

## $K^+p$ Interactions at 2 BeV/c—Elastic Scattering and Total Cross Sections\*

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Study of 1.96-BeV/c  $K^+p$  interactions in the Brookhaven 20-in. liquid-hydrogen bubble chamber has yielded a measurement of the elastic-scattering cross section  $\sigma = 7.5 \pm 0.7$  mb. A fit to the differential cross section versus momentum transfer of the form  $\sigma_0 e^{-\alpha t}$  gives, for the interval  $0.01 (\text{BeV}/c)^2 < t < 0.60 (\text{BeV}/c)^2$ ,  $\alpha = 3.1 \pm 0.3 (\text{BeV}/c)^{-2}$ , and  $\sigma_0 = 4.8 \pm 0.5$  mb/sr. The total cross section obtained is  $19.4 \pm 2.0$  mb with single-pion production dominant.

### I. INTRODUCTION

AT energies sufficiently above the threshold for inelastic processes, all elastic scattering shows a characteristic large forward "diffraction" peak. It is thought that the characteristics of the diffraction scattering may not depend on the properties of the particular particles involved. Unified descriptions of high-energy elastic scattering have been suggested by various authors and asymptotic formulas have been constructed.<sup>1</sup> To determine the validity of such descriptions, it is of interest to compare the diffraction scattering of various particles on protons, in the same regime of momentum transfer. We present here the results of a study of  $K^+p$  elastic interactions in the 20-in. hydrogen bubble chamber exposed to a separated  $K^+$  beam of momentum  $1.96 \pm 0.02$  BeV/c from the Brookhaven alternating gradient synchrotron.<sup>2</sup> We include also cross sections for the various inelastic reactions.

### II. SELECTION OF ELASTIC EVENTS

All two-prong events were measured on digitized projectors and analyzed with the reconstruction and fitting program PACKAGE.<sup>3</sup> Two major difficulties arise in the selection of the elastic events: (1) a low efficiency for finding events with short recoil protons, and (2) distinguishing between elastic scattering and  $\pi^0$  production  $K^+p \rightarrow K^+p\pi^0$ . To eliminate the first, a minimum projected track length was determined which made the scanning efficiency independent of track length. This determined a minimum scattering-angle cutoff  $\bar{\theta} = 7^\circ$  corresponding to a momentum-transfer-squared  $t = 2\bar{p}^2(1 - \cos\bar{\theta}) \cong 0.01 (\text{BeV}/c)^2$ , where  $\bar{\theta}$  and  $\bar{p}$  are,

respectively, the c.m. scattering angle and momentum, as well as a  $55^\circ$  maximum cutoff in the azimuthal angle  $\phi$  between the plane of the outgoing tracks and the plane of the four cameras (the plane of zero "dip" angle). This restriction on  $\phi$  also eliminates steep tracks, making the evaluation of relative bubble densities more reliable. Events satisfying these criteria and the kinematics of elastic scattering were usually kinematically consistent also with the reaction  $K^+p \rightarrow K^+\pi^0p$  and often with the inelastic reactions yielding  $\pi^+K^0p$  and  $\pi^+K^+n$ . In all cases  $\pi^+$  production could be distinguished from elastic scattering by observation of track bubble densities. This procedure was not useful for the hypothesis of single  $\pi^0$  production if neither outgoing track had momentum between about 400 and 1300 MeV/c or stopped in the chamber. The ambiguous  $K^+p\pi^0$  fits, however, almost invariably posited a  $\pi^0$  of  $\cos\theta < -0.95$  and  $\bar{p} < 100$  MeV/c. Since this pattern was also exhibited by the  $K^+p\pi^0$  fits rejected on the basis of bubble density and was inconsistent with smooth extrapolation of the distribution of uniquely identified  $K^+p\pi^0$  events, it is likely that such fits are spurious, resulting from inaccuracies of momentum measurement. The 80 ambiguous events of this type were thus included in the elastic-scattering group. In this way 636 events were accepted as elastic scatters. The  $\chi^2$  distribution is in good agreement with that expected for four constraint events. Of the two-prong events which failed to fit the hypotheses of either two or three particles in the final state, 34 were classified to have two neutral particles in the final state and 49 were immeasurable for technical reasons. The four-body final states were identified by computing the missing mass from the unbalance in momentum and energy. The number of events whose missing mass is consistent with either a neutral  $K^*$  or a neutral  $N^*$  is in agreement with the number expected from charge independence and the number of such resonant states found in the analysis of the four-charged-prong events.<sup>4</sup> Assuming the observed ratio of elastic to inelastic two-prong events to be the same for the immeasurable events, we take 22 of these to be elastic in our cross-section determination.

We evaluated the  $\pi^+$  contamination in the beam by fitting all four-prong events to the hypotheses of an

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<sup>1</sup> S. Fernbach, R. Serber, and T. B. Taylor, Phys. Rev. **75**, 1352 (1949); P. T. Matthews and A. Salam, Nuovo Cimento **21**, 127 (1961); **21**, 823 (1961); R. Serber, Phys. Rev. Letters **10**, 357 (1963); S. Minami (see Ref. 8); G. F. Chew and S. C. Frautschi, *ibid.* **7**, 394 (1961); S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962); G. F. Chew, S. C. Frautschi, and S. Mandelstam, *ibid.* **126**, 1202 (1962); R. Blankenbecler and M. L. Goldberger, *ibid.* **126**, 766 (1962); B. M. Udgaoonkar, Phys. Rev. Letters **8**, 142 (1962).

<sup>2</sup> T. A. O'Halloran, Jr., thesis, Lawrence Radiation Laboratory Report No. UCRL-11068, 1963 (unpublished).

<sup>3</sup> A. H. Rosenfeld, Lawrence Radiation Laboratory Report No. UCRL-9099, 1961 (unpublished).

<sup>4</sup> G. Goldhaber, S. Goldhaber, W. Chinowsky, W. Lee, and T. O'Halloran, Phys. Letters **6**, 62 (1963).

incident  $\pi^+$ . Four-prong events were used because the  $\pi^+p$  cross section for this topology is about three times as great as that for  $K^+p$ , thus giving an enriched sample of beam contaminants. Only one event satisfying entrance-angle and momentum criteria was found to favor incoming pion kinematics, yielding an estimated  $\pi^+$  beam contamination of  $(0.25_{-0.25}^{+0.5})\%$  which we neglect. This estimate is consistent with that obtained from the measured  $K-\pi$  separation<sup>2</sup> in the beam.

### III. TOTAL CROSS SECTION

Extrapolating the observed distribution of Fig. 1 to zero momentum transfer, assuming for the differential cross section an exponential dependence  $e^{-\alpha t}$ , which is in excellent agreement with observation for  $t < 0.6$  (BeV/c)<sup>2</sup>, gave 19 events to be added to the sample. Correcting for the azimuthal angle cutoff and the estimated fraction of immeasurable events which were elastic, we arrive at a total  $N_T = 1094$  events, for all  $t$  and  $\phi$ , in a predetermined fiducial volume of the hydrogen chamber. In the same volume were 164  $\tau$  decays from which we find the incident  $K^+$  flux.<sup>5</sup> From these data we evaluate the total elastic  $K^+p$  scattering cross section

$$\sigma_E = 7.5 \pm 0.7 \text{ mb},$$

where the error is statistical. Cook *et al.*<sup>6</sup> have reported a total elastic cross section of  $5.6 \pm 0.4$  mb at 1.97 BeV/c, not in gross disagreement with the present result.

### IV. DIFFERENTIAL CROSS SECTION

The observed angular distribution of the elastic scattering is shown in Figs. 1 and 2. We find the expected predominant diffraction peak in the forward direction. In the backward hemisphere the data are too sparse for detailed study. No attempt is made at a phase-shift analysis. Guided by "Regge" theory and an optical model discussed below, we seek to fit the observed angular distribution (below some upper limit of the momentum-transfer-squared  $t$ ) by an exponential form

$$d\sigma/d\Omega = \sigma_0 e^{-\alpha t}. \quad (1)$$

This is seen in Fig. 1 to fit the data well in the forward hemisphere. Because the average measurement error of  $t$  is not negligibly small compared with the width of the

<sup>5</sup> All three-prong  $K^+$  decays are included in this group. Branching ratios of Roe *et al.* [Phys. Rev. Letters 7, 346 (1961)], with corrections for Dalitz pair decays, were used in determining cross sections in this paper. Very recently two new determinations of the  $\tau$  branching ratio have appeared [Callahan *et al.* (1964) and Shaklee *et al.* (1964) as quoted in Rosenfeld *et al.*, Lawrence Radiation Laboratory Report No. UCRL-3030, I, 1964 (unpublished)]. Comparing all available results Rosenfeld *et al.* find a  $\tau$  branching ratio of  $0.055 \pm 0.001$  as compared with  $0.057 \pm 0.003$  of Roe *et al.* used in the present paper. The effect of this would be to decrease all the cross sections given in the present paper by a factor 59/61.

<sup>6</sup> V. Cook, D. Keefe, L. T. Kerth, P. G. Murphy, W. A. Wenzel, and T. F. Zipf, Phys. Rev. 129, 2743 (1963).

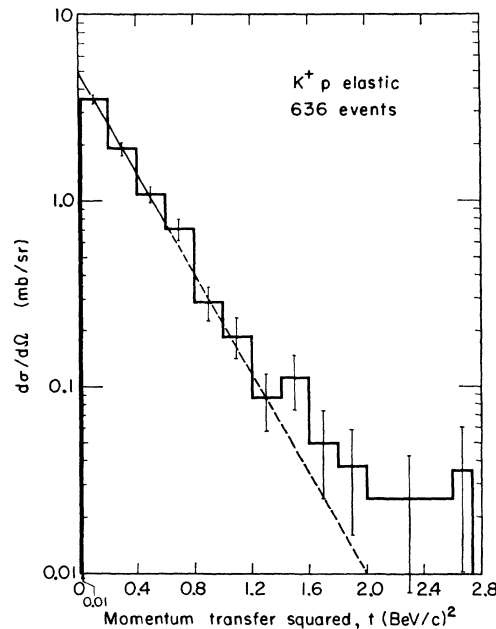


FIG. 1. Logarithmic distribution of 636 elastic-scattering events versus momentum-transfer-squared  $t$ . The straight line shows  $d\sigma/d\Omega = \sigma_0 \exp(-\alpha^* t)$  where  $\alpha^* = 3.1$  (BeV/c)<sup>-2</sup> is the maximum likelihood estimator [Eqs. (2)-(6)] for interval I [ $0.01 \leq t \leq 0.6$  (BeV/c)<sup>2</sup>], and  $\sigma_0 = 4.8$  mb/sr. The line is dashed in the extrapolated region beyond the fitted interval.

rapidly decreasing angular distribution, it is incorrect to obtain  $\alpha$  merely from the slope of a least-squares straight-line fit to the histogram of Fig. 1. The finite momentum-transfer resolution results in a net shift of the measured distribution relative to the true distribution in the direction of increasing  $t$ , that is, in the direction of decreasing cross section. We therefore obtain  $\alpha^*$ , the best estimate of  $\alpha$ , from a maximum-likelihood procedure which takes explicit account of the measurement errors and makes optimum use of the experimental information. The likelihood function has the form

$$L(\alpha) = \prod_j P_j(t_j, \sigma_j, \alpha), \quad (2)$$

where  $\prod_j$  is a product over all events in the  $t$  interval  $t_{\min}$  to  $t_{\max}$  being fitted to  $e^{-\alpha t}$ ;  $t_j$  and  $\sigma_j$  are, respectively, the measured  $t$  (from the kinematic fitting program) and its uncertainty for the  $j$ th event. For convenience we take the error distribution in  $t$  to be a truncated Gaussian, cut off on each side at either three standard deviations or the kinematic limit, whichever is reached first, and normalized accordingly. Then

$$P(t, \sigma, \alpha) = \frac{1}{N_\sigma(\alpha)} \int_{\xi = \max(0, t-3\sigma)}^{\min(\xi_{\max}, t+3\sigma)} Q(t, \xi, \sigma, \alpha) d\xi, \quad (3)$$

where  $\xi$  is the true value of  $t$  for the  $j$ th event, and  $\xi_{\max} = 2.74$  (BeV/c)<sup>2</sup> is the kinematic upper limit of  $\xi$  at 1.96 BeV/c. Here  $[1/N_\sigma(\alpha)]Q(t, \xi, \sigma, \alpha)d\xi dt$  is the probability that for a sample of events with arbitrary cutoffs

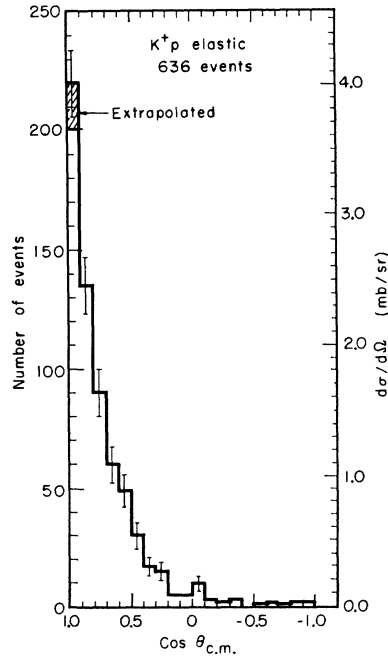


FIG. 2. Angular distribution of 636 elastic-scattering events. Shaded area indicates events deduced by extrapolation below cutoff  $\theta_{c.m.} = 7^\circ$ , assuming angular distribution  $\sim \exp(-\alpha t)$ .

$t_{min}, t_{max}$  an event will have true momentum transfer squared in the interval  $d\xi$ , and measured momentum transfer squared in the interval  $dt$ .  $Q$  has the form

$$Q(t, \xi, \sigma, \alpha) = e^{-\alpha \xi} \frac{\exp[-\frac{1}{2}(t-\xi)^2/\sigma^2]}{M(\sigma, \xi)}. \quad (4)$$

The normalization factors are

$$M(\sigma, \xi) = \int_{\tau=\max(0, \xi-3\sigma)}^{\min(\xi_{max}, \xi+3\sigma)} \exp[-\frac{1}{2}(\tau-\xi)^2/\sigma^2] d\tau, \quad (5)$$

and

$$N_\sigma(\alpha) = \int_{t=t_{min}}^{t_{max}} \int_{\xi=\max(0, t-3\sigma)}^{\min(\xi_{max}, t+3\sigma)} Q(t, \xi, \sigma, \alpha) dt d\xi. \quad (6)$$

This maximum-likelihood procedure yielded the results shown in Table I for three different intervals of momentum transfer.

If  $L(\alpha)$  has the Gaussian form

$$L(\alpha) \sim \exp[-\frac{1}{2}(\alpha - \alpha^*)^2/\Sigma^2]$$

expected for good statistics,  $\ln L$  will have the parabolic form  $\ln L = -\frac{1}{2}(\alpha - \alpha^*)^2/\Sigma^2 + \text{const}$ . Then  $\Sigma$ , the rms vari-

TABLE I. Results of fitting elastic angular distribution to  $e^{-\alpha t}$ .

	Interval I	Interval II	Interval III
$t_{min}$ (BeV/c) <sup>2</sup>	0.0103 ( $\theta = 7^\circ$ )	0.0103	0.025 ( $\bar{\theta} = 11^\circ$ )
$t_{max}$	0.6	1.0	0.4
$N$ (number of events)	510	590	394
$\alpha^* \pm \Sigma$ (BeV/c) <sup>-2</sup>	$3.1 \pm 0.3$	$2.9 \pm 0.14$	$3.3 \pm 0.5$
$P(\chi^2)$	0.7	0.4	...

ance of  $L$ , is given by the half-width of the parabola at  $\ln L(\alpha) = \ln L(\alpha^*) - \frac{1}{2}$ . The uncertainties  $\Sigma$  quoted in Table I were determined in this way.<sup>7</sup> The likelihood curve for interval I is shown in Fig. 3. It is seen to be parabolic, as are also the curves  $\ln L(\alpha)$  for the other two intervals.

A peculiarity of the exponential hypothesis is that the numerical value of  $\ln L(\alpha^*)$  depends only on  $\alpha^*$  and so cannot be used as a measure of goodness of fit. We therefore calculated  $\chi^2$ 's for intervals I and II by comparing the histogram of Fig. 1 with  $\sigma_0 e^{-\alpha^* t}$ . The forward-scattering cross sections  $\sigma_0$  were obtained from the fractional cross sections for the intervals considered, and are given in Table II. The  $\alpha^*$ 's and  $\chi^2$ 's are of course independent of the over-all normalization.<sup>5</sup> The probability  $P(\chi^2)$  that the  $\chi^2$  would be greater than that calculated if the hypothesis is correct is listed in Table I. The curve  $\sigma_0 e^{-\alpha^* t}$  for interval I has been superimposed on Fig. 1.

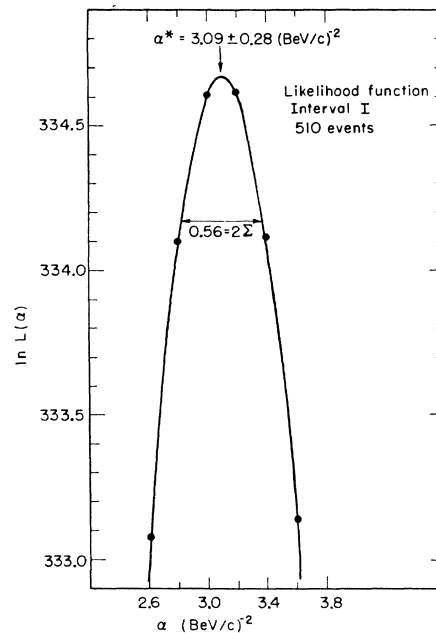


FIG. 3. Logarithm of likelihood function  $L(\alpha)$  (Eqs. 2-6) used to fit interval I [ $0.01 \leq t \leq 0.6$  (BeV/c)<sup>2</sup>] to the hypothesis  $d\sigma/d\Omega \sim \exp(-\alpha t)$ . The estimator  $\alpha^*$  is obtained from the maximum of the likelihood function, and  $\Sigma$ , the rms variance of  $L(\alpha)$ , is the half-width of  $\ln L(\alpha)$  at  $\ln L(\alpha^*) - \frac{1}{2}$ .

<sup>7</sup> As an additional measure of goodness of fit we calculated the expected variance of  $\alpha^*$ :

$$\text{Expected } \Sigma = \frac{1}{\sqrt{N}} \left[ \int_{t=t_{min}}^{t_{max}} \frac{1}{\bar{P}} \left( \frac{\partial P}{\partial \alpha} \right)_\alpha^2 dt \right]^{-1/2}$$

using the approximation

$$P(t, \sigma, \alpha) \approx \frac{\alpha}{[e^{-\alpha t_{min}} - e^{-\alpha t_{max}}]} e^{-\alpha t}.$$

This gave the results 0.28, 0.17, and 0.48 (BeV/c)<sup>-2</sup> for intervals I, II, and III, respectively. The agreement with the rms variances given in Table I serves as evidence for the correctness of the form assumed for the likelihood function.

Using  $\alpha^*$  for interval I we obtain the forward differential cross section

$$\sigma_0 = 4.8 \pm 0.5 \text{ mb/sr.} \quad (7)$$

From the optical theorem, with total cross section  $\sigma_T = 19.4 \pm 2.0$  mb obtained in the present experiment, we have

$$\sigma_0 \geq 4.3 \pm 0.9 \text{ mb/sr.} \quad (8)$$

It is seen that the forward-scattering amplitude is predominantly imaginary as expected, and consistent with being purely imaginary. We have compared our data with a purely imaginary high-energy scattering amplitude of the form

$$F(\bar{\theta}) = \frac{ik}{\sqrt{\pi}} [a_0 e^{-iA_1 t} + C(E) \pm b_0(E) e^{-iB_1(\xi_{\max} - t)}] \quad (9)$$

suggested by Minami<sup>8</sup> to explain both the increase of  $\alpha$  with energy for some systems (e.g.,  $K^+p$ ,  $p\bar{p}$ ) and the absence of this effect for others (e.g.,  $\pi p$ ,  $\bar{p}p$ ).  $A_1$  is here taken to be energy-independent, as are  $a_0$  and  $B_1$ . From higher energy  $K^+p$  elastic-scattering data, then, Minami finds  $a_0 = 18.2$  (mb)<sup>1/2</sup> (BeV/c)<sup>-1</sup> and  $A_1 = 5.6$  (BeV/c)<sup>-2</sup>. Using these values we conclude that Eq. (9), for which Minami finds evidence, in the region 7 to 15 BeV/c, of  $K^+p$  elastic scattering, is inconsistent with our data at 2 BeV/c.

It is interesting to note that the exponential dependence on  $t$ ,

$$d\sigma/d\Omega = \sigma_0 e^{-\alpha t} \quad (1)$$

follows also from an optical model in which it is assumed that the transmitted amplitude  $a(b)$ , for impact parameter  $b$ , is given by

$$1 - a(b) = (A + Bi) e^{-b^2/(b^2)}. \quad (10)$$

Setting<sup>9</sup>  $a(b) = e^{i\chi(b)}$  in

$$f(\bar{\theta}) = -i \int_{b=0}^{\infty} J_0(2kb \sin \frac{1}{2} \bar{\theta}) \{e^{i\chi(b)} - 1\} b db, \quad (11)$$

it follows that

$$d\sigma/d\Omega = \sigma_0 e^{-\langle b^2 \rangle t}, \quad (12)$$

where  $\langle b^2 \rangle = 2\alpha$  is a measure of the mean-square radius of the  $K^+p$  interaction. The region of validity of the approximation Eq. (11) is  $\bar{\theta} \lesssim 23^\circ$ . For  $\alpha = 3.1 \pm 0.3$  (BeV/c)<sup>-2</sup>, the rms interaction radius ( $\langle b^2 \rangle^{1/2} = 0.49 \pm 0.02$  F).

For comparison we show in Table II the results of various elastic-scattering experiments of  $K^-$ ,  $\pi^+$ ,  $\pi^-$ ,  $p$ , and  $\bar{p}$  on protons at approximately the same total barycentric energy, parametrized in terms of  $\alpha$ . It is quite clear that the simplest models of diffraction

TABLE II. Elastic diffraction peaks of various systems at  $E_{\text{c.m.}} \approx 2.2$  BeV fitted to  $d\sigma/d\Omega = \sigma_0 e^{-\alpha t}$ .

Experiment	$P_{\text{lab}}$ (BeV/c)	Interval fitted $t \lesssim$	$\alpha$ (BeV/c) <sup>-2</sup>	$\sigma_0$ (mb/sr)
$K^+p$ this expt.	1.96	1.0 (BeV/c) <sup>2</sup>	$2.9 \pm 0.14$	$4.6 \pm 0.4$
$K^+p$ this expt.	1.96	0.6	$3.1 \pm 0.3$	$4.8 \pm 0.5$
$K^+p$ this expt.	1.96	0.4	$3.3 \pm 0.5$	
$K^+p^b$	1.97	0.21	$3.9 \pm 1.2$	$3.6 \pm 0.2$
$K^-p^a$	2.00	0.4	9.1 <sup>a</sup>	12.9
$K^-p^d$	1.95	0.6	$7.9 \pm 0.6$	$12.5 \pm 1.0$
$\pi^+p^a$	2.02	0.4	$5.7 \pm 0.4$	
$\pi^+p^f$	2.0		$5.0 \pm 0.4$	
$\pi^-p^a$	2.01	0.4	$7.8 \pm 0.2$	
$\pi^-p^d$	1.95	0.36	$8.1 \pm 0.2$	$10.4 \pm 2.0$
$\pi^-p^g$	2.05	0.25	$7.84 \pm 0.7$	
$p\bar{p}^b$	1.45	0.15	10 <sup>a</sup>	
$\bar{p}p^i$	1.61	0.17	13 <sup>a</sup>	

<sup>a</sup> Our fits to data presented by the quoted authors.

<sup>b</sup> See Ref. 6.

<sup>c</sup> R. Crittenden, H. Martin, W. Kernan, L. Liepuner, A. C. Li, F. Ayer, L. Marshall, and M. L. Stevenson, Phys. Rev. Letters 12, 429 (1964).

<sup>d</sup> V. Cook, B. Cork, T. F. Hoang, D. Keefe, L. Kerth, W. Wenzel, and T. Zipf, Phys. Rev. 123, 320 (1961).

<sup>e</sup> D. E. Damouth *et al.*, University of Michigan, Ann Arbor, Technical Report No. 12, 1963 (unpublished).

<sup>f</sup> V. Cook, B. Cork, W. Holly, and M. Perl, Phys. Rev. 130, 762 (1962).

<sup>g</sup> L. D. Jacobs and D. Miller (private communication).

<sup>h</sup> T. Morris, E. Fowler, and J. Garrison, Phys. Rev. 103, 1472 (1956).

<sup>i</sup> G. Lynch, R. Foulks, G. Kalbfleisch, S. Limentani, J. B. Shafer, M. L. Stevenson, and N. Xuong, Phys. Rev. 131, 1276 (1963).

scattering fail at this energy. The experimental results, summarized in Table II, indicate that the forward-scattering angular distributions are well fitted by an exponential dependence on momentum-transfer-squared, but the widths of the distributions are not all equal. Of course one expects such behavior only in some asymptotic high-energy limit, assuming the validity of the Regge-pole hypothesis, and this is presumably not the case here. A regularity in the data appears in comparing the widths of the distributions for particle and antiparticle. In all cases,  $K^\pm$ ,  $\pi^\pm$ , and  $p^\pm$ , the positive particle's angular distribution is broader. Further, while the parameter  $\alpha$  varies from 3 (BeV/c)<sup>-2</sup>, for  $K^+p$  scattering, to 13 (BeV/c)<sup>-2</sup> for  $\bar{p}p$  scattering, the differences

TABLE III. Cross sections observed for the  $K^+p$  interaction at 1.96 BeV/c.

Reaction product	Cross section (mb)
$K^+p$ elastic	$7.5 \pm 0.7$
$K^0\pi^+p$	$4.6 \pm 0.6$
$K^+\pi^0p$	$2.0 \pm 0.3$
$K^+\pi^+n$	$1.6 \pm 0.3$
$K^+\pi^-p\pi^+$	$1.7 \pm 0.2$
$K^0\pi^0p\pi^+$	$1.3 \pm 0.2$
$K^0\pi^+n\pi^+$	$0.33 \pm 0.1$
$K^+\pi^0n\pi^+$	$\sim 0.3$
$K^+\pi^-\pi^0\pi^+p$	$0.05 \pm 0.02$
$K^0\pi^+\pi^-\pi^+p$	$0.02 \pm 0.01$
$K^+\pi^-\pi^+\pi^+p$	$0.01 \pm 0.006$
$K^+K^+A$	$\leq 0.01$
$\sigma_{\text{total}}$	$19.4 \pm 2.0$

<sup>8</sup> S. Minami, Phys. Rev. 135, B1263 (1964).

<sup>9</sup> R. J. Glauber, *Lectures in Theoretical Physics I*, Boulder 1958, (Interscience Publishers, Inc., New York, 1959).

between the widths for the positive and negative particle's scattering are not inconsistent with a constant value,  $(\alpha_- - \alpha_+) \approx 4 (\text{BeV}/c)^{-2}$ .

### V. INELASTIC CROSS SECTIONS

In Table III are listed the partial cross sections for all open channels.<sup>2</sup> Identifications were made on the basis of kinematic fit and bubble-density estimates. The quoted errors are statistical only and do not include uncertainties due to incorrect identification, believed small compared to the purely statistical uncertainties. Detailed characteristics of these reactions are discussed

elsewhere.<sup>2,4</sup> The  $\Lambda^0$ -production cross section is an upper limit, no hyperons having been observed.

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## Possible C-Noninvariant Effects in the $3\pi$ Decay Modes of $\eta^0$ and $\omega^0$ †

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In this paper, the observed  $CP$  noninvariance in  $K_2^0 \rightarrow \pi^+ + \pi^-$  is assumed to be due to the existence of a new strangeness-conserving but  $C$ -noninvariant and  $T$ -noninvariant interaction called  $H_F$ , whose coupling constant  $F$  is  $\sim 10^3 G$  where  $G$  is the Fermi coupling constant in the usual  $CP$ -invariant weak interaction. A phenomenological analysis of the possible forms of the energy asymmetry between  $\pi^+$  and  $\pi^-$  in the  $3\pi$  decay modes of  $\eta^0$  and  $\omega^0$  is made. It is pointed out that a study of such an asymmetry can be used to test the possible existence of  $H_F$  as well as its isotopic-spin selection rules.

### 1. INTRODUCTION

IT has been suggested recently,<sup>1,2</sup> in connection with the observation<sup>3</sup>

$$K_2^0 \rightarrow \pi^+ + \pi^-, \quad (1)$$

that the violation of  $CP$  invariance is due not to the usual weak interaction, but rather to the possible existence of a new  $CP$ -noninvariant interaction<sup>3a</sup> called

$H_F$ . If  $H_F$  conserves the strangeness quantum number, then its coupling constant  $F$  must be much stronger than the Fermi constant  $G$  for the usual weak interaction, called  $H_G$ . It is estimated that

$$F \sim 10^3 G; \quad (2)$$

or the dimensionless constant  $(Fm_p)^2$  is given by

$$Fm_p^2 \sim 10^{-2}, \quad (3)$$

where  $m_p$  is the mass of the proton. The usual weak interaction  $H_G$  violates  $C$  invariance and  $P$  invariance, but it is assumed to be invariant under  $CP$  and  $T$ . The new interaction  $H_F$  is assumed to violate  $C$  invariance and  $T$  invariance, but is invariant under  $CT$  and  $P$ , where  $C$ ,  $P$ , and  $T$  denote, respectively, the usual three operators: charge conjugation, space inversion, and time reversal. Reaction (1) can occur only through the second-order  $CP$ -noninvariant term  $H_F H_G$ ; thus, its amplitude is much smaller than that of  $K_1^0 \rightarrow \pi^+ + \pi^-$ , which can occur through  $H_G$  alone.

The possible existence of such a new interaction can

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<sup>1</sup> T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965).

<sup>2</sup> L. B. Okun (to be published). See also J. Prentki and M. Veltman, Phys. Letters **15**, 88 (1965), in which they consider the possibility that the  $C$ -,  $T$ -noninvariant interaction is simply the usual  $SU_3$ -violating but  $SU_2$ -conserving part of the strong interaction. This possibility seems to encounter several difficulties, especially in view of the present accuracy ( $\sim 2\%$  in relative amplitude) of  $T$  invariance in many nuclear reactions (see Refs. 4 and 5), all of which violate the  $SU_3$  symmetry.

<sup>3</sup> J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

<sup>3a</sup> Note added in proof. Recently, J. Bernstein, G. Feinberg, and T. D. Lee [Phys. Rev. (to be published)] proposed that the electromagnetic interaction  $H_\gamma$  has a large violation of  $C$  conservation. In this case,  $H_F$  is regarded as the radiative correction effect and  $(Fm_p)^2$  is simply the fine-structure constant  $\alpha$ .

It should be noted that by measuring the  $\pi^+$ ,  $\pi^-$  asymmetry in  $\eta^0$  (or  $\omega^0$ )  $\rightarrow \pi^+ \pi^- \pi^0$  one can conclude that the observed  $C$  non-conservation is not due to the weak interaction, but one cannot decide whether the  $C$ -noninvariant interaction  $H_F$  is due to the second-order effects of  $H_\gamma$ , or it is simply an integral, but small, part of the strong interaction  $H_{st}$ . To differentiate between these

two possibilities, it is necessary to study reactions involving photons or charged lepton pairs such as  $\eta^0 \rightarrow \pi^+ \pi^- \gamma$ , which would exhibit a  $\sim (kR)^2 \sim 10\%$  fractional asymmetry in the energy distributions of  $\pi^+$  and  $\pi^-$  if  $H_\gamma$  has large violations of  $C$  conservation, but this fractional asymmetry is reduced to only  $\sim (Fm_p)^2 \times 10\% \sim 10^{-3}$  if  $H_F$  is independent of  $H_\gamma$  and, therefore, may be regarded as an integral part of  $H_{st}$ .