Electromagnetic Interactions and the Subgroups of $SU(6)$

H. HARARI
Israel Atomic Energy Commission, Soreq Nuclear Research Center, Yavne, Israel

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Various sets of relations concerning the electromagnetic properties of the constituents of the $SU(6)$ supermultiplets were recently derived by various authors. A systematic study of these relations is presented here, using subgroups of $SU(6)$ which are generalizations of the U-spin subgroup of $SU(3)$. It is shown that two different assumptions on the transformation properties of the electromagnetic contributions to the particle masses are not in contradiction with the present data: (a) Invariance to all orders under $SU(2)_U\otimes SU(2)_R\otimes SU(2)_P$, where R and P are the total quark spins of the $Q=-\frac{1}{3}$ and $Q=\frac{2}{3}$ quarks, respectively. (b) Second-order contributions from an $SU(2)_U$-invariant interaction which transforms like a **35**. Moreover, the relation $p-n=N^{*+}-N^{*0}$ is obtained from all the various assumptions which are stronger than the usual U-spin invariance. This relation may serve as a criterion for the usefulness of any discussion of electromagnetic mass differences according to $SU(6)$.

THE study of electromagnetic phenomena within the framework of the $SU(6)$ symmetry scheme[1-3] has recently led to various consequences concerning magnetic moments,[4,5] electromagnetic mass differences[5-8] and form factors.[9,10] In this paper we present a systematic study of the various possible assumptions on the $SU(6)$ transformation properties of electromagnetic operators. Using different subgroups of $SU(6)$, we derive in each case a set of predictions which may serve as criteria for testing the appropriate assumptions. We show that the best description of the experimental masses follows from the straightforward assumption that electromagnetic mass differences are due to second-order contribution from a U-spin-invariant interaction which transforms like the **35** representation of $SU(6)$. We present a general mass formula for the baryons, which describes both electromagnetic and "medium strong" mass differences.

We start by reviewing the results of a similar analysis based on $SU(3)$.[11] This is most easily done by using the U-spin[12] invariance of electromagnetic interactions. Assuming that the electromagnetic mass operator $H_{\rm em}$ *is a U-spin scalar,* we obtain[13] (particle label = particle mass):

$$n-p+\Xi^--\Xi^0=\Sigma^--\Sigma^+\,, \tag{1}$$

$$N^{*-}-N^{*0}=Y^{*-}-Y^{*0}=\Xi^{*-}-\Xi^{*0}\,, \tag{2}$$

$$N^{*0}-N^{*+}=Y^{*0}-Y^{*+}\,. \tag{3}$$

For every $SU(3)$ multiplet we may have contributions from all the possible $U=Q=0$ tensor operators of $SU(3)$. In the case of $SU(3)$-octets, this amounts only to components of the **1**, **8**, and **27** representations, leading to the result:

$$M_{\rm em}=a+bQ+c[U(U+1)-\tfrac{1}{4}Q^2]+dQ^2\,. \tag{4}$$

For the pseudoscalar and vector meson octets we get no electromagnetic mass relations as $b=0$. For the decuplet an additional contribution of the **64** is possible, and the most general U-spin-invariant expression is

$$M_{\rm em}=a+bQ+dQ^2+eQ^3\,. \tag{5}$$

If we allow only *second-order graphs* in the symmetry-breaking mechanism, we remain only with tensor operators of the **1**, **8**, and **27** representation. Equation (4) is then the general formula for *every* $SU(3)$ representation, predicting $e=0$ in Eq. (5). This implies[14]

$$N^{*++}-N^{*-}=3(N^{*+}-N^{*0})\,. \tag{6}$$

A much more restrictive assumption is that of an *octet dominance* of the symmetry-breaking electromagnetic interaction. This leaves us only with the $I=0$ and $I=1$ operators, leading to the following relations[15]:

$$\Sigma^+-\Sigma^0=\Sigma^0-\Sigma^-\,, \tag{7}$$

$$\pi^\pm=\pi^0\,, \tag{8}$$

$$\rho^\pm=\rho^0\,, \tag{9}$$

$$N^{*++}-N^{*+}=N^{*+}-N^{*0}=N^{*0}-N^{*-}\,. \tag{10}$$

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[11] For a discussion of electromagnetic mass differences within $SU(3)$ see, e.g.: S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961); A. J. Macfarlane and E. C. G. Sudarshan, Nuovo Cimento **31**, 1176 (1964); S. P. Rosen, Phys. Rev. Letters **11**, 100 (1963); H. J. Lipkin, Argonne National Laboratory Report, 1963 (unpublished); L. A. Radicati, L. E. Picasso, D. P. Zanello, and J. J. Sakurai, Phys. Rev. Letters **14**, 160 (1965).
[12] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters **10**, 361 (1963).
[13] Equation (1) is the Coleman-Glashow relation (Ref. 11). For a derivation of (1)–(3) from U-spin invariance see, e.g., H. J. Lipkin, Ref. 11.
[14] Equation (6) is most easily derived from I-spin considerations. The isospin transformation properties of the second-order electromagnetic interaction are those of $I=0,1,2$. This leads, for every isomultiplet, to the relation $M=a+bI_z+cI_z^2$ from which (6) is immediately obtained.
[15] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964). In I-spin language we say that $M=a+bI_z$. For mesons $b=0$.

TABLE I. Classification of low-lying baryons and mesons according to the subgroups of $SU(6)$. $(\Sigma^0\Lambda)_0$ and $(\Sigma^0\Lambda)_1$ are, respectively, the $U=0$ and $U=1$ states of $\Sigma^0\Lambda$. The same goes for $\pi^0\eta$. ω', ω'' and ω''' are the three orthogonal combinations of ρ^0, ω, and φ which are eigenstates of the Casimir operators of $SU(4)_U$ with pure U-spin.

$SU(6)$	Group $SU(4)_U\times SU(2)_P$	$SU(2)_U\times SU(2)_R\times SU(2)_P$	Particles
56	(1,4)	(1,1,4)	N^{*++}
	(4,3)	(2,2,3)	N^{*+},Y^{*+},p,Σ^+
	(10,2)	(3,3,2)	$N^{*0},Y^{*0},\Xi^{*0},n,(\Sigma^0\Lambda)_1,\Xi^0$
		(1,1,2)	$(\Sigma^0\Lambda)_0$
	(20,1)	(4,4,1)	$N^{*-},Y^{*-},\Xi^{*-},\Omega^-$
		(2,2,1)	Σ^-,Ξ^-
35	(4,2)	(2,2,2)	K^{*+},ρ^+,K^+,π^+
		(3,3,1)	$K^{*0},\omega',\bar{K}^{*0}$
	(15,1)	(3,1,1)	$K^0,(\pi^0\eta)_1,\bar{K}^0$
		(1,3,1)	ω''
	(1,1)	(1,1,1)	$(\pi^0\eta)_0$
	(1,3)	(1,1,3)	ω'''
	(4,2)	(2,2,2)	K^{*-},ρ^-,K^-,π^-

Note that (10) together with (2) and (3) implies an equality among all six $\Delta Q=1$ mass differences within the $SU(3)$ decuplet. For an arbitrary $SU(3)$ multiplet, octet dominance predicts

$$M_{\mathrm{em}}=a+bQ+c[U(U+1)-\tfrac{1}{4}Q^2]. \qquad (11)$$

However, Eqs. (7) and (8) are not well satisfied by experimental data, hinting that the octet-dominance assumption may serve only as a crude approximation.

Concerning other electromagnetic properties we mention only two facts:

(a) *U-spin invariance* predicts equal electromagnetic form factors for members of the same U-spin multiplet, e.g.,

$$\mu(p)=\mu(\Sigma^+);\quad F_{\mathrm{el}}(n)=F_{\mathrm{el}}(\Xi^0). \qquad (12)$$

(b) If we require *octet transformation properties* for the form factors we find

$$\mu=\alpha Q+\beta[U(U+1)-\tfrac{1}{2}Q^2-\tfrac{1}{6}C_2^{(3)}], \qquad (13)$$

where $C_2^{(3)}$ is the quadratic Casimir operator of $SU(3)$. Equation (13) leads, among other things, to[16]

$$\mu(\Lambda)=\tfrac{1}{2}\mu(n). \qquad (14)$$

We now proceed to discuss the $SU(6)$ symmetry scheme. We denote the three basic quarks by p', n', λ'. p' is a $Q=\frac{2}{3}$, $U=0$ state while (n',λ') form a $Q=-\frac{1}{3}$, $U=\frac{1}{2}$ doublet. $SU(6)$ can be decomposed in the following way:

$$SU(6)\supset SU(4)_U\otimes SU(2)_P\supset \supset SU(2)_U\otimes SU(2)_R\otimes SU(2)_P. \qquad (15)$$

$\mathbf{P}$ is the total quark spin of all the p' quarks and $\mathbf{R}$ is the total quark-spin of the n' and λ' quarks. The total spin $\mathbf{J}$ satisfies: $\mathbf{J}=\mathbf{P}+\mathbf{R}$. $SU(2)_U$, $SU(2)_R$, and $SU(2)_P$ are the groups of U-spin, R-spin, and P-spin, respectively. $SU(4)_U$ is the U-spin analog[9] of $SU(4)_I$ of Bég and Singh.[17] The classification of low-lying baryons and mesons according to these subgroups is presented in Table I. The weakest plausible assumption on the $SU(6)$ character of H_{em} is that it transforms like a combination of all $U=J=0$ components of all possible representations of $SU(6)$. This does not lead to any mass relations, apart from Eqs. (1)–(3). We may now continue in two alternative ways:

(a) We assume invariance of electromagnetic terms of all orders under the subgroups defined in (15).

(b) We restrict ourselves to usual U-spin invariance, but allow only low-order contributions to the symmetry-breaking mechanism.

Following the first approach, we note that *invariance under* $SU(2)_U\otimes SU(2)_R\otimes SU(2)_P$ implies equal electromagnetic contributions to the mass of all members of the same $SU(2)_U\otimes SU(2)_R\otimes SU(2)_P$ multiplet, leading only to one new relation

$$n-p=N^{*0}-N^{*+}. \qquad (16)$$

The following set of relations[18] is predicted in a similar way by *invariance under* $SU(4)_U\otimes SU(2)_P$:

[16] See S. Coleman and S. L. Glashow, Ref. 11.

[17] M. A. B. Bég and V. Singh, Phys. Rev. Letters 13, 418 (1964).

[18] Equations (17) and (18) were first derived by Sakita (Ref. 5) using the stronger assumption of second-order contributions. The same results were obtained by Chan and Sarker (Ref. 6) whose assumption coincides with ours. In both papers complicated tensorial methods were used. Equation (19) is a new result.

$$\Sigma^- - \Sigma^0 = \Xi^- - \Xi^0 = N^{*-} - N^{*0} = Y^{*-} - Y^{*0} = \Xi^{*-} - \Xi^{*0}, \quad (17)$$

$$n - p = \Sigma^0 - \Sigma^+ = N^{*0} - N^{*+} = Y^{*0} - Y^{*+}, \quad (18)$$

$$K^{*0} - K^{*+} = K^0 - K^+ = \rho^0 - \rho^+. \quad (19)$$

Equations (17) and (18) are not in good agreement with experimental data. The experimental situation with respect to Eq. (19) is not clear.

If, on the other hand, we restrict ourselves to invariance under $SU(2)_U$ and allow only *second-order processes* of an interaction which transforms like the **35** we obtain Eq. (1)–(3), (6), (16) and

$$N^{*-} - N^{*0} + \Xi^- - \Xi^0 = 2(\Sigma^- - \Sigma^0). \quad (20)$$

None of these relations contradict the *known* data. Our last assumption is essentially equivalent to that of Kuo and Yao[7] who have suggested the following expression for H_{em}:

$$H_{\text{em}} = aQ^2 + b\mathbf{M}\cdot\mathbf{M}. \quad (21)$$

This will generally include components of the **1**, **35**, **189**, **280**, $\overline{\mathbf{280}}$, and **405** representations of $SU(6)$ with (**1**,**1**), (**8**,**1**), or (**27**,**1**) transformation properties under $SU(3)\otimes SU(2)_J$. However, for the baryons in the **56**, the contributions of **189**, **280**, and $\overline{\mathbf{280}}$ are absent and we find the following electromagnetic mass formula:

$$M_{\text{em}} = a_0 + a_1 J(J+1) + bQ + c[U(U+1) - \tfrac{1}{4}Q^2] + dQ^2. \quad (22)$$

Note that the first two terms in (22) do not contribute to the mass differences within the isomultiplets. The other terms in (22) coincide with those of Eq. (4) with the additional prediction of identical sets of values of b, c, and d for the **8** and **10** representations of $SU(3)$ which construct the **56**.

Finally we may try to assume in analogy with $SU(3)$ octet dominance, an $SU(6)$ **35** *dominance* of the mass-splitting processes. This immediately leads to the general formula

$$M_{\text{em}} = a + bQ, \quad (23)$$

which predicts equal mass differences for all possible $\Delta Q = 1$ pairs within the isomultiplets of every $SU(6)$ representation. For mesons this implies a complete degeneracy of π and ρ masses [Eqs. (8) and (9)] which clearly contradicts the experimental facts. However, (23) may still serve as a sort of qualitative relation for the baryons, possibly connected with the hitherto unexplained, consistent decrease of baryon masses when electric charge increases.

Comparison of the above predictions with experimental mass values leaves us with two different assumptions which do not contradict the data:

(a) invariance to all orders, under $SU(2)_U \otimes SU(2)_R \otimes SU(2)_P$;

(b) second-order contributions from an $SU(2)_U$-invariant interaction which transforms like a **35**.

Assumption (a) leads to (1)–(3) and (16) while (b) provides the general formula (22), and consequently Eqs. (1)–(3), (6), (16), and (20). Assumption (b) is also more appealing from the physical point of view. We now combine it with the "medium strong" mass formula[17] in order to obtain a general mass expression for the baryons in the **56**:

$$M = a + bY + c[I(I+1) - \tfrac{1}{4}Y^2] + dQ + e[U(U+1) - \tfrac{1}{4}Q^2] + fQ^2 + gJ(J+1). \quad (24)$$

Only accurate measurements of electromagnetic mass differences within the $SU(3)$ decuplet will enable us to test the various possibilities seriously. In particular, Eq. (16) may serve as a criterion for the usefulness of the $SU(6)$ symmetry with respect to electromagnetic mass differences. If (16) is not obeyed by nature, all assumptions which are stronger than simple U-spin invariance fail.

Finally we would like to make the following remark concerning the $SU(6)$ properties of electric form factors: Assuming that the electric form factor transforms under $SU(6)$ like Q (i.e., the $U=Q=J=0$ component of a **35** we obtain the following trivial relation for the **56** representation[19]:

$$F_{\text{el}}(t) = \alpha(t)Q. \quad (25)$$

t is the (momentum transfer)^2; $\alpha(t)$ is the same function of t for all baryons in the **56**. Equation (25) predicts, among other things, a vanishing electric form factor for the neutron. This last result is also obtainable from the following weaker assumption[20]: F_{el} is a scalar under $SU(4)_U$[21] and transforms like an octet under $SU(3)$. The derivation goes as follows: Since F_{el} is scalar under $SU(4)_U$ and $SU(2)_J$, it is also a scalar under $SU(4)_U \otimes SU(2)_P$. Hence

$$F_{\text{el}}(\Lambda) \equiv F_{\text{el}}(n). \quad (26)$$

However, from the octet transformation properties of F_{el} we conclude [see Eq. (14)]: $F_{\text{el}}(\Lambda) = \frac{1}{2}F_{\text{el}}(n)$. Hence

$$F_{\text{el}}(\Lambda) \equiv F_{\text{el}}(n) \equiv 0. \quad (27)$$

A word of caution must be added here concerning the validity of such relations at high values of t, for which the "static" $SU(6)$ approximation is not adequate.[22]

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[19] This result, which is so trivial for $SU(6)$ is not obtainable from $SU(3)$, because the F/D ratio which vanishes for $t=0$ may have any arbitrary value for other t's, leading to an equation of the form (13), where both α and β are independent functions of t.

[20] This was assumed in Ref. 9. However, the result (25), also stated there, cannot be obtained without assuming explicitly that F_{el} is a component of the **35**.

[21] This is made plausible by the fact that the electric charge operator is an $SU(4)_U$ scalar, hence terms like Q^2, etc. will also possess this property.

[22] For a discussion of this problem see: R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London), **A284**, 146 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965).