1-\alpha)^2-\frac{2}{3}\alpha^2+3\Gamma]-2\alpha(1-\alpha)\Gamma+\frac{10}{27}\alpha^4 \end{bmatrix}\gamma_8 \quad (25)$$

where $\Gamma=\gamma_{10}/\gamma_8$. Since this must be diagonal, we get an equation connecting Γ and α by putting the off-diagonal term equal to zero. We now have one-channel problems for the diagonal elements and an application of Eq. (8) leads to setting the 11 element of Eq. (23) equal to the 11 element of Eq. (25). This gives a second equation connecting Γ and α. If we now solve these two equations we get $\Gamma=1.13$ and $\alpha=0.68$. With these values, the 22 element of Eq. (25) is negative; this corresponds to a repulsive force.[11]

If we now look at the $(\frac{3}{2})^+$ decimet state, Eq. (8) gives[10]

$$\Gamma=(16/33)[\tfrac{4}{3}\alpha^2+4\alpha(1-\alpha)]. \quad (26)$$

With the above value of α this leads to $\Gamma=0.72$, which is roughly consistent with the value obtained in the octet calculation. If we now use Eq. (9), we find that the F's are small in all other states (compared to the values in the $\frac{1}{2}^+$ octet and $(\frac{3}{2})^+$ decimet). Thus it is consistent to have the $\frac{1}{2}^+$ octet and $(\frac{3}{2})^+$ decimet reciprocally bootstrapping in 0^- octet $-\frac{1}{2}^+$ octet scattering.

The above bootstrap can be extended to all $l>1$, just as in the $SU(2)$ case. The only change is to replace the spin crossing matrix (10) with the matrix (13).

5. OTHER BOOTSTRAP POSSIBILITIES

We shall now look at other pairs of multiplets to see whether any of them can support themselves. We follow exactly the same procedure as before. In looking at the 1, 10, $\overline{10}$ and 27 states we simply use the $SU(3)$ analog of Eq. (8), since these are one-channel problems. When dealing with an octet state, we assume that all exchanged particles have the same mass; this permits us to diagonalize the problem and use Eq. (8) in the new representation. If we follow this procedure for $l>0$, we find that the various possibilities can be grouped into the following cases:

(a) Suppose we write (F,j) for a state with $SU(3)$ dimensionality F and total angular momentum j. The cases $(1,l-\frac{1}{2})$-$(10,l+\frac{1}{2})$, $(1,l-\frac{1}{2})$-$(\overline{10},l+\frac{1}{2})$, $(1,l-\frac{1}{2})$-$(27,l-\frac{1}{2})$, $(1,l+\frac{1}{2})$-$(10,l-\frac{1}{2})$, $(1,l+\frac{1}{2})$-$(10,l+\frac{1}{2})$, $(1,l+\frac{1}{2})$-$(\overline{10},l-\frac{1}{2})$, $(1,l+\frac{1}{2})$-$(\overline{10},l+\frac{1}{2})$, $(10,l-\frac{1}{2})$-$(\overline{10},l-\frac{1}{2})$, $(10,l-\frac{1}{2})$-$(27,l-\frac{1}{2})$, $(\overline{10},l-\frac{1}{2})$-$(27,l-\frac{1}{2})$, $(8,l+\frac{1}{2})$-$(\overline{10},l-\frac{1}{2})$, $(8,l+\frac{1}{2})$-$(\overline{10},l+\frac{1}{2})$, $(8,l-\frac{1}{2})$-$(10,l-\frac{1}{2})$, $(8,l-\frac{1}{2})$-$(\overline{10},l+\frac{1}{2})$ and $(8,l-\frac{1}{2})$-$(27,l-\frac{1}{2})$ lead simply to negative ratios of coupling constants if we apply Eq. (8). They are thus inadmissible.

(b) The cases $(1,l-\frac{1}{2})$-$(1,l+\frac{1}{2})$, $(1,l-\frac{1}{2})$-$(10,l-\frac{1}{2})$, $(1,l-\frac{1}{2})$-$(\overline{10},l-\frac{1}{2})$, $(1,l-\frac{1}{2})$-$(27,l+\frac{1}{2})$, $(1,l+\frac{1}{2})$-$(27,l-\frac{1}{2})$, $(1,l+\frac{1}{2})$-$(27,l+\frac{1}{2})$, $(10,l-\frac{1}{2})$-$(10,l+\frac{1}{2})$, $(10,l-\frac{1}{2})$-$(\overline{10},l+\frac{1}{2})$, $(10,l-\frac{1}{2})$-$(27,l+\frac{1}{2})$, $(10,l+\frac{1}{2})$-$(\overline{10},l-\frac{1}{2})$, $(10,l+\frac{1}{2})$-$(\overline{10},l+\frac{1}{2})$, $(10,l+\frac{1}{2})$-$(27,l-\frac{1}{2})$, $(10,l+\frac{1}{2})$-$(27,l-\frac{1}{2})$, $(10,l+\frac{1}{2})$-$(27,l+\frac{1}{2})$, $(\overline{10},l-\frac{1}{2})$-$(\overline{10},l+\frac{1}{2})$, $(\overline{10},l-\frac{1}{2})$-$(27,l+\frac{1}{2})$, $(\overline{10},l+\frac{1}{2})$-$(27,l-\frac{1}{2})$, $(\overline{10},l+\frac{1}{2})$-$(27,l+\frac{1}{2})$, $(27,l-\frac{1}{2})$-$(27,l+\frac{1}{2})$, $(8,l+\frac{1}{2})$-$(10,l+\frac{1}{2})$, $(8,l-\frac{1}{2})$-$(\overline{10},l-\frac{1}{2})$, and $(8,l+\frac{1}{2})$-$(27,l+\frac{1}{2})$ give residue ratios which are completely different, depending on whether one uses Eq. (8) for the first state or the second in each case. They are thus inconsistent. In the last of these cases, there are actually two possibilities corresponding to $\alpha=0$ or 1, but both are ruled out for the same reason.

(c) In the cases $(8,l+\frac{1}{2})$-$(1,l-\frac{1}{2})$, $(8,l+\frac{1}{2})$-$(1,l+\frac{1}{2})$, $(8,l-\frac{1}{2})$-$(1,l-\frac{1}{2})$, and $(8,l-\frac{1}{2})$-$(1,l+\frac{1}{2})$, there are again two possibilities, one of which is excluded for the same reason as in (a) and the other for the same reason as in (b).

(d) In the case $(8,l-\frac{1}{2})$-$(8,l+\frac{1}{2})$ there are four possibilities corresponding to $\alpha=0$, 1 and $\beta=0$, 1; three of these are excluded for the same reason as in case (a) and one for the same reason as in (b).

(e) In each of the cases $(8,l+\frac{1}{2})$-$(27,l-\frac{1}{2})$ and $(8,l-\frac{1}{2})$-$(27,l+\frac{1}{2})$ there are two possibilities again. In both cases, however, if we follow the procedure of Sec. 4, we find that, in the diagonalized octet state, the 11 and 22 matrix elements of the input residue matrix are comparable in magnitude. This means that two octet states come out of the calculation, although only one is exchanged in the crossed channel. Thus it is inconsistent to have just the above pairs supporting each other.

(f) We are finally left with the case $(8,l+\frac{1}{2})$-$(10,l-\frac{1}{2})$. Here the result is very similar to the case $(8,l-\frac{1}{2})$-$(10,l+\frac{1}{2})$ which we considered in the preceding section. If, however, we calculate the F's of Eq. (9), we find that, at least for low l, F is quite large in the $(1,l-\frac{1}{2})$ state. This would imply that an extra particle comes out of the calculation and so it would be inconsistent to have just the $(8,l+\frac{1}{2})$-$(10,l-\frac{1}{2})$ pair supporting itself. Of course, this does not happen for very large values of l. For such values, however, our model is meaningless anyway. We conclude therefore that the $(8,l-\frac{1}{2})$-$(10,l+\frac{1}{2})$ pair is the only one capable of supporting itself in 0^- octet $-\frac{1}{2}^+$ octet scattering for $l>0$.

[10] See, e.g., V. Singh, Nuovo Cimento 33, 763 (1964).

[11] The method of Ref. 2 leads to $\alpha=0.57$ and $\alpha=0.78$. It thus gives values different from the value $\alpha=0.68$ which one gets if one assumes octet-deciment degeneracy.

For $l=0$, the main difference is that the $j=l-\frac{1}{2}$ state is absent. If, however, we look at all pairs not involving this angular-momentum state, we find that we reach the same conclusions as for $l>0$ except for the three cases: $(1,\frac{1}{2})$-$(27,\frac{1}{2})$, $(8,\frac{1}{2})$-$(27,\frac{1}{2})$, and $(8,\frac{1}{2})$-$(10,\frac{1}{2})$. We shall proceed to consider these one by one.

(i) For $(1,\frac{1}{2})$-$(27,\frac{1}{2})$ an approximately self-consistent solution can be found if we use Eq. (8). However, the residue of the $(27,\frac{1}{2})$ state turns out to be much smaller than that of the $(1,\frac{1}{2})$ state. $\Gamma_{21}/\Gamma_1=5/33$ or $7/27$ depending on which strap one looks at. From Eq. (9) this means that the corresponding F for that state is also very small. But this, as we discussed already, means that the corresponding isobar either does not exist or has too high a mass to play any role in our simple model. The $(1,\frac{1}{2})$-$(27,\frac{1}{2})$ pair thus is inadmissible.

(ii) For $(8,\frac{1}{2})$-$(27,\frac{1}{2})$ we have two solutions. One of these can be excluded for the same reasons as in case (b). The other solution is more or less consistent by itself. If, however, we calculate the F's using Eq. (9) we find that it is large in the $(1,\frac{1}{2})$ state. Thus, an extra particle would come out of the calculation, and it is not consistent to consider just the above pair as supporting itself.

(iii) It now remains to consider the case $(8,\frac{1}{2})$-$(10,\frac{1}{2})$. If Γ_{10} and Γ_8 are the residues in the 10 and 8 states, respectively, and β is the D to F ratio in the 8 state, we obtain $\beta=0.67$ if we follow the method of the preceding section. For this value $\Gamma_{10}/\Gamma_8\approx 2$ in the 8 state and $\Gamma_{10}/\Gamma_8\approx\frac{4}{3}$ in the 10 state; these values are roughly consistent with each other. If we use Eq. (9), we find that the F's are fairly small in all the other states. Thus our model does seem to suggest the possibility of an 8-10 bootstrap in the s wave.

Now it is well known that purely attractive forces cannot produce a resonance in the s wave. One can readily see that the forces in both the $(8,\frac{1}{2})$ and $(10,\frac{1}{2})$ states here are purely attractive, so, if they bootstrap each other, they can only exist as bound states. Further, since we are considering s-wave scattering, the parity of these states would be opposite to that of the external $(8,\frac{1}{2})$ baryon, which we take by definition to be positive. We shall use the obvious notation $(8,\frac{1}{2}\pm)$ to distinguish the different parity states. Since the $(8,\frac{1}{2}^-)$ and $(10,\frac{1}{2}^-)$ states would be bound states and so lie close to the external $(8,\frac{1}{2}^+)$ state, one would then be forced to consider a considerably enlarged problem involving all these states as external particles. This enlarged problem may or may not have a solution. Even if it does, a solution not involving the s-wave states would be singled out as a much simpler one.[6]

Suppose we now assume only the existence of a 0^- octet and ask the question: What is the simplest set of baryons which can be self-supporting? Now the simplest possibility which suggests itself is the scattering of the 0^- octet with a spin-$\frac{1}{2}$ $SU(3)$ singlet. However, there is obvioulsy no way of producing this singlet as a bound-state pole in this scattering process. We, therefore, turn to the next most complicated problem, which is the scattering of the 0^- octet with a $\frac{1}{2}^+$ octet. But this as we have seen, does lead to a self-supporting system. Thus, just as in the $SU(2)$ case, the physically interesting case is also the simplest.

CONCLUSION

In most bootstrap calculations, one normally assumes the existence of certain particles and tries to calculate some of their parameters. Within the bootstrap philosophy, however, one should also be able to predict the existence or nonexistence of sets of particles. In practice it is difficult to see how this can ever be done (even in a very limited approximate framework), since it involves trying out an infinite number of possibilities. However, the above calculations suggest that this may be possible if we also bring in some criterion of simplicity to limit the number of possibilities. We have shown how, within a drastically oversimplified scheme, the bootstrap approach when combined with such a criterion does seem to lead to a unique self-supporting system. Moreover, this also happens to be the physically interesting case.[12]

Once we have found such a self-supporting set, we can ask whether it leads to additional particles and if so whether they do not have any important effect on that set (if they did, they would have to be considered as part of that original set). For instance, in our model, the 0^- octet, $\frac{1}{2}^+$ octet, and $\frac{3}{2}^+$ decimet form the original set. Having established from a p-wave calculation that this set is self-supporting we looked at other l waves in 0^- octet$-\frac{1}{2}^+$ octet scattering and found a chain of pairs of mutually supporting particles. In the static model, these do not affect the results of the p-wave calculation. We find then that one has an octet-decimet bootstrap in all other waves. One, therefore, has a whole series of "dependent" particles, which owe their existence to particles established in the p-wave calculation, but which do not affect the p-wave particles themselves.

Another example of such "dependent" particles is the chain considered in Ref. 5. Here again one takes the p-wave octet-decimet as given and obtains further particles which do not affect the basic set very much. It is quite trivial to generalize the results of Ref. 5 and obtain similar chains in which the octet $(l-\frac{1}{2})$ and decimet $(l+\frac{1}{2})$ isobars are the lowest members.

ACKNOWLEDGMENT

One of us (L. A. P. B.) would like to thank Professor M. G. K. Menon for his kind hospitality at the Tata Institute.

[12] For a somewhat different criterion of simplicity, see R. Capps Nuovo Cimento 34, 932 (1964).

APPENDIX

We wish to give here an argument to show that the inclusion of vector mesons does not affect the above conclusions. For simplicity, we shall consider the πN case, although the argument can be trivially generalized to the $SU(3)$ case.

Let $\Delta B_{IJ}{}^{(\rho)}$ be the contribution of ρ-meson exchange to the force term B_{IJ}. Then the contribution to N_{IJ} is

$$\Delta N_{IJ}{}^{(\rho)}(\omega)=\frac{1}{\pi}\int_L \frac{\mathrm{Im}\Delta B_{IJ}{}^{(\rho)}(\omega')}{\omega'-\omega}D_{IJ}(\omega').$$

With the linear D approximation (Eq. 7) this becomes

$$\Delta N_{IJ}{}^{(\rho)}(\omega)=\Delta B_{IJ}{}^{(\rho)}(\omega)D_{IJ}(\omega)+\frac{1}{\omega_{IJ}-\omega_0}\lim_{\omega\to\infty}(\omega\Delta B_{IJ}{}^{(\rho)}(\omega)).$$

Then

$$\Delta g_{IJ}{}^{(\rho)}(\omega)=\Delta B_{IJ}{}^{(\rho)}(\omega)+\frac{1}{\omega_{IJ}-\omega}\lim_{\omega\to\infty}(\omega\Delta B_{IJ}{}^{(\rho)}(\omega)).$$

This gives for the change in the reduced width γ_{IJ}

$$\Delta\gamma_{IJ}{}^{(\rho)}=\lim_{\omega\to\infty}(\omega\Delta B_{IJ}{}^{(\rho)}(\omega)).$$

With the expression for $\Delta B_{IJ}{}^{(\rho)}(\omega)$ given by Chew,[1] viz.

$$\Delta B_{IJ}{}^{(\rho)}(\omega)=\frac{C^{(\rho)}}{4k^2}\ln\left(1+\frac{4k^2}{m_\rho{}^2}\right),$$

we have

$$\lim_{\omega\to\infty}(\omega\Delta B_{IJ}{}^{(\rho)}(\omega))=0.$$

PHYSICAL REVIEW VOLUME 139, NUMBER 5B 6 SEPTEMBER 1965

Precession of Relativistic Particles of Arbitrary Spin in a Slowly Varying Electromagnetic Field

DANIEL ZWANZIGER*
Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvette (S et O), Saclay, France
(Received 19 April 1965)

It is shown that, under accelerator or bubble-chamber conditions, the passage of a particle of arbitrary spin through an electromagnetic field effects a Lorentz transformation on its momentum and polarization, and a linear differential equation determining this transformation is given. We also give explicitly the decay-time dependence of the angular distribution that describes the decay of a particle moving in an electromagnetic field, and thereby obtain a method, explained in detail, of measuring the magnetic moment of an unstable, higher spin particle like the Ω^-. It is noted that the gyromagnetic ratio $g=2$ leads to particularly simple equations of motion for all spins, and not only for spin $\frac{1}{2}$. In an appendix we use a novel covariant algebraic method to solve the equations of motion and obtain the finite Lorentz transformation, in the case of a constant and homogeneous electromagnetic field. The method involves the introduction of an algebra of 4-by-4 matrices that plays the same role for 4-vectors as the Dirac algebra for 4-spinors.

I. RELATIVISTIC LARMOR THEOREM

WE wish to describe the time evolution of the polarization matrix, or density matrix in spin space, of a relativistic particle of arbitrary spin in a slowly varying electromagnetic field. This matrix is perhaps most directly observable if the particle decays, for it determines the angular distribution of the decay products, a function, $I(p_1,p_2,\cdots)$, of the 4-momenta p_1, $p_2\cdots$ of the daughter particles. Knowledge of the momentum and polarization matrix at a time $t=0$, and of its subsequent time evolution, allows one to predict the dependence $I(p_1p_2\cdots t)$ of the decay angular distribution on the decay time t. We will obtain this dependence explicitly.

The equation of motion of the dipole polarization, corresponding to spherical harmonics of order 1 in the decay angular distribution, has been described in the literature,[1] and is known most familiarly in covariant form as the Bargmann-Michel-Teledgi (BMT) equation.[2] However, particles of spin $j>\frac{1}{2}$ also have higher multipole polarization, corresponding to harmonics of all orders up to $2j$ in the angular distribution. The new content of the description given here is that it is applied to these higher moments as well. It takes the form of a simple generalization of Larmor's theorem which, however, when stated relativistically is found to apply to the momentum as well as to the polarization.

* Present address: Courant Institute of Mathematical Sciences, New York University, New York, New York.

[1] H. Bacry, Nuovo Cimento 3, 1164 (1962). This article contains many references to earlier work on the subject.

[2] V. Bargmann, L. Michel, and V. Teledgi, Phys. Rev. Letters 2, 435 (1959).