Effect of Mass Splittings on Symmetry Relations between Scattering Amplitudes*

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The question of the violation of relations predicted by $SU(3)$ invariance between scattering amplitudes that is due to the kinematical effect of mass splittings alone, with no other symmetry breaking in the Hamiltonian, is considered. On the basis of qualitative considerations and a simple potential scattering model, it is argued that these effects can be very severe, and can produce order-of-magnitude disagreements with the theoretical predictions unless the total center-of-mass energy is large compared with the masses of the particles involved. It seems likely that all existing disagreements between $SU(3)$ predictions and scattering experiments could be accounted for on the grounds of mass splittings alone. A study is also made of whether to compare cross sections at the same initial kinetic energies, or at the same Q values, as has been done heretofore. It appears that there is comparatively little to choose between the two methods, or between either of them and other procedures which might be used. If the result depends sensitively on the method adopted for comparing the cross sections, this in itself is an indication that the effects of mass splittings are so large that agreement with theory cannot be expected.

SEVERAL authors[1-7] have studied the problem of deriving relationships among scattering amplitudes, and consequent equalities or inequalities among cross sections, from $SU(3)$ symmetry, and have compared some of the resulting predictions with experiment. One knows, of course, that there is at least one sizable violation of $SU(3)$, namely, the mass splittings within $SU(3)$ multiplets. Even if no other violations are present, the mass splittings will produce deviations from the relations which would hold in the limit of exact symmetry, and one must somehow try to take these purely kinematic effects into account in comparing theory and experiment. It is the purpose of this note to suggest that the effects of the mass splittings are probably so large at energies for which data currently exist as to make comparison of theory and experiment essentially meaningless, and that data at much higher energies will be required if scattering experiments are to give any insight at all into $SU(3)$. This suggestion is based on qualitative estimates, and on calculations in a potential scattering model.

To attempt to gain some understanding of the situation, let us consider two reactions

$$A+B\to C+D \tag{1}$$

and

$$A'+B'\to C'+D' \tag{2}$$

for which the amplitudes would be equal in the limit of complete $SU(3)$ invariance. We denote by k and K the magnitude of the initial and final three-momenta in the center-of-mass system in reaction (1), and by k' and K' the corresponding quantities for reaction (2); if there were no mass splittings, we would have $K=K'$ when $k=k'$. We let t_1 and t_2 be the invariant amplitudes for the two reactions, so that the symmetry prediction is

$$t_1(k,K)=t_2(k,K). \tag{3}$$

t_1 is related to the cross section for reaction (1) by the relation

$$|t_1|^2=(kE^2/K)\sigma_1\equiv F\sigma_1, \tag{4}$$

where E is the total center-of-mass energy. We may now ask how to try to verify Eq. (3) experimentally. The first point, as has been noted by Meshkov *et al.*,[4] is that one should not compare the cross sections themselves, but rather the cross sections multiplied by the factor F, i.e., the square of the invariant amplitudes; this takes phase-space effects into account. The question still remains at what energies one should compare the two cross sections; because of the mass splittings, it will no longer be possible to make $k=k'$ and $K=K'$ simultaneously. One can do one or the other, and hope that one of the equations

$$t_1(k,K)=t_2(k',K) \tag{3a}$$

or

$$t_1(k,K)=t_2(k,K') \tag{3b}$$

will be approximately correct in the presence of mass differences. Equation (3a), which is similar to the prescription for comparing cross sections for different reactions suggested by Meshkov *et al.*[4] and which we will discuss below, has the advantage that it carries out the comparison so that the thresholds of the two final states coincide. However, as we shall see, we cannot expect the theory to be valid unless we are well above thresholds anyway. Equation (3b) thus seems equally plausible. This latter prescription seems especially plausible in the case that reactions (1) and (2) involve the same initial state. In this case the same eigenphase

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[1] P. G. O. Freund, H. Ruegg, D. Speiser, and A. Morales, Nuovo Cimento 25, 307 (1962).
[2] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters 10, 361 (1963); C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Letters 1, 44 (1962).
[3] A. E. Everett, Phys. Rev. 132, 2278 (1963).
[4] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters 12, 87 (1964).
[5] M. Konuma and Y. Tomozawa, Phys. Rev. Letters 12, 425 (1964).
[6] H. Harari and H. J. Lipkin, Phys. Rev. Letters 13, 208 (1964).
[7] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters 13, 212 (1964).

shifts will contribute to both reactions with, in general, comparable strength. The prescription for comparing the two reactions at the same incident momentum or total energy, then insures, for example, that if one of the eigenphases has a maximum at some energy, then we compare the two cross sections at the energy where each receives a contribution from the maixmum. Since approximations (3a) and (3b) seem more or less equally reasonable, and would be equivalent in the absence of mass splittings, the situation is likely to be that if the quality of the agreement with experiment depends sensitively on how one compares cross sections this in itself indicates that the effect of mass splittings is large enough that agreement between theory and experiment should not be expected; this is indeed the case in the simple model discussed below.

Of course Eqs. (3a) and (3b) are not the only possible approximations to make in attempting to solve the ambiguity in how to compare Eq. (3) to experiment in the presence of mass splittings. In the model which we discuss below, we shall actually study two similar, but not identical, approximations, namely the assumptions that

$$t_1(q,Q)=t_2(q',Q) \tag{3a$'$}$$

and

$$t_1(q,Q)=t_2(q,Q'), \tag{3b$'$}$$

where q and Q are the initial and final total kinetic energies in the center-of-mass system for reaction (1), and q' and Q' are similarly defined for reaction (2). We adopted these approximations because, following the suggestion in Ref. (4), most of the comparisons so far actually made with experimental data have compared cross sections at the same Q values, i.e., they have assumed the validity of equations such as (3a$'$) if Eq. (3) held in the absence of mass splittings. We also note that if there are other channels present with the same quantum numbers as those of reactions (1) and (2) then the eigenphases and hence the amplitudes t_1 and t_2 will depend on the Q values in these channels as well as those in the initial and final states of reactions (1) and (2) themselves. If we choose to make $q=q'$ or $Q=Q'$, then, in general, the Q values in the other channels will not be equal for the two reactions. There is no reason to expect that a particular amplitude will depend especially strongly on any one of these variables in comparison to the others if the coupling between channels is strong. There will therefore be additional complexities in a multichannel problem which are not dealt with in the model which we discuss. We assume that these will not affect the general nature of our conclusions.

Let us now ask under what circumstances (3a) or (3b) will be reasonable approximations to (3). Let us observe that, if we could solve the problem exactly, we would find that t_1 depended on sines and cosines (i.e., spherical Bessel functions) of ka and Ka, where a is the radius of interaction, as a result of matching interior and exterior (plane-wave) solutions of the Schrödinger equation. In units with $\hbar=c=1$, the momenta k and K satisfy the equation

$$\begin{aligned}E&=(k^2+m_A)^{1/2}+(k^2+m_B^2)^{1/2}\\&=(K^2+m_C^2)^{1/2}+(K^2+m_D^2)^{1/2}.\end{aligned} \tag{5}$$

If we hold, for example, K fixed then a change dm in, say, m_C produces a change in k given by

$$dk=(m_Cdm/k)(E_AE_B/(E_A+E_B)E_C), \tag{6}$$

where E_A, E_B, and E_C are the total energies of particles A, B, and C. Since the energies are comparable, Eq. (6) yields the order-of-magnitude estimate

$$dk\approx m_Cdm/k \tag{6$'$}$$

for the change in k produced by a mass splitting dm. If Eq. (3a) is to hold when the masses of C and C' differ by an amount dm, then the changes in sin ka and cos ka produced by a change in k of magnitude given by Eq. (6$'$) must be much less than 1; that is, we need

$$adk=am_Cdm/k\ll 1. \tag{7}$$

For a of the order of a pion Compton wavelength, and dm of the order of the observed mass splittings within $SU(3)$ multiplets, adm is of order 1, so that Eq. (7) requires $k\gg m_C$. Put another way, the initial kinetic energy in the center-of-mass system must be large compared to the rest energy. Similarly, for Eq. (3b) to be valid the final kinetic energy will in general have to be large compared to the rest energy. In an actual case, of course, new channels continue to open up as the energy gets higher, and we will never have the situation where the kinetic energy is large compared to the masses in all channels. We can hope, however, that at high energy the coupling to the comparatively small number of channels with nearby thresholds may be small enough that the amplitude will not be greatly changed by the fact that the momenta in those channels do not have the values they would have if there were no mass splittings.

To attempt to confirm these conclusions, we have studied a simple model of two interacting spin-$\frac{1}{2}$ particles. (It makes no difference whether the "spin" is ordinary spin, iso-spin, or U spin.) The Schrödinger equation in the center-of-mass system is, outside the range of interaction,

$$[(p^2+m^2)^{1/2}+(p^2+M^2)^{1/2}]\psi=E\psi, \tag{8}$$

where m and M are mass operators for the two particles and p is one-half the relative momentum operator. We take the eigenvalues of m to be $m_1\pm dm_1$ for the cases where the spin of particle 1 is down and up respectively; similarly, the eigenvalues of M are $m_2\pm dm_2$. Within the region of interaction, the Schrödinger equation has the form

$$[(p^2+m^2)^{1/2}+(p^2+M^2)^{1/2}+H']\psi=E\psi, \tag{8$'$}$$

where H' is the interaction. Our model consists in taking H' to be a simple square well. This is, of course, not what one would have in a relativistic theory, but it seems reasonable to suppose that the conclusions we will draw as to the kinematic effects of mass splittings should not be sensitive to the details of the interaction. Also, in order to get the strongest possible set of relations between amplitudes in the absence of mass splittings, we shall take H' to be nonzero only in triplet states. Thus, we take $H'=-V$ for r (the relative coordinate, which is conjugate to p) less than some range a in triplet spin states, and $H'=0$ in singlet states. It should be noted that it is important to include relativistic kinematics in the Schrodinger equation, since if one used the nonrelativistic relation $k=(2\mu E)^{1/2}$ between energy and center-of-mass momentum, than in the presence of mass splittings the momenta for coupled spin states do not become equal at high energies. Thus the use of nonrelativistic kinematics might well badly overestimate the effect of the mass splittings, which is what we are concerned with studying.

With our square-well potential, Eq. (8′) can be solved. Because V is constant, the interior spatial wave functions for definite orbital angular momentum are eigenstates of the operator p^2, so that Eq. (8′) involves only the square roots of numbers rather than operators. Let us designate by 1, 2, and 3 the spin states in which both particles have their spins up, particle 1 has spin up and particle 2 spin down, and particle 1 spin down and 2 spin up, respectively. [The reader is again reminded that the term "spin" need not refer to ordinary spin in our discussion, but can be thought of as referring to the U spin of $SU(3)$ defined in Ref. 2.] We sketch the solution of Eq. (8′) in the case that the total spin is zero, in which case spin states 2 and 3 are coupled. The interior solution will then have the form, for the case of S waves

$$\psi=\sin kr(a\chi_t+b\chi_s)/kr\,,$$

where the spin wave functions χ_t and χ_s are given by

$$\chi_t=(|2\rangle+|3\rangle)/\sqrt{2}\quad \chi_s=(|2\rangle-|3\rangle)/\sqrt{2}\,.$$

Equation (8′) becomes

$$H\psi=(\sin kr/kr)[(\gamma a+\eta b+V)\chi_t+(\gamma b+\eta a)\chi_s]$$
$$=E(\sin kr/kr)(a\chi_t+b\chi_s)\,,\quad (8'')$$

where

$$\gamma=0.5(\alpha+\beta)\quad \eta=0.5(\alpha-\beta)$$

and

$$\alpha=[k^2+(m_1-dm_1)^2]^{1/2}+[k^2+(m_2+dm_2)^2]^{1/2}\,,$$
$$\beta=[k^2+(m_1+dm_1)^2]^{1/2}+[k^2+(m_2-dm_2)^2]^{1/2}\,.$$

Equating the coefficients of $|2\rangle$ and $|3\rangle$ on the two sides of (8″) gives a pair of simultaneous homogeneous equations for a and b. Requiring that the determinant of the coefficients be zero, so that a nontrivial solution exists, then gives an equation for k^2 whose two roots give the two possible values of the momentum squared inside the well. In practice, because of the many square roots in Eq. (8″), the eigenvalue equation for k^2 is quite intractable algebraically. The method actually adopted was to guess approximate answers for the two values of k^2 by solving a problem in which the mass splitting within the well is replaced by an additional term in H' of the form

$$H'_{\rm ef}=-dm_1s_{z1}-dm_2s_{z2}\,,$$

where s_{z1} and s_{z2} are the operators corresponding to the z components of the spin of the two particles. Use of these values of k^2 in the interior wave function is equivalent to solving the actual problem, i.e., with a mass splitting instead of $H'_{\rm ef}$, but with effective triplet and singlet potentials $V_{\rm ef}$ and $V_{s\rm ef}$ instead of V and 0; $V_{\rm ef}$ and $V_{s\rm ef}$ may be computed straightforwardly by requiring that nontrivial solutions for a and b in Eq. (8″) exist when either of our two roots for k^2 is used and the actual potentials are replaced by the effective ones. The shift in the value of the triplet potential is of no consequence since we are only interested in examining the properties of the solutions for typical potentials with reasonable orders of magnitude; the effective triplet potential is quite as good for our proposes as the original potential. If V is independent of energy, then $V_{\rm ef}$ will have an energy dependence, which turns out to be slight. There is little one can say about what the energy dependence of a potential which might approximate the strong interactions ought to be. It seems unlikely, however, that the energy dependence will have much effect on our conclusions as to the kinematic effects of mass splittings. We do desire, however, that the problem we solve be one in which the actual singlet potential should be 0. Therefore an iteration procedure was used in which the original value of the singlet potential, used in obtaining the approximate roots for k^2, was varied until $V_{s\rm ef}$ was sufficiently close to 0.

The exterior solutions, from Eq. (8), are simply free-particle space wave functions multiplying spin wave functions in which the z component of the spin of each of the particles is specified, i.e., the spin wave functions $|2\rangle$ or $|3\rangle$. The magnitude of the exterior momentum depends, of course, on the spin state because of the mass splitting. Imposing in the usual way continuity between the interior and exterior solutions, as well as an asymptotic boundary condition that there be an incoming wave in one specified spin state only, then determines the solution and allows one to obtain the contribution of the partial wave in question to the scattering amplitude by reading off the coefficient of the outgoing waves. The amplitudes obtained are the exact amplitudes for scattering by a potential with parameters $V_{\rm ef}$ and $V_{s\rm ef}$, since we have used our approximate values for k^2 in constructing the interior wave function.

The solution in the case that the z component of the total spin is 1 can be carried out straightforwardly. There is now only one spin state involved, so that the solution of Eq. (8″) does not involve coupled equations, and the algebra is manageable. One must, of course, take the depth of the potential for the spin 1 case to be given by V_{ef}, as found in the solution of the $s_z=0$ case.

We consider a case in which particles 1 and 2 are a "meson" and a "baryon" with $m_1=300$ MeV, $m_2=1100$ MeV, $dm_1=150$ MeV, and $dm_2=-150$ MeV. The magnitudes of these masses and mass splittings are, of course, of the order of magnitude of those in the physical meson and baryon U-spin multiplets. We represent the S—wave amplitudes for elastic scattering in spin states 1 and 2 by f_{11} and f_{22}, and for scattering from state 2 to state 3 by f_{32}. The amplitudes are normalized so that the S-wave contribution to the differential cross section for scattering from spin state i to j is $(k_j/k_i)|f_{ji}|^2$; they thus differ from the invariant amplitudes defined earlier by a factor 1/(total energy).

If there were no mass splitting, and the masses were independent of the z component of the spin, then the problem would be invariant under rotations in spin space. (This is true since the problem we are solving is one in which the masses depend on the z components of the spins, but the potential depends only on the total spin. It is, of course, important that our technique of approximate solution corresponds only to changing the values of the singlet and triplet potentials, and does not introduce any dependence of the potentials, as opposed to the masses, on the z components of the spins.) The invariance under rotation leads to the relation

$$f_{11}=f_{22}+f_{32}. \tag{9}$$

In addition, if the potential and hence the scattering amplitude is zero in the singlet state, we have the additional relation

$$f_{22}=f_{32} \tag{10}$$

in the limit of complete symmetry. As will be discussed below, we have verified that the violations of Eq. (10) due to the fact that our effective singlet potential is not exactly zero are quite small. If we now introduce mass differences between states with spin up and spin down, the z direction in spin space is singled out by the mass splittings, although the remainder of the Hamiltonian is still rotationally invariant. The scattering can then depend on the z components of the spins, it is only the z component of the total spin, and not its magnitude, which is conserved, and Eqs. (9) and (10) will no longer hold exactly. We have examined the extent to which Eqs. (9) and (10) remain valid in our model when the mass differences are introduced. We have studied several sets of potential parameters with the general order of magnitude that one would expect to characterize an effective strong interaction potential between elementary particles. The qualitative conclusions as to the effect of the mass splittings were not at all sensitive to a rather wide variation in the potential parameters and hence we present, in Tables I and II, the result only for one typical case. Moreover, for simplicity, we have confined the calculation to the S-wave case; except for possible accidental cancellations, the changes in the total amplitude due to the mass differences will obviously be of the same general magnitude as the changes in the individual terms in the partial-wave sum.

Table I gives a comparison of Eq. (10) with the results of the calculations at a representative set of energies; we compare f_{32} at total energy E and final-state kinetic energy Q_3 (all energies refer to the center-of-mass system, and the units are chosen so that $\hbar=c=100$ MeV$=1$; i.e., the unit of length is the Compton wavelength of a particle of mass 100 MeV) with f_{22} evaluated at the same total energy, and hence at the same initial kinetic energy, and also with f_{22} evaluated at a total energy E' such that the kinetic energy in the final state equals Q_3, which is the procedure suggested in Ref. 4. Since f_{22} differs from the invariant amplitude, as already noted, in the latter case f_{22} has been multiplied by E'/E before being compared with f_{32}. Table I

TABLE I. Values of the real and imaginary parts of the scattering amplitudes f_{22} and f_{32}. (See text for notation.) We take the central meson and baryon masses to be 3.0 and 11.0, and the mass difference between states with spin up and spin down to be -3.0 and $+3.0$ for the meson and baryon, respectively, in the system of units used in which $\hbar=c=\lambda=1$, where λ is the Compton wavelength of a particle of mass 100 MeV. E is the total energy, and Q_3 the kinetic energy in spin state 3. (All energies are in the center-of-mass system.) The effective triplet potential, after the iterative solution of the Schrödinger equation, as discussed in the text, ranges from 5.7 at $E=21.7$ to 6.2 at $E=506$; V_s, the singlet potential at each energy, is given in the table. The range of both potentials is 0.7. f_{32} and f'_{22} are the S-wave scattering amplitudes at energy E; f''_{22} equals f_{22} evaluated at energy E'', and multiplied by E''/E, where $E''=E-6$ is the total energy at which the kinetic energy in state 2 is equal to Q_3. In the absence of mass splittings, and neglecting differences of 10% or less due to the nonzero value of the singlet potential after iteration, $f'_{22}=f''_{22}=f_{32}$. The prescription for comparing theory and experiment given in Ref. 4 corresponds to the assumption $f''_{22}=f_{32}$.

E	Q_3	V_s	$\mathrm{Re}f'_{22}$	$\mathrm{Im}f'_{22}$	$\mathrm{Re}f_{32}$	$\mathrm{Im}f_{32}$	$\mathrm{Re}f''_{22}$	$\mathrm{Im}f''_{22}$
21.7	4.7	−0.05	−0.0394	0.0987	−0.0084	0.0048	−0.0255	0.1458
24.7	7.7	−0.06	−0.0366	0.0749	−0.0133	0.0104	−0.0249	0.1090
30.7	13.7	−0.08	−0.0297	0.0502	−0.0125	0.0109	−0.0296	0.0606
36.7	19.7	−0.09	−0.0199	0.0400	−0.0119	0.0158	−0.0248	0.0419
48.7	31.7	−0.10	−0.0160	0.0241	−0.0091	0.0112	−0.0144	0.0273
106	89	−0.12	−0.0058	0.0087	−0.0040	0.0065	−0.0061	0.0086
506	489	−0.13	−0.0011	0.0015	−0.0008	0.0014	−0.0011	0.0015

TABLE II. Values of the scattering amplitude f_{11} and of the sum $f_{22}+f_{33}$ for the same potential and masses as in Table I. f_{11} is evaluated at total energy E and kinetic energy Q_1. If E' and E'' are the values of the total energy at which the kinetic energy in spin states 2 and 3, respectively, equals Q_1, then $\Sigma'=(E'/E)(f'_{22}+f'_{32})$ and $\Sigma''=E'f'_{22}/E+E''f''_{32}/E$, where f'_{22} and f'_{32} are evaluated at total energy E', and f''_{32} at total energy E''. In the absence of mass splittings, $f_{11}=\Sigma'=\Sigma''$. The prescription of Ref. 4 corresponds to the assumption $f_{11}=\Sigma''$.

E	Q_1	ReΣ'	ImΣ'	Ref_{11}	Imf_{11}	ReΣ''	ImΣ''
18.7	4.7	...	...	−0.0753	0.0343	−0.0387	0.1742
24.7	10.7	−0.0406	0.0894	−0.0481	0.0285	−0.0525	0.1089
30.7	16.7	−0.0368	0.0762	−0.0386	0.0323	−0.0367	0.0759
36.7	22.7	−0.0330	0.0558	−0.0300	0.0384	−0.0328	0.0536
45.7	31.7	−0.0248	0.0428	−0.0230	0.0295	−0.0250	0.0409
103	89	−0.0105	0.0149	−0.0093	0.0131	−0.0103	0.0155
503	489	−0.0019	0.0029	−0.0019	0.0027	−0.0019	0.0029

also gives the values of the effective potential in the singlet state. The iteration was carried out to the point that the magnitude of Vs_{ef} was less than or about equal to 0.1 in the units used, and thus small compared to the mass differences. To check the extent of the disagreement with Eq. (10) due to the nonzero value of V_{sef} rather than to the mass differences, we calculated f_{11} and f_{22} using the same triplet and singlet potentials and central meson and baryon masses as those in Table I, but taking the mass differences to be zero. Throughout the range of energies, f_{22} and f_{32} differed in this case by perhaps 10%; for example, at $E=24.7$, one finds in the absence of mass splitting $f_{22}=-0.0254+0.0164i$, and $f_{32}=-0.0229+0.0164i$. Differences of this order of magnitude between f_{22} and f_{32} are obviously much smaller than those shown in Table I; hence, almost the entire discrepancy between Eq. (10) and the results of the calculations must be attributed to the mass differences, and not to the fact that our approximation method results in our solving a problem in which the singlet potential is not quite zero.

In Table II we compare Eq. (9) with the calculated results. Again, we compare f_{11} with $f_{22}+f_{33}$ evaluated at an energy E' such that the initial kinetic energy is the same for all three reactions, and also with the sum of f_{22} and f_{33} each evaluated at an energy such that the final kinetic energies for all of the reactions are equal; f_{22} and f_{32} are again multiplied by the appropriate ratios of total energies before the comparisons are made. Since Eq. (9) holds, in the limit of complete symmetry, for all values of the singlet potential, there is no need to consider the nonzero value of V_{sef} as far as the results of Table II are concerned.

It is obvious from Tables I and II that Eqs. (9) and (10) are very badly violated until one gets to total center-of-mass energies that are quite large compared to the masses of the particles, in accordance with our previous estimate based on Eq. (6′). It should, of course, be borne in mind that we are here talking about amplitudes which must be squared to obtain cross sections, so that the differences between observed cross sections will be worse still; if the conclusions drawn from our model are qualitatively correct, cross sections which would be equal in the absence of mass splittings may differ by an order of magnitude even though the center-of-mass energy is 2 BeV or so above threshold. As to the question of whether Eqs. (9) and (10) are more nearly correct if we evaluate the amplitudes at energies such that the initial kinetic energies or the final kinetic energies are the same for all the reactions involved, there seems to be little to choose; there certainly seems to be no clear evidence in favor of the procedure suggested in Ref. (4) as opposed to the alternative procedure [approximation (3b′)] of comparing cross sections for different reactions at energies such that the initial kinetic energy is the same for each reaction; if anything the latter procedure seems to work somewhat better in our examples. What does appear to be true, however, is that if it makes much difference which procedure is adopted, then this is in itself, in accordance with our earlier expectations, an indication that the effect of the mass splitting is severe and that whichever of the two procedures for comparing theory and experiment is adopted, one will not find good agreement with theoretical relations based on the assumption of complete symmetry, even if the Hamiltonian is completely symmetric aside from the mass differences. In Table I, for example, the difference between f'_{22} and f''_{22} gives an indication of the sensitivity of the invariant amplitudes to changes in their arguments of the magnitude produced by the mass splittings. Since f_{22} and f'_{22} seem to be about equally good approximations to f_{32} (that is, the amplitude is about as sensitive to changes in k, or q, as to changes in K, or Q), the difference between either one of them and f_{32} due to the mass splittings will be at least of comparable magnitude to their differences from one another; in fact, in our examples, f_{22} and f'_{22} are in much better agreement with one another than either one of them is with f_{32}. This is in addition to any further differences between the amplitudes for the two different channels caused by a symmetry breaking term in the interaction Hamiltonian. Hence, if the ambiguity as to how one compares cross sections for different reactions is important, i.e., if quantities such as f'_{22} and f''_{22} in our example are very different form one another, this is likely to be an indication that there are large violations of the symmetry relations, regardless of the approximation adopted in comparing with experiment, due to the mass splittings whether or not there are additional symmetry breaking effects present.

Let us now turn to the question of what the implications of the foregoing discussion are for the existing attempts to compare SU(3) scattering predictions with experiment.[4–7] These have yielded a mixture of agreement in some reactions and disagreement in others. We wish to suggest that, because of the comparatively low center-of-mass energies at which data are available, the disagreement between experiment and the SU(3) predictions is in no case worse than might be expected on the grounds of the mass splittings alone, and constitutes no evidence for additional violation of SU(3) over and above the mass differences within multiplets. Let us take the most extreme case first. SU(3) predicts that the cross sections for the reactions

$$K^- + p \to \Xi^\circ + K^\circ \tag{11a}$$

and

$$K^- + p \to \Sigma^- + \pi^+ \tag{11b}$$

should be equal. The only available data on Ξ° production[8] gives a cross section of 0.01 ± 0.005 mb at center-of-mass energy 2.0 BeV and Q value 185 MeV. The cross section for reaction (11b) at the same Q value is about 8 mb,[9,10] so that the SU(3) prediction fails, after taking the F factors of Eq. (4) into account, by about a factor of 300. However, it must be remembered that we are very close to threshold, where it seems quite possible that the mass differences may produce such catastrophic disagreements. For example, note the situation at $E = 21.7$ in Table I, where a violation of the symmetry relations of nearly this magnitude occurs in our model. Moreover, there is strong indication that the effects of the mass splitting are very important in the case of reactions (11a) and (11b), for if one compares the cross sections at the same total energy and initial kinetic energy, one finds the cross section for reaction (11b) is down to 0.4 ± 0.1 mb, so that the discrepancy changes from a factor of about 17 to a factor of about 6 in the amplitudes, again after taking account of the F's, if one changes the way in which the two reactions are compared. As we have seen, the fact that the two cross sections for reaction (11b) are so different from one another is very likely to mean that there are differences of comparable size between either of them and the cross section for reaction (11a) caused simply by the mass differences. Hence even this drastic disagreement between theory and experiment probably could be accounted for on the basis of mass differences alone. [Of course, there are probably other symmetry breaking effects also. We argue only that their existence is not established by the current data on reactions (11a) and (11b), and that these other deviations from complete $SU(3)$ symmetry need not necessarily be large despite the great discrepancy between theory and experiment when the two reactions are compared at the same Q value.]

Meshkov *et al.*[7] have also pointed out that the cross sections for the reactions

$$\pi^- + p \to Y_1^{*-} + K^+, \tag{12a}$$

$$K^- + p \to Y_1^{*-} + \pi^+, \tag{12b}$$

$$K^- + p \to \Xi^{*-} + K^+, \tag{12c}$$

should all be equal and equal to one third of the cross section for

$$\pi^- + p \to \Xi^{*-} + \pi^+. \tag{12d}$$

In fact these predictions are badly violated. If one compares the cross sections at the same Q values (always taking into account the phase space factor, F) one finds that the cross section for reaction (12b) is roughly equal to one third that for (12d), but is an order of magnitude or more larger than the cross sections for reactions (12a) and (12c). (The relevant data are given in Ref. 7.) Again, however, the highest energies at which data are available are of the order of 2500 MeV, only about 500 MeV above the thresholds for reactions (12a) and (12c), and our model suggests that the mass splittings alone could cause order-of-magnitude discrepancies in the cross sections that close to threshold. Moreover, the extent of the discrepancy is again quite sensitive to the way in which one carries out the comparison. For example, the cross section, multiplied by F, for reaction (12c) at 2314 MeV is 0.062 ± 0.015 mb. This is to be compared with the similar quantity for reaction (12b), which is 6.0 ± 1.0 mb at the same Q value, but only 0.24 ± 0.05 mb at the same total energy and initial kinetic energy, so that the discrepancy changes from a factor of 100 to a factor of 4 depending on which way one makes the comparison, again suggesting that we are not above the energy range where the kinematic effects of the mass splittings are very large.

The one case in which there is a discrepancy at center-of-mass energies considerably larger than the masses of the particles is pointed out in Ref. 6. The cross sections for

$$\pi^+ + p \to \pi^+ + p\,, \tag{13a}$$

$$K^+ + p \to K^+ + p\,, \tag{13b}$$

at center-of-mass energies of 4 to 5 BeV differ by about a factor of 2, when they are predicted to be essentially equal because of the small cross section for $\pi^+ + p \to K^+ + \Sigma^+$ at these energies. Since these are both elastic scattering reactions, in this case one can make $k = k'$ and $K = K'$ simultaneously. There are, however, other coupled channels for which the channel momenta in the two reactions will not be equal, and, as we have mentioned earlier, the amplitudes for (13a) and (13b) will depend strongly on these other variables as well. It therefore seems reasonable that the effects of the mass splittings will be comparable to those observed in our model, in which case they could well account for differences in the amplitudes of the order of 50% at this energy. Even at 5 BeV the parameter m/k in Eq. (6′) is 20%, so that effects of this order of magnitude due to the different momenta in channels coupled to (13a) and (13b) can certainly not be excluded. There is also a violent discrepancy between the cross sections for reactions (13a) and (13b) at low energies. This, however, seems to be clearly due to the fact that the 3-3

[8] A. Friedman, thesis, CERN, 1964 (unpublished).

[9] M. Ferro-Luzzi, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters 8, 28 (1962).

[10] W. A. Cooper, H. Courant, H. Filthuth, E. I. Malamud, A. Minguzzi-Reinzi, H. Schneider, A. M. Segar, G. A. Snow, W. Willis, E. S. Gelsema, J. C. Kluyver, A. G. Tenner, K. Browning, I. S. Hughes, and R. Turnbull, *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 298.

resonance occurs below threshold for reaction (13b), while it is above threshold for (13a) due to the fact that the resonance is also coupled to the $K^+\Sigma^+$ channel with its high threshold. This is a particularly clear example of the large deviations from symmetry that can be produced by the mass differences.

The remaining discrepancy which has been reported[5] is in $p\bar{p}$ interactions at a center-of-mass energy of 2700 MeV. As it involves only a factor of about 2 in cross sections, and the energy is not particularly large compared to the masses involved, it is clear that this can easily be accounted for by the mass differences.

To summarize, it would appear that none of the reported discrepancies between $SU(3)$ and the results of scattering experiments are so large, considering the energies at which the experiments have been done, that they might not be due entirely to the effects of the mass differences within multiplets, with no other large symmetry breaking mechanism required. Conversely, the cases in which agreement has been found[4,6] are quite probably fortuitous. Because our results would indicate that there are uncertain, but probably quite large, effects due simply to the mass differences, it would seem that scattering experiments may not be a very fruitful way either of gaining evidence for $SU(3)$ or of studying the nature of its violations. In any event, data will be needed at considerably higher center-of-mass energies than those at which experiments have now been done.

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Quadrilocal Model of Baryons and Unitary Symmetry

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A unified theory of baryons is proposed based on a spinor wave equation that depends on four space-time points or equivalently on the center of mass and three relative-coordinate vectors. The associated subsidiary condition and the structure of the mass operator are such that the four-point association is maintained within a small region of Minkowski space-time with characteristic length and that the theory has $U(9)$ symmetry in the full symmetry limit. By the couplings of internal motions this symmetry is reduced to the direct product of the usual unitary-spin group $U(3)$ and the other unitary group $U(3)'$ characteristic of spherical-oscillator-type motions, and then this latter is further reduced to simple rotational invariance. Baryonic states are assigned to the 165-dimensional irreducible representation (IR) of the $U(9)$ corresponding to the first excited shell with respect to the oscillatory motions of relative coordinates. These states are subgrouped according to the IR of the usual $SU(3)$ and to the eigenvalue of the relative angular momentum. Identifications with known levels are then made. The whole treatment is carried out covariantly, and minimum violation of causality is implied inside the particle.

INTRODUCTION

We propose in this paper a unified theory of elementary particles, specifically of baryons, based on the hypothesis that a particle has a configuration represented by four space-time points $y_\mu{}^\alpha$ ($\alpha=1, \cdots, 4$). This just doubles the coordinates describing an elementary particle as compared with the bilocal model[1] of Yukawa.

The attractive feature of our theory lies in the fact that it represents the simplest possible model endowing an elementary particle with *full* and finite extension in space-time in conformity with relativistic covariance, that the usual $U(3)$ symmetry together with its breakdown is directly ascribed to this space-time nature of particles (rather than to the characteristics of meson-baryon interactions), and that internal attributes such as charge and hypercharge are reduced to quantized internal motions themselves,[2] in contradistinction with the viewpoint of the usual composite models.[3]

Furthermore, our model implies underlying broken $U(9)$ symmetry such that its irreducible representation (IR) $(3,0,0,\cdots,0)$ groups together, baryon supermultiplets belonging to different relative orbital angular-momentum states.

[1] H. Yukawa, Phys. Rev. **77**, 219 (1950); **80**, 1047 (1950); **91**, 415 (1953); Progr. Theoret. Phys. (Kyoto) **31**, 1167 (1964); M. Markov, Nuovo Cimento Suppl. **3**, 760 (1956).

[2] T. Takabayasi, Nuovo Cimento **33**, 668 (1964).

[3] S. Sakata, Prog. Theoret. Phys. (Kyoto) **16**, 686 (1956); M. Gell-Mann, Phys. Letters **8**, 214 (1964).