Self-Consistent Particle Sets in a Baryon Bootstrap Model*

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(Received 26 April 1965)

An $SU(n)$-symmetric bootstrap model is considered in which the baryons are meson-baryon bound states. A postulate used previously in similar models of mesons is applied: that a solution to the dynamical equations should exist in which particles of the same type are degenerate. It is shown that this postulate is difficult to satisfy unless the baryons correspond to a single irreducible representation of the $SU(n)$ group. The π-N-N^* system is discussed in detail. The condition that the baryons correspond to a single irreducible representation does not favor a particular value of n for the underlying $SU(n)$ symmetry, but does rule out the fundamental representation for the baryons, and favors the group $SU(mn)$ over $SU(m)\otimes SU(n)$. Several relations between $SU(4)$ and $SU(6)$ symmetry schemes are pointed out.

I. INTRODUCTION

One of the principal goals of bootstrap theory is to predict the quantum numbers of the existing set of strongly interacting particles. Progress toward this goal may result from approximate bootstrap models, if reasonable self-consistency conditions are formulated that are satisfied by a system of particles that corresponds to reality, and are not satisfied by a large class of hypothetical systems of particles. If every particle is allowed to have a different mass, it is difficult to achieve this type of progress at the present time. The difficulty results from the inaccuracy of existing dispersion-theoretic techniques, and from the fact that the self-consistency equations must depend on the detailed energy dependence assumed for some of the amplitudes, if all the masses are different.

In the last few years a certain amount of progress concerning consistent and inconsistent sets of particles has been achieved, particularly in models involving only mesons.[1] These results depend on the assumption that particles produced from the same type of process in amplitudes involving the same phase-space factors are equivalent in two ways. First, all cutoff and subtraction constants introduced into the partial-wave dispersion relations that represent the dynamics must be taken to have the same value for equivalent particles. Second, a solution in which all equivalent particles are degenerate is assumed to exist. If this equivalence assumption is made, self-consistency conditions may be formulated that do not depend on the detailed nature of the dispersion relations. We are not concerned here with the mechanism that leads to the observed mass-splitting of multiplets of particles.

In this paper we formulate self-consistency conditions that are based on the above equivalence assumption, and apply the conditions to a bootstrap model of the baryons assumed to correspond to meson-baryon bound states. The main purpose is to find some properties of the meson and baryon multiplets that may be determined from self-consistency. The nature of the model and the conditions are discussed in Sec. II. In Sec. III the π-N-N^* system is examined and found not to satisfy the conditions. Some other systems of particles based on simple representations of $SU(n)$ are discussed in Sec. IV. The $SU(6)$-symmetric system discussed previously by one of the authors is a solution.[2,3]

II. THE SELF-CONSISTENCY CONDITIONS

We consider a set of baryons B_i that correspond to bound-state poles in the various reaction amplitudes involving meson-baryon states of the type μ_j+B_k. The forces that produce the poles are assumed to result from the exchange of the baryons. The baryons are all equivalent, and the mesons are equivalent, in the sense of Sec. I. The orbital angular momenta of the different μB states must be the same, but different total angular momenta (baryon spins) are allowed, if the static approximation is made. The scattering amplitudes may be represented by a matrix T; the amplitudes are defined so that they do not possess poles or zeros at the μB threshold.

We are concerned only with the questions of the self-consistency of various schemes of particles, and the ratios of interaction constants. These questions depend only on the salient features of the dynamics. Each of the elements of the scattering matrix possesses a right-hand cut and left-hand cut. The discontinuity across the right-hand cut is given by the many-channel two-particle unitarity condition, and that across the left-hand cut is given by the B exchange mechanism. Because of the equivalence assumption, the left-hand discontinuity matrix ΔT may be written in the form

$$\Delta T = i\beta(E)F\,, \tag{1}$$

where $\beta(E)$ is a scalar function of the total energy and F is a constant matrix. The elements of F are quadratic functions of the interaction constants. It is not neces-

* Supported in part by the National Science Foundation.
[1] See, for example, R. H. Capps, Phys. Rev. Letters 10, 312 (1963); R. E. Cutkosky, Phys. Rev. 131, 1888 (1963); Chan Hong-Mo, P. C. De Celles, and J. E. Paton, Phys. Rev. Letters 11, 521 (1963).
[2] R. H. Capps, Phys. Rev. Letters 14, 31 (1965).
[3] J. G. Belinfante and R. E. Cutkosky, Phys. Rev. Letters 14, 33 (1965).

sary to know the form of $\beta(E)$; one needs only the condition that the potential is of definite sign in the energy region of the bound-state poles. The specific condition is that there is an unambiguous choice of sign of the function $\beta(E)$ such that a positive diagonal element of F corresponds to an attractive force.

The matrix F may be diagonalized by a real, orthogonal transformation. If F is diagonal, the inelastic amplitudes have zero left-hand discontinuities and must vanish, so that T is also diagonal. A resonance must correspond to a positive eigenvalue of F. We assume that the resonance energy is a monotonic decreasing function of the strength of the force. Hence, the equivalence of the different particles in B_i implies that the appropriate eigenvalues of F are equal; furthermore the eigenvalues corresponding to all nonresonating channels must be smaller than those corresponding to the resonances.

We define the interaction constant $\gamma_{\alpha i}$, where α refers to a baryon B_α, and i refers to a μB state, by the equation

$$\gamma_{\alpha i}\gamma_{\alpha i'}=-KR_{ii'}{}^{\alpha},$$

where $R_{ii'}{}^{\alpha}$ is the residue of the pole corresponding to B_α in the scattering amplitude $(\mu B)_i \rightarrow (\mu B)_{i'}$, and K is a conveniently chosen positive constant. If the label i refers to a simple product state, i.e., $(\mu B)_i=\mu_j+B_k$, then $\gamma_{\alpha i}$ is a conventional three-particle interaction constant. The symbol λ_α is used to denote the element of F associated with the channel α in the representation in which F is diagonal. If a bound state occurs in the channel α, then F_α must be positive. The definition of $\gamma_{\alpha i}$ leads to the relation,

$$\gamma_{\alpha i}=C(\lambda_\alpha)^{1/2}O_{\alpha i},$$

where C is a constant, and $O_{\alpha i}$ is the element of the orthogonal transformation matrix that relates the physical state i with the bound state eigenstate α. The self-consistency conditions discussed above then lead to the equations

$$\gamma_{\alpha i}/\gamma_{\alpha i'}=O_{\alpha i}/O_{\alpha i'}\,, \tag{2}$$

$$\lambda_\alpha=\lambda_{\alpha'} \tag{3}$$

$$\sum_i \gamma_{\alpha i}{}^2=\sum_i \gamma_{\alpha' i}{}^2\,, \tag{4}$$

$$\lambda_\nu<\lambda_\alpha\,, \tag{5}$$

where α and α' refer to particles in the B multiplet, and λ_ν is an eigenvalue of F associated with a channel in which no bound state occurs. These conditions are applied to various specific models in Sec. III and Sec. IV.

III. THE π-N-N^* MODEL

In this section we apply the techniques of Sec. I to the static, reciprocal bootstrap model of the N and N^* particles, in the approximation in which the N and N^* are degenerate.[4] Our treatment differs from the treatment of Ref. 4, in that we include πN^* scattering states, as well as πN states. The N and N^* are equivalent. The underlying symmetry is $SU(2)\otimes SU(2)$.

In order to compute the F matrix of Eq. (1), we must consider both N and N^* exchange forces. The strengths of these contributions are determined by the crossing submatrices corresponding to the 11 and 33 states (where the indices denote twice the isospin and twice the spin), and by the coupling constants $\gamma_{N\pi N}$, $\gamma_{N^*\pi N}$, and $\gamma_{N^*\pi N^*}$. The crossing matrices for the different processes, determined from standard group-theoretical techniques, are

$$\begin{aligned} C(\pi N\rightarrow\pi N)&=\frac{1}{9}\times\begin{pmatrix}1 & 16\\ 4 & 1\end{pmatrix},\\ C(\pi N\rightarrow\pi N^*)&=\frac{1}{18}\times\begin{pmatrix}8 & 20\\ 5 & 8\end{pmatrix},\\ C(\pi N^*\rightarrow\pi N^*)&=\frac{1}{900}\times\begin{pmatrix}25 & 400\\ 100 & 484\end{pmatrix},\end{aligned} \tag{6}$$

where the first row and column refer to the 11 state and the second row and column to the 33 state. It is convenient to reduce the force matrix into two submatrices, referring to the $N(11)$ and $N^*(33)$ states. These matrices, determined from Eq. (6) are

$$\begin{aligned} F_N&=\frac{1}{9}\times\begin{pmatrix}a^2+16b^2 & 8ab+10bc\\ 8ab+10bc & b^2+4c^2\end{pmatrix},\\ F_{N^*}&=\frac{1}{9}\times\begin{pmatrix}4a^2+b^2 & 5ab+4bc\\ 5ab+4bc & 4b^2+(121/25)c^2\end{pmatrix},\\ a&=\gamma_{N\pi N}\,,\quad b=\gamma_{N^*\pi N}\,,\quad c=\gamma_{N^*\pi N^*}\,,\end{aligned} \tag{7}$$

where the first and second rows and columns now refer to πN and πN^* states, respectively. The condition $\gamma_{N\pi N^*}=2\gamma_{N^*\pi N}$, which results from the greater statistical weight of the 33 state, has been used in the derivation of Eq. (7).

We now apply the self-consistency conditions of Sec. I. Henceforth, elements of the F matrices are denoted by three subscripts, the first referring to the spin and isospin, the second to the initial scattering configuration, and the third to the final configuration. The index 1 refers to the 11 state and to the πN configuration. The condition of Eq. (5) requires that the eigenvalues λ_1 and λ_2 corresponding to the N and N^* poles must be the largest eigenvalues of F_1 and F_2, i.e.,

$$\lambda_\alpha=\tfrac{1}{2}(F_{\alpha 11}+F_{\alpha 22})+[\tfrac{1}{4}(F_{\alpha 11}-F_{\alpha 22})^2+F_{\alpha 12}{}^2]^{1/2}. \tag{8}$$

The coefficients $O_{\alpha i}$ of the column of the diagonalizing matrix corresponding to λ_α may be determined by

[4] Geoffrey F. Chew, Phys. Rev. Letters 9, 233 (1962).

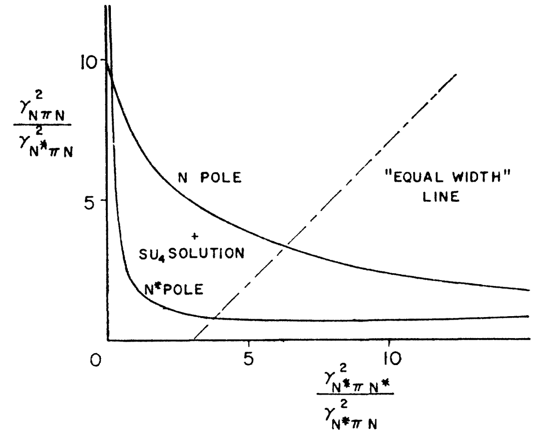

FIG. 1. The three self-consistency equations for the π-N-N^* model. The N pole, N^* pole, and "equal width" equations are Eqs. (9), (10), and (11), respectively.

standard methods. The self-consistency equation, Eq. (2), then leads to the two conditions,

$$\gamma_{N\pi N}/(2\gamma_{N^*\pi N}) = F_{121}/(\lambda_1 - F_{111}) = [X_1 + (X_1^2+1)^{1/2}]^{-1}, \quad (9)$$

$$\gamma_{N^*\pi N}/\gamma_{N^*\pi N^*} = F_{221}/(\lambda_2 - F_{211}) = [X_2 + (X_2^2+1)^{1/2}]^{-1}, \quad (10)$$

$$X_i = \tfrac{1}{2}(F_{i22} - F_{i11})/F_{i12}.$$

The phases of the various states may be chosen so that $\gamma_{N\pi N}$ and $\gamma_{N^*\pi N}$ are positive. It may be shown from Eqs. (8), (9), and (10) that $\gamma_{N^*\pi N^*}$ must then also be positive.

The third self-consistency condition follows from Eq. (4); i.e.,

$$\gamma_{N\pi N}^2 + 4\gamma_{N^*\pi N}^2 = \gamma_{N^*\pi N}^2 + \gamma_{N^*\pi N^*}^2. \quad (11)$$

This equation, and Eqs. (9) and (10), may be regarded as three equations for the two ratios $\gamma_{N\pi N}^2/\gamma_{N^*\pi N}^2$ and $\gamma_{N^*\pi N^*}^2/\gamma_{N^*\pi N}^2$. The solution to each of the equations is plotted in Fig. 1; it is seen that the equations are not compatible.

The failure of the π-N-N^* model is easily understood, since there are more self-consistency relations than variables. It is difficult to satisfy all the relations in such a model unless some underlying symmetry is present. If a triplet of vector mesons (interacting in states of total angular momentum 0) and a pseudoscalar singlet, are added to the model, a solution does exist, in which the ratios of interaction constants are given by $SU(4)$ symmetry. This solution is analogous to the $SU(6)$-invariant solution discussed in Refs. 2 and 3. If the V triplet is denoted by ρ, the ratios of the ρNN, ρN^*N^*, πNN, πNN^*, and πN^*N^* interaction constants are the same in the $SU(4)$ and $SU(6)$ models. This follows from the fact that $SU(4)$ is a subgroup of $SU(6)$, together with the fact that these particles of the $SU(4)$ model may be identified unambiguously from their quantum numbers with specific particles of the $SU(6)$ model. On the other hand, the squares of the various X_0/π interaction ratios of the $SU(4)$ model (where X_0 is the P singlet) are equal to the sum of the squares of the corresponding X_0/π and η/π ratios of the $SU(6)$ model. The $SU(4)$ solution for the π interaction ratios is denoted by a cross in Fig. 1. This solution is discussed further in Sec. IV.[5]

It is interesting to compare the present model with that introduced by Chew, in which only πN scattering states are included.[4] The self-consistency conditions often used in the πN model involve a specific assumption concerning the energy dependence of the amplitudes; these conditions are quite different from those used here. The conditions used in Ref. 4 are equivalent to the requirement that the crossing submatrix referring to the 11 and 33 channels possesses an eigenvalue of one.[6] This condition is satisfied in the πN model. On the other hand, if πN^* states are included and πN states neglected, this type of model is inconsistent, as the maximum eigenvalue of the crossing submatrix is ≈ 0.62. In this case, the coefficients of Eq. (6) may be used to show that the two self-consistency equations of the type used in Ref. 4 lead to very different values for the ratio $\gamma_{N\pi N^*}^2/\gamma_{N^*\pi N^*}^2$, i.e., 16/35 and 104/25.

The fact that the crossing submatrix for the πN channels has an eigenvalue of one results from the circumstance that there are only two πN states involved in the spin and isospin crossing matrices, together with the fact that these matrices are identical. The two-by-two spin (or isospin) crossing matrix C' satisfies the condition $C'^2 = 1$. Since C' is not a multiple of the unit matrix, it must have one eigenvalue of (1) and one of (-1), and thus must be traceless. This matrix may be written,

$$C' = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}.$$

The condition that $C'^2 = 1$ implies the relation, $\alpha^2 + \beta\gamma = 1$. If the α^2 term is transposed and the equation is squared, the result may be written,

$$\text{determinant}\begin{pmatrix} \alpha^2 - 1 & \beta^2 \\ \gamma^2 & \alpha^2 - 1 \end{pmatrix} = 0.$$

This is just the condition that the πN crossing submatrix referring to the 11 and 33 channels possess an

[5] K. Y. Lin and R. E. Cutkosky, Phys. Rev. (to be published), have investigated the π-N-N^* bootstrap model without assuming N-N^* degeneracy. Certain reasonable approximations to the dispersion integrals are made. A solution is found, in which the N^*-N mass difference is positive. If this solution does not correspond to a solution of a more exact (future) dispersion theory, a violation of the hypothesis of Sec. I is not implied. The $SU(4)$ solution may be regarded as the degeneracy solution of the π-N-N^* model, because the mesons are not bootstrapped, so that the extra mesons of the $SU(4)$ model play only the role of modifying the various forces.

[6] See I. S. Gerstein and K. T. Mahanthappa, Nuovo Cimento 22, 239 (1964).

eigenvalue of one, since the elements of this matrix are the squares of the corresponding elements of C'.

We conclude that the consistency of the Chew model of the πN amplitudes, and the inconsistency of a similar model of the πN^* amplitudes, are not related directly to the positive, physical N^*-N mass difference (the reason that the πN channel is preferred in the conventional treatment). There may be an indirect connection, however, for the positive N^*-N mass difference may result from the different structures of the πN and πN^* crossing matrices.[7]

IV. VARIOUS MESON-BARYON SCHEMES

As pointed out in Sec. III, there are more self-consistency equations than there are coupling constant ratios in our model, so that self-consistency is difficult to achieve unless each set of equivalent particles corresponds to a single irreducible representation of a Lie group. In this section, we postulate such a requirement and examine some schemes in which the mesons and baryons each correspond to a single irreducible representation of $SU(n)$ or of $SU(m)\otimes SU(n)$. It is assumed that the interactions are invariant to the group transformations, and that baryon exchange forces produce the baryon poles in the coupled μB states. The group symmetry implies that Eqs. (2), (3), and (4) are satisfied automatically, so that only the one self-consistency equation, Eq. (5), need be examined. This equation is equivalent to the condition that the largest element of that column of the μB crossing matrix associated with B exchange must occur in the representation of the B.

We consider only small representations of the groups. Since the $BB\mu$ interaction exists, the mesons must be associated with a self-conjugate representation of plurality zero. The Young diagrams for such representations contain mn boxes, where m is an integer. The simplest representation of this type (representation with the smallest dimension) corresponds to n boxes in a single column. This is the identity representation. If there is only one meson state, different baryons are not coupled, so there is no group structure at all to the interactions. Hence, we identify the mesons with the next simplest representation of the proper class, which involves a Young diagram with $(n-1)$ boxes in the first column, and one in the second. This is the regular representation, with (n^2-1) states.

The baryons must be identified with a representation B such that the direct product $B\otimes B^*$ contains the regular representation at least once. (The symbols B and μ are used to denote the baryon and meson representations, as well as the particles themselves.) This requirement allows any representation for B except the identity representation. This follows from the fact that the matrix elements of the n^2-1 generators of $SU(n)$ in the space of B all vanish if and only if B is the identity representation.

We consider first the simplest schemes based on $SU(2)$-different nucleon multiplets and a meson triplet. The quantum number may be regarded as the z component of the spin in the static model. The meson may be a pseudoscalar meson, with the unit orbital angular momentum playing the role of the meson spin. The simplest possibilities for B are the doublet, triplet, and quadruplet representations. The elements of the crossing matrices associated with baryon exchange for these cases are

$$\begin{array}{llll} B\text{ doublet}, & C_2=-\tfrac{1}{3}, & C_4=\tfrac{2}{3} & \\ B\text{ triplet}, & C_1=-1, & C_3=\tfrac{1}{2}, & C_5=\tfrac{1}{2} \\ B\text{ quadruplet}, & C_2=-\tfrac{2}{3}, & C_4=11/15, & C_6=\tfrac{2}{5}, \end{array} \tag{12}$$

where the subscript denotes the multiplicity of the meson-baryon state. A positive C_i corresponds to an attractive force. It is seen that the self-consistency condition of Eq. (5) is violated if the nucleon spin is $\frac{1}{2}$ or 1, and is satisfied if the nucleon spin is $\frac{3}{2}$. Thus, the simplest solution associated with $SU(2)$ occurs when the baryons correspond to the completely symmetric representation formed from trilinear combinations of the states of the fundamental representation. We call this the cubic representation.

We next consider some hypothetical schemes in which the particles are associated with irreducible representations of $SU(n)$, where $n>2$. We are not concerned with the physical meaning of the quantum numbers. There are many possible representations that may be identified with the baryons in such schemes. Only two are considered here, the fundamental and cubic representations. The elements of the crossing matrices corresponding to baryon exchange may be computed from the formula of Belinfante and Cutkosky,[3] i.e.,

$$C_i=R[X_i\delta_{i,B}+\tfrac{1}{2}(X_i-X_B-X_\mu)], \tag{13}$$

where X_i is the value of the quadratic Casimir operator for the representation i, and R is a normalization constant. We now determine R from the sum rule $\sum_i W_iC_i=W_B$, where W_i is the multiplicity of the representation i.[8] If one chooses n^2-1, properly normalized, Hermitian operators J_α as generators of the group, the quadratic Casimir operator is $\sum_\alpha J_\alpha^2$, and the bracket term $\frac{1}{2}(X_i-X_B-X_\mu)$ of Eq. (13) is the eigenvalue in the state i of the operator $\sum_\alpha J_\alpha^\mu J_\alpha^B$. This bracket term does not contribute to $\sum_i W_iC_i$, since this sum is the trace of the operator C over the μB product space. From these results, it is clear that

[7] This occurs in the calculation of Lin and Cutkosky, Ref. 5. A simple reason that the state of smaller statistical weight tends to be lightest in "reciprocal" bootstrap models is pointed out by R. H. Capps, Nuovo Cimento 34, 932 (1964).

[8] R. H. Capps, Phys. Rev. 134, B460 (1964).

$R=X_B^{-1}$. A general expression for X_i is given by Whippman.[9]

We denote an irreducible representation of $SU(n)$ (for $n>2$), by the conventional symbol $(\lambda_1,\lambda_2,\cdots\lambda_{n-1})$, where λ_i is the difference in the lengths of the i and $i+1$ rows of the appropriate Young tableau. A calculation shows that if the baryons correspond to the fundamental representation $(1,0,\cdots 0)$, the elements of the crossing matrix associated with baryon exchange are,

$$C_{(1,0,\cdots 0)}=-1/(n^2-1)\,,$$
$$C_{(2,0,\cdots 1)}=-C_{(0,1,\cdots 1)}=n/(n^2-1)\,. \qquad (14)$$

Our notation is such that if $n=3$, the integer b in the representation symbol (a,b) must be taken as the sum of the second and last integers of the general symbol used here. If $n\geqslant 4$, the dots represent $n-4$ zeros. Since $C_{(1,0,\cdots 0)}$ is negative, this general scheme is inconsistent.

If the baryons correspond to the cubic representation $(3,0,\cdots 0)$, the elements of the crossing matrix corresponding to baryon exchange are,

$$C_{(3,0,\cdots 0)}=(2n+6-9/n)N\,,\quad C_{(2,1,\cdots 1)}=-N\,,$$
$$C_{(1,1,\cdots 0)}=-(n+3)N\,,\qquad C_{(4,0,\cdots 1)}=3N\,,\qquad (15)$$
$$N=n/[3(n+3)(n-1)]\,.$$

These schemes are consistent, since the largest positive element is $C(3,0,\cdots 0)$. If $n=6$, this is the scheme of Refs. 2 and 3; if $n=4$, this is the ρ, π, X_0, N, N^* model discussed in Sec. III. It is seen that the bootstrap hypothesis does not lead to a favored value of n, but does lead to a preference of the cubic to the fundamental representation for the baryons.

We next consider the direct-product group $SU(2)\otimes SU(3)$. The $SU(2)$ and $SU(3)$ may be associated with the spin and internal symmetry, respectively. The mesons are identified with the regular representation i.e., the spin-1 octet. In this type of scheme a crossing matrix element is the product of the appropriate elements of $SU(2)$ and $SU(3)$ crossing matrices. The principal new feature is that an attractive force may result from the product of negative elements of the $SU(2)$ and $SU(3)$ matrices. Because of this feature, self-consistency is difficult to achieve. All cases have been examined in which the baryon spin is $\frac{1}{2}$, 1, or $\frac{3}{2}$ and the $SU(3)$ representation is $(1,0)^3$, $(2,0)^6$, $(1,1)^8$, $(3,0)^{10}$, or $(2,1)^{15}$. If the representation μ is contained more than once in $B\otimes B^*$, the μBB interactions are taken to be F type (proportional to the appropriate matrix elements of the generators of the group). There is only one exception to the following rule: If the baryon-exchange force in the representation B is attractive, there is another representation I such that the crossing matrix inequality $C_I/C_B>0.8$ is satisfied. The exception is the case in which the $SU(3)$ and $SU(2)$ multiplicities of B are 10 and 4. In this case, the most attractive forces are in the states of multiplicities (10,4) and (8,2). The ratio $C_{(8,2)}/C_{(10,4)}$ is approximately equal to 0.6. Thus, this scheme is consistent, but not very satisfactory. It would be difficult to construct a dynamical model in which the spin-$\frac{3}{2}$ decuplet bootstrapped itself and the spin-$\frac{1}{2}$ octet did not resonate.

On the other hand, in the "physical" $SU(6)$ scheme, with the baryons identified with representation $(3,0,0,0,0)^{56}$, the attractive force in the representation B is $5\frac{1}{2}$ times as large as the only other attractive force, as seen from Eq. (15). Thus, $SU(6)$ is favored over $SU(2)\otimes SU(3)$. However, the bootstrap hypothesis does not favor $SU(6)$ over all other $SU(n)$ as the basic symmetry group. Furthermore, there are many possible identifications of B within the $SU(6)$ scheme that lead to consistency. It can be shown, however, that if B is limited to representations that may be formed from the direct product of one, two, or three of the fundamental representation $(1,0,0,0,0,)^6$, and $C_B>0$, and C_I is the largest crossing matrix element other than C_B, then the greatest value of C_B and the smallest value of C_I/C_B occur if B is the 56-fold representation $(3,0,0,0,0,)^{56}$. This latter distinction is not very great in all cases. For example, if B is the 20-fold multiplet $(0,0,1,0,0,)^{20}$, then it can be shown that $C_B=3/7$ and $C_I/C_B=2/9$

We now turn our attention to a brief discussion of mass splitting. The physically observed mass splitting of the meson and baryon multiplets is characterized by the fact that the Clebsch-Gordan coefficients corresponding to the mutual interactions among the lightest of the various $SU(3)$ multiplets are very large.[10] It was pointed out in Ref. 10 that in certain hypothetical schemes of particles based on representations of Lie groups, no type of mass splitting could lead to such large mutual interactions of the light particles. It was postulated that this fact may be one of the basic reasons that the physical set of particles exists.

The above considerations may be stated in a more explicit manner, in terms of representations of $SU(6)$ and its subgroup $SU(4)$. All representations of $SU(6)$ that may be formed from three or fewer fundamental representations [one of which may be of the conjugate type $(0,0,0,0,1)^6$] are symbolized by $(a,b,c,0,d)$, where either c or d is zero. For each such representation there is an $SU(4)$ representation with a similar structure, the representation $(a,b,c+d)$. The $SU(4)$ meson-baryon assignments, $\mu=(1,0,1)^{15}$ and $B=(3,0,0)^{20}$ correspond to the physical scheme based on $SU(6)$. The $SU(2)\otimes SU(2)$ structure of the representations, $(1,0,1)^{15}$ and $(3,0,0)^{20}$ are $(3,3)+(3,1)+(1,3)$ and $(4,4)+(2,2)$, respectively. These multiplets correspond exactly to the lightest isotopic spin multiplets in each of the meson and baryon $SU(3)$ multiplets, i.e., to the π, ρ, X_0, N^*,

[9] M. L. Whippman (to be published). The authors would like to thank J. G. Belinfante and R. E. Cutkosky for this information.

[10] R. H. Capps, Phys. Rev. 137, B125 (1965).

and N.[11] This type of mass-splitting is impossible if the baryon representation is any of the other three-quark representations of $SU(6)$, i.e., $(1,1,0,0,0)^{70}$, $(0,0,1,0,0)^{20}$, $(2,0,0,0,1)^{120}$ or $(0,1,0,0,1)^{84}$, for in these cases the number of $SU(2)\otimes SU(3)$ multiplets in the $SU(6)$ representation is greater than the number of $SU(2)\otimes SU(2)$ multiplets in the corresponding representation of $SU(4)$.

In conclusion, the requirement that the baryon multiplet correspond to a single irreducible representation favors strongly the group $SU(6)$ to the direct product group $SU(2)\otimes SU(3)$ as the basic symmetry group. The 56-fold representation is favored over other simple representations for the baryons, but not very strongly in some cases. However, the bootstrap hypothesis does provide a possible reason for the nonexistence of the fundamental multiplet of $SU(6)$.

[11] Since the mesons are emitted in P waves in the static limit, it is convenient to identify the spin-1 singlet with the pseudoscalar X_0 particle, rather than with a vector meson. This point is discussed in detail in Refs. 2 and 3.

ACKNOWLEDGMENT

The authors would like to thank Professor P. T. Matthews for the hospitality at Imperial College, London, where part of this work was done.

PHYSICAL REVIEW VOLUME 139, NUMBER 5B 6 SEPTEMBER 1965

Dynamical Basis for η-Baryon Interactions*

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(Received 23 April 1965)

The recent experimental evidence for a rapid rise in η production near threshold in the reactions $\pi^-+p\rightarrow\eta^0+n$ and $K^-+p\rightarrow\eta^0+\Lambda$ is analyzed on the basis of a constant-K-matrix formalism. Representative scattering lengths for the η-baryon systems are introduced to illustrate pertinent features of the presently known experimental data. A dynamical basis for the physical situation is discussed in terms of possible new baryon states due to virtual or bound states of the η-baryon systems. Comparison of the results with expectations from a conjectured η octet of baryon ($\frac{1}{2}$-) states for $SU(3)$ symmetry is briefly referred to.

1. INTRODUCTION

It is well known that because of the lack of orbital angular momentum to give centrifugal barrier containment, S-wave pseudoscalar meson-baryon interactions do not in general form favorable situations for resonances or peak enhancements. Some years back Dalitz and Tuan[1] showed that for a coupled two-channel problem, owing to strong attractive forces in the closed channel in the S state, a quasi-bound state can be formed which will *manifest* itself as an S-wave resonance in the open channel. Likewise, several papers have discussed threshold effects or cusps[2] in strong interactions involving particle reactions. It has been noted[3,4] that both classes of phenomena are attributable to poles of the S matrix in appropriate unphysical sheets of the two-channel problem. To establish an unambiguous terminology we call the former the quasi-bound-state problem with a corresponding *bound*-state pole, the latter, a virtual-state problem with a *virtual*-state pole. Threshold effects involving only particles stable in strong interactions do produce large cusp effects, when there exists a virtual-state pole in the S matrix *close* to an S-wave threshold on the unphysical sheet reached by passing through the branch cut associated with the threshold. Note that in both classes of phenomena, the existence of an $S_{1/2}$ resonance or cusp effect for an open meson-baryon channel is due to the dynamical influence from an additional second channel present. A simple illustration of the physics involved is to take the analogous situation of the ^3S (bound-state deuteron) and ^1S (virtual state of deuteron) of the n-p system; here only one channel, corresponding to the second of the above channels, is present.

Recently, Berley *et al.*[5] have raised the question whether the rapid rise and subsequent sharp drop in η^0 production (over a small range of energy) from $K^-+p\rightarrow\eta^0+\Lambda^0$ is a possible dynamical manifestation of a quasi-bound-state of the η-Λ system. Bulos *et al.*[6]

* Work supported in part by the National Science Foundation and the U. S. Air Force Office of Scientific Research.
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