between the widths for the positive and negative particle's scattering are not inconsistent with a constant value, $(\alpha_- - \alpha_+) \approx 4$ (BeV/c)$^{-2}$.

V. INELASTIC CROSS SECTIONS

In Table III are listed the partial cross sections for all open channels.[2] Identifications were made on the basis of kinematic fit and bubble-density estimates. The quoted errors are statistical only and do not include uncertainties due to incorrect identification, believed small compared to the purely statistical uncertainties. Detailed characteristics of these reactions are discussed elsewhere.[2,4] The Λ^0-production cross section is an upper limit, no hyperons having been observed.

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Possible C-Noninvariant Effects in the 3π Decay Modes of η^0 and ω^0 †

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In this paper, the observed CP noninvariance in $K_2^0 \to \pi^+ + \pi^-$ is assumed to be due to the existence of a new strangeness-conserving but C-noninvariant and T-noninvariant interaction called H_F, whose coupling constant F is $\sim 10^3 G$ where G is the Fermi coupling constant in the usual CP-invariant weak interaction. A phenomenological analysis of the possible forms of the energy asymmetry between π^+ and π^- in the 3π decay modes of η^0 and ω^0 is made. It is pointed out that a study of such an asymmetry can be used to test the possible existence of H_F as well as its isotopic-spin selection rules.

1. INTRODUCTION

It has been suggested recently,[1,2] in connection with the observation[3]

$$K_2^0 \to \pi^+ + \pi^-, \quad (1)$$

that the violation of CP invariance is due not to the usual weak interaction, but rather to the possible existence of a new CP-noninvariant interaction[3a] called H_F. If H_F conserves the strangeness quantum number, then its coupling constant F must be much stronger than the Fermi constant G for the usual weak interaction, called H_G. It is estimated that

$$F \sim 10^3 G; \quad (2)$$

or the dimensionless constant (Fm_p^2) is given by

$$Fm_p^2 \sim 10^{-2}, \quad (3)$$

where m_p is the mass of the proton. The usual weak interaction H_G violates C invariance and P invariance, but it is assumed to be invariant under CP and T. The new interaction H_F is assumed to violate C invariance and T invariance, but is invariant under CT and P, where C, P, and T denote, respectively, the usual three operators: charge conjugation, space inversion, and time reversal. Reaction (1) can occur only through the second-order CP-noninvariant term $H_F H_G$; thus, its amplitude is much smaller than that of $K_1^0 \to \pi^+ + \pi^-$, which can occur through H_G alone.

The possible existence of such a new interaction can

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[1] T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965).

[2] L. B. Okun (to be published). See also J. Prentki and M. Veltman, Phys. Letters **15**, 88 (1965), in which they consider the possibility that the C-, T-noninvariant interaction is simply the usual SU_3-violating but SU_2-conserving part of the strong interaction. This possibility seems to encounter several difficulties, especially in view of the present accuracy ($\sim$2% in relative amplitude) of T invariance in many nuclear reactions (see Refs. 4 and 5), all of which violate the SU_3 symmetry.

[3] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[3a] *Note added in proof.* Recently, J. Bernstein, G. Feinberg, and T. D. Lee [Phys. Rev. (to be published)] proposed that the electromagnetic interaction H_γ has a large violation of C conservation. In this case, H_F is regarded as the radiative correction effect and (Fm_p^2) is simply the fine-structure constant α.

It should be noted that by measuring the π^+, π^- asymmetry in η^0 (or ω^0) $\to \pi^+\pi^-\pi^0$ one can conclude that the observed C nonconservation is not due to the weak interaction, but one cannot decide whether the C-noninvariant interaction H_F is due to the second-order effects of H_γ, or it is simply an integral, but small, part of the strong interaction H_{st}. To differentiate between these two possibilities, it is necessary to study reactions involving photons or charged lepton pairs such as $\eta^0 \to \pi^+\pi^-\gamma$, which would exhibit a $\sim(kR)^2 \sim 10\%$ fractional asymmetry in the energy distributions of π^+ and π^- if H_γ has large violations of C conservation, but this fractional asymmetry is reduced to only $\sim(Fm_p^2) \times 10\%$ $\sim 10^{-3}$ if H_F is independent of H_γ and, therefore, may be regarded as an integral part of H_{st}.

be tested by examining the limits of T invariance and C invariance in the strong interactions. For example, the reciprocity relation for any strong reaction may be violated with a fractional difference of the order of 10^{-2}–10^{-3}. The current accuracy of the reciprocity relation[4,5] is about 2% for

$$p+t \leftrightarrows d+d. \quad (4)$$

All nuclear matrix elements in either γ or β transitions may also contain a small admixture of T-noninvariant amplitude which is $\sim(10^{-2}$–$10^{-3})$ times the T-invariant amplitude. Such a T-noninvariant amplitude can be observed by measuring the relative phase[6] between G_V and G_A in β decay, or by studying, e.g., the $E2$, $M1$ interference term through the detection of a $[\mathbf{k}_\beta\cdot(\mathbf{k}_\gamma\times\mathbf{k}_\gamma')]\times(\mathbf{k}_\gamma\cdot\mathbf{k}_\gamma')$ term in an appropriate β-γ-γ transition.[7]

It has been pointed out[8] that another possible test of such a relatively strong C-noninvariant interaction is to study the energy distributions of π^+ and π^- in the decay

$$\eta^0 \to \pi^+ + \pi^- + \pi^0. \quad (5)$$

The decay of η^0 is known to violate G parity. Reaction (5) and

$$\eta^0 \to 3\pi^0 \quad (6)$$

can occur through the second order electromagnetic interaction to a final (3π) state with $I=1$ and $C=+1$, where $I=$ total isospin value of the 3π system. The η^0 can also decay, now, through H_F to a final state with $C=-1$ and $I=0$ or 2. The interference between the $C=\pm1$ amplitudes may result in an asymmetry in the energy distribution of π^+ and π^-. The detection of such an asymmetry would be an absolute proof of C noninvariance in the η^0 decay.

The present note contains some remarks concerning the phenomenological analysis of the 3π final states. A number of simple statements are established. These results can be used to study the possible existence of C noninvariance in η^0 decay, as well as to test the selection rule

$$|\Delta\mathbf{I}|<2, \quad (7)$$

which may be satisfied by H_F. For clarity, these statements are expressed in the form of mathematical theorems in Sec. 2, and their proofs are given in Sec. 3. Estimates of the magnitude and the form of $\pi^\pm$ asymmetry are made in Sec. 4. If the results from η^0 decay, indeed, establish the existence of the C-noninvariant interaction H_F, the remaining question whether H_F contains a $|\Delta\mathbf{I}|=1$ part can be answered by analyzing the $\pi^\pm$ distributions in ω^0 decay. A brief discussion concerning the ω^0 decay is given in Sec. 5.

[4] L. Rosen and J. E. Brolley, Jr., Phys. Rev. Letters 2, 98 (1959).
[5] See also D. Bodansky *et al.*, Phys. Rev. Letters 2, 101 (1959), in which they studied the reactions $C^{12}+\alpha \leftrightarrows N^{14}+d$.
[6] M. T. Burgy *et al.*, Phys. Rev. Letters 1, 324 (1958).
[7] For a detailed discussion, see E. M. Henley and B. A. Jacobsohn, Phys. Rev. 113, 225 (1959); B. A. Jacobsohn and E. M. Henley, Phys. Rev. 113, 234 (1959).
[8] R. Friedberg, T. D. Lee, and M. Schwartz (unpublished).

2. SOME ELEMENTARY THEOREMS

Let Q_+, Q_-, and Q_0 be, respectively, the kinetic energy of the final π^+, π^-, and π^0 in the rest system of η^0. The polar coordinates r,θ in the usual Dalitz plot[9] are defined by

$$Q_0=Q[1+r\cos\theta] \quad (8)$$

and

$$Q_\pm=Q[1+r\cos(\tfrac{2}{3}\pi\mp\theta)], \quad (9)$$

where

$$Q=\tfrac{1}{3}m_\eta-m_\pi \quad (10)$$

and m_η, m_π are, respectively, the masses of η^0 and π. The physical region is the entire area within the curve

$$r\leqq R(\theta), \quad (11)$$

where the function $R(\theta)$ is given by

$$R^2=[1/(1+\epsilon)][1-\epsilon R^3\cos 3\theta] \quad (12)$$

and

$$\epsilon=2m_\eta(m_\eta-3m_\pi)/(m_\eta+3m_\pi)^2\cong 0.16. \quad (13)$$

Correspondingly, we can also introduce the same coordinates (r,θ) for the decay $\eta^0\to3\pi^0$, provided Q_+ and Q_- in Eq. (9) are replaced by the kinetic energies of the other two π^0. Because of Bose statistics, only the sector $0\leqq\theta<\frac{1}{3}\pi$ is needed for the $3\pi^0$ system.

Let $A(r,\theta)$ and $B(r,\theta)$ be, respectively, the amplitudes for the decays $\eta^0\to\pi^++\pi^-+\pi^0$ and $\eta^0\to3\pi^0$. It is useful to consider the Fourier analysis of these amplitudes:

$$A(r,\theta)=\alpha_0(r)+\sum_{l=1}^{\infty}[\alpha_l(r)\cos l\theta+\alpha_l'(r)\sin l\theta] \quad (14)$$

and

$$B(r,\theta)=\sum_{l=0}^{\infty}\beta_{3l}(r)\cos(3l\theta). \quad (15)$$

The amplitudes $A(r,\theta)$, $B(r,\theta)$ and their Fourier components satisfy a number of simple conditions[10]:

Theorem 1. If at any r

$$A(r,\theta)\neq A(r,-\theta), \quad (16)$$

then C invariance is not valid in η^0 decay.

Theorem 2. If the total isospin I of the 3π system satisfies $I\neq2$, then at any r,

$$A(r,\theta)=A(r,-\theta), \quad (17)$$

[9] R. Dalitz, Phil. Mag. 44, 1068 (1953); Phys. Rev. 94, 1046 (1954); E. Fabri, Nuovo Cimento 11, 479 (1954).
[10] Identical considerations can be applied to the 3π decay modes of the neutral K mesons. For the K_2^0 decay, the dominant 3π states are $C=+1$ and $I=1$. The corresponding $C=-1$ amplitudes, which can occur only through H_FH_G, are expected to be smaller by a factor $\sim(10^{-2}$–$10^{-3})\times(Q/m_\pi)^n$ where $n=3$ or 1 depending on whether H_F satisfies Eq. (7) or not. For the K_1^0 decay, the $C=-1$ states can be reached through H_G, but the $C=+1$ states can occur only by using H_GH_F. However, among the $C=-1$ states, the amplitudes of the allowed $I=2$ states are reduced because of the $|\Delta\mathbf{I}|=\frac{1}{2}$ selection rule of H_G, and that of the $I=0$ state is reduced because of the centrifugal barrier effect. Thus, the $C=\pm1$ amplitudes may become somewhat comparable in their magnitudes. The absolute rate of the decay $K_1^0\to3\pi$ is, however, extremely small, because of the limited phase space available and these additional reduction factors.

provided

$$\theta=0, \quad \pm\tfrac{2}{3}\pi, \quad \pm\tfrac{4}{3}\pi \text{ and } \pi. \tag{18}$$

Theorem 3. If $I\neq 3$, then[11]

$$\beta_{3l}(r)=-3\alpha_{3l}(r) \tag{19}$$

where $l=0$, 1, 2, etc.

Theorem 4. If the final-state interactions between pions are neglected, then, by using CPT invariance,

$$\alpha_l(r)=\text{real}, \quad \beta_{3l}(r)=\text{real},$$

and

$$\alpha_l'(r)=\text{pure imaginary}, \tag{20}$$

where, for convenience, $\alpha_0(r)$ is chosen to be real.

Thus, a detection of

$$|A(r,\theta)|^2\neq|A(r,-\theta)|^2 \tag{21}$$

at any (r,θ) implies that not only C invariance is violated but that the strong pion interactions cannot be neglected.

In view of the limited number of presently available data on η^0 decay, it may be useful to project the two-dimensional distribution into a one-dimensional representation. We define

$$N_\pm(\theta)=\frac{1}{2}\int_0^{R(\theta)}[|A(r,\theta)|^2\pm|A(r,-\theta)|^2]rdr \tag{22}$$

where $R(\theta)$ is given by Eq. (12). In this one dimensional representation, to test C noninvariance is to measure the function $N_-(\theta)$. The selection rule, Eq. (7), can be tested by examining whether or not $N_-(\theta)=0$ along the lines given by Eq. (18).

All the above theorems are completely general but elementary. Their proofs are given in the next section.

3. PROOF OF THE THEOREMS

Let us consider a state of three pions with momenta $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$, which correspond to kinetic energies Q_1, Q_2, and Q_3 where

$$Q_1>Q_2>Q_3. \tag{23}$$

There are altogether seven such states; the 3π system can be either $3\pi^0$ or $(\pi^0\pi^+\pi^-)$, and if it is $(\pi^0\pi^+\pi^-)$, their energies can be either

$$Q_0=Q_1, \quad Q_+=Q_2, \text{ and } Q_-=Q_3, \tag{24}$$

or any other permutations between (Q_0,Q_+,Q_-). The particular assignment, Eq. (24), determines that the corresponding point (r,θ) must be within the sector $0<\theta<\frac{1}{3}\pi$ in the Dalitz plot. The other 5 permutations between (Q_0,Q_+,Q_-) correspond to the 5 images of this point (r,θ), formed by reflections with respect to the lines given by Eq. (18).

[11] Equation (19) holds in general, provided that the C-noninvariant interaction H_F does not contain an additional $C=+1$ but $|\Delta\mathbf{I}|=3$ part.

For a given set (Q_1,Q_2,Q_3) the amplitudes of the η^0 decay into these seven final states can be represented by a column matrix ψ:

$$\psi=\begin{bmatrix} A(r,\theta) \\ A(r,-\theta) \\ A(r,-\frac{2}{3}\pi+\theta) \\ A(r,\frac{2}{3}\pi-\theta) \\ A(r,\frac{2}{3}\pi+\theta) \\ A(r,-\frac{2}{3}\pi-\theta) \\ B(r,\theta) \end{bmatrix}, \tag{25}$$

where

$$0\leqq\theta<\tfrac{1}{3}\pi. \tag{26}$$

In Eq. (26) the equality sign is for the case that some of these energies may be degenerate.

We note that for the $(\pi^0\pi^+\pi^-)$ system the charge conjugation operator C is the reflection

$$\theta\rightarrow-\theta. \tag{27}$$

By using expression (27) and the fact that C of η^0 is $+1$, theorem 1 is proved.

An alternative representation of these seven amplitudes is to use the isotopic spin. For a given set (Q_1,Q_2,Q_3), there are one $I=0$ state, three $I=1$ states, two $I=2$ states and one $I=3$ state. The charge conjugation C of these states is given by

$$C=-(-1)^I. \tag{28}$$

Among these states, the $I=0$ state is completely antisymmetric with respect to these three pions. Thus, if $I\neq 2$, the $C=-1$ part of the 3π system consists of only the $I=0$ state; therefore, in the Fourier expansion, Eq. (14)

$$\alpha'_{3l+1}=\alpha'_{3l+2}=0, \tag{29}$$

where l is any positive integer including zero. Theorem 2, now, follows.

To construct explicitly the seven eigenstates of the total isospin, let us denote the isospin vector of the pion with energy Q_i by $\mathbf{I}_i$, and the sum of two different isospin vectors $\mathbf{I}_i$ and $\mathbf{I}_j$ by

$$\mathbf{I}_{ij}=\mathbf{I}_i+\mathbf{I}_j. \tag{30}$$

We define[12]

$$|I_{ij},I\rangle\equiv|(I_i,I_j)I_{ij},I_k=1;I,I_z=0\rangle \tag{31}$$

which is an eigenstate of I_{ij} and $I(i\neq j\neq k\neq i)$. The z component I_z of the total isospin is always zero. The column matrix ψ can also be represented by using these $|I_{ij},I\rangle$ states as base vectors:

$$\langle I_{ij},I|\psi\rangle=\sum_{\lambda=1}^{7}\langle I_{ij},I|\lambda\rangle\psi_\lambda \tag{32}$$

[12] For definiteness, the relative phases between the different $|I_{ij},I\rangle$ states are fixed by adopting the notations used in A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957).

where, according to Eq. (25), $\psi_1 = A(r,\theta)$, $\psi_2 = A(r,-\theta)$, $\cdots$, and $\psi_7 = B(r,\theta)$. The matrix elements $\langle I_{ij},I|\lambda\rangle$ can be readily evaluated by using the usual Clebsch-Gordan coefficients. We find

$$\langle I_{ij}, I=3|\lambda\neq 7\rangle = \tfrac{1}{2}\langle I_{ij}, I=3|7\rangle. \quad (33)$$

Theorem 3 is, then, proved by setting $\langle I_{ij}, I=3|\psi\rangle = 0$ in Eq. (32).

To obtain theorem 4, we note that upon neglecting the final-state pion interactions and by using Eq. (28), the state $|I_{ij},I\rangle$ for any given energy distribution (Q_1,Q_2,Q_3) transforms under the CPT operation as

$$CPT|I_{ij},I\rangle = e^{i\delta}(-1)^I|I_{ij},I\rangle, \quad (34)$$

where δ is independent[12] of I_{ij} and I. Let H_{eff} be the effective Hamiltonian for the η^0 decay, whose matrix element gives the combined effects of H_F and the second-order electromagnetic interactions. Both H_F and the electromagnetic interactions are invariant under P and CT. The same holds for H_{eff}, and, in particular,

$$CPTH_{\text{eff}}T^{-1}P^{-1}C^{-1} = H_{\text{eff}}. \quad (35)$$

By noticing that T represents the joint operation of a complex conjugation and a unitary operator, we find

$$\langle I_{ij},I|H_{\text{eff}}|\eta^0\rangle^* = e^{i\delta'}(-1)^I\langle I_{ij},I|H_{\text{eff}}|\eta^0\rangle, \quad (36)$$

where δ' is independent of I and I_{ij}. Thus, theorem 4 is established.

4. SIMPLE PHENOMENOLOGICAL ANALYSIS

Assuming that H_F exists, we will try to estimate the approximate magnitude of $\pi^+\pi^-$ asymmetry in η^0 decay by assuming some simple forms of $A(r,\theta)$ and $B(r,\theta)$. Clearly, these forms depend on whether or not H_F satisfies the $|\Delta\mathbf{I}|<2$ rule.

Case I. We consider first the case that H_F satisfies the $|\Delta\mathbf{I}|<2$ rule. In this case, Eqs. (19) and (29) are both valid. From Eqs. (8) and (9), it follows that as $r\to 0$, $\alpha_l(r)\to \text{constant}\times(Qr)^l$ and $\alpha_l'(r)\to\text{constant}\times(Qr)^l$. Since $Q\cong 48$ MeV is reasonably smaller than any of the intrinsic energy scale that characterizes either η or π, this suggests, at least for orientation purposes, a simple phenomenological form

$$A(r,\theta) = a + be^{i\phi_b}r\cos\theta + ce^{i\phi_c}r^3\sin 3\theta, \quad (37)$$

and

$$B(r,\theta) = -3a, \quad (38)$$

where ϕ_b, ϕ_c are real and a, b, c are real and positive. For convenience, these functions are normalized, so that[13]

$$\int_0^{\pi/3} d\theta \int_0^{R(\theta)} r\,dr\,\psi^\dagger\psi = 1, \quad (39)$$

where ψ is given by Eq. (25). Thus, $A(r,\theta)$ and $B(r,\theta)$ are characterized by *four* independent *real* parameters; among these, the parameter c shows the violation of C invariance and the phase angles ϕ_b and ϕ_c give measures for the final state pion interactions. [$\phi_b=0$ or π and $\phi_c=\pm\frac{1}{2}\pi$, if there is no final-state pion interaction.] By using Eq. (32), the final state ψ may be expressed in terms of the eigen-states of I_{ij} and I:

$$\psi = \sqrt{3}a\sum_{(i,j)}|I_{ij}=0, I=1\rangle - \tfrac{2}{3}be^{i\phi_b}\sum_{(i,j)}Q^{-1}(Q_i-Q_j)|I_{ij}=1, I=1\rangle - \tfrac{4}{3}\sqrt{2}ce^{i\phi_c}[Q^{-3}\prod_{(i,j)}(Q_i-Q_j)]|I_{12}=1, I=0\rangle, \quad (40)$$

where the sums and the product extend over the three cyclic pairs $(i,j)=(1,2)$, $(2,3)$, and $(3,1)$, and the state $|I_{12}=1, I=0\rangle = |I_{23}=1, I=0\rangle = |I_{31}=1, I=0\rangle$.

Let P_1, P_1', and P_0 be, respectively, the probabilities that the final three pions are all in the s states with a total $I=1$, that the pions are in some partly antisymmetric states, such as p states, but with a total $I=1$ and that the three pions are in the $I=0$ state. By using Eqs. (37)–(40) we obtain[13]

$$P_1 \cong 7.35a^2, \quad (41)$$

$$P_1' \cong 0.60b^2, \quad (42)$$

and

$$P_0 \cong 0.23c^2, \quad (43)$$

where

$$P_0 + P_1 + P_1' = 1. \quad (44)$$

The branching ratio of the neutral to the charged mode is given by

$$R = \text{Rate}(\eta^0\to 3\pi^0)/\text{Rate}(\eta^0\to\pi^+ + \pi^- + \pi^0) \cong 0.627P_1. \quad (45)$$

The corresponding density distributions in θ become

$$N_+(\theta) = \tfrac{1}{2}a^2R^2(\theta) + \tfrac{2}{3}ab\cos\phi_b R^3(\theta)\cos\theta + \tfrac{1}{4}b^2R^4(\theta)\cos^2\theta + \tfrac{1}{8}c^2R^8(\theta)\sin^2 3\theta, \quad (46)$$

and

$$N_-(\theta) = \tfrac{2}{5}ac\cos\phi_c R^5(\theta)\sin 3\theta + \tfrac{1}{3}bc\cos(\phi_b-\phi_c)R^6(\theta)\cos\theta\sin 3\theta, \quad (47)$$

where $R(\theta)$ and $N_\pm(\theta)$ are given by Eqs. (12) and (22).

The relative magnitudes of a, b, c may be estimated by making the *ad hoc* choice of m_π to be the characteristic energy scale (or, the inverse of the "radius" of η^0 decay). From dimensional considerations, we may estimate, though with somewhat questionable reliability,

$$P_1'/P_1 \sim (Q/m_\pi)^2, \quad (48)$$

and

$$P_0/P_1 \sim (Fm_p^2/\alpha)^2(Q/m_\pi)^6, \quad (49)$$

where Q is the average kinetic energy of the pion,

[13] Equation (39) is valid only if the mass difference between $\pi^\pm$ and π^0 is neglected. The phase space of $3\pi^0$ is about 1.12 times that of $\pi^0\pi^+\pi^-$ in the η^0 decay. This factor is included in the subsequent Eqs. (41) and (45).

$Fm_p^2 \cong 10^{-2}$ is the dimensionless coupling constant of H_F, and $\alpha = (137)^{-1}$. Substituting Eqs. (41)-(43) in the above expressions, we find

$$b/a \sim 1 \tag{50}$$

and

$$c/a \sim 0.3. \tag{51}$$

From Eqs. (46) and (47), an integrated $\pi^+\pi^-$ asymmetry value $\sim(8/15\pi)(c/a) \sim 5\%$ may be possible in the η^0 decay. The contribution of the c^2 term in $N_+(\theta)$ is relatively unimportant.

Among the published data on η^0 decay,[14–19] except for Ref. 16, only the folded distributions $|A(r,\theta)|^2 + |A(r,-\theta)|^2$ are given. By comparing these data with Eq. (46), and neglecting the c^2 term in $N_+(\theta)$, we find[20]

$$b/a \sim \tfrac{4}{5} \quad \text{and} \quad \phi_b \sim \pm 120°. \tag{52}$$

The corresponding branching ratio is $R \sim 1.45$. The determination of the parameters c/a and ϕ_c must wait till the complete unfolded distribution becomes available.

It is important to note that the asymmetry function $N_-(\theta)$ may change rather drastically as θ varying from 0 to π. As an illustration, one may use Eq. (52) and set arbitrarily $\phi_c = \mp 60°$. The resulting π^+, π^- asymmetry has the approximate relative integrated values

$$\int_0^{\pi/3} N_-(\theta)d\theta : \int_{\pi/3}^{2\pi/3} N_-(\theta)d\theta : \int_{2\pi/3}^{\pi} N_-(\theta)d\theta \sim 0 : -9 : 20, \tag{53}$$

showing that the asymmetry can change both its sign and its magnitude, if H_F satisfies the $|\Delta\mathbf{I}| < 2$ rule.

Case II. If the C-noninvariant interaction H_F contains a $|\Delta\mathbf{I}| = 2$ part, then, instead of Eqs. (37) and (38), we may assume

$$A(r,\theta) = a + be^{i\phi_b} r \cos\theta + ce^{i\phi_c} r \sin\theta \tag{54}$$

and

$$B(r,\theta) = -3a, \tag{55}$$

where, for simplicity, the contribution of the $|\Delta\mathbf{I}| = 0$ part of H_F is neglected. Otherwise, an additional $r^3 \sin 3\theta$ term should be included in Eq. (54). The distribution functions $N_+(\theta)$ are given by

$$N_+(\theta) = \tfrac{1}{2}a^2R^2(\theta) + \tfrac{2}{3}ab \cos\phi_b R^3(\theta) \cos\theta + \tfrac{1}{4}R^4(\theta)[b^2 \cos^2\theta + c^2 \sin^2\theta] \tag{56}$$

and

$$N_-(\theta) = \tfrac{2}{3}ac \cos\phi_c R^3(\theta) \sin\theta + \tfrac{1}{4}bc \cos(\phi_b - \phi_c) R^4(\theta) \sin 2\theta. \tag{57}$$

Let P_2 be the probability of the 3π system in the $I = 2$ state, and P_1, P_1' have the same definitions as that in the previous case. We find that Eqs. (41) and (42) remain valid, and

$$P_2 \cong 0.60c^2 \tag{58}$$

where

$$P_2 + P_1 + P_1' = 1. \tag{59}$$

The corresponding expression of the branching ratio R in terms of P_1 is still given by Eq. (45). The magnitude of b/a and c/a can be estimated to be both ~ 1. Thus, the c^2 term cannot be neglected in $N_+(\theta)$, and one may expect $N_-(\theta)$ to be a relatively slowly varying function but with a larger magnitude than that in the previous case.

5. ω^0 DECAY

If a statistically significant amount of charge asymmetry is indeed found experimentally in the η^0 decay, then it is important to know whether the C-noninvariant interaction H_F contains a $|\Delta\mathbf{I}| = 1$, $C = -1$ part. Such a term cannot contribute to the 3π decay of η^0. On the other hand, in the ω^0 decay,

$$\omega^0 \to \pi^+ + \pi^- + \pi^0, \tag{60}$$

an asymmetry between the $\pi^+\pi^-$ distribution can occur only if H_F does violate the isospin conservation by an amount $|\Delta\mathbf{I}| = 1$ or 3.

To analyze the pion distributions in the Dalitz plot for the ω^0 decay, it is convenient to define an axial vector $\mathbf{P}$ which, because of invariance under rotations and space inversion, must be parallel to the initial polarization direction of ω^0:

$$\mathbf{P} \equiv (\mathbf{k}_1 \times \mathbf{k}_2) + (\mathbf{k}_2 \times \mathbf{k}_3) + (\mathbf{k}_3 \times \mathbf{k}_1), \tag{61}$$

where $\mathbf{k}_i$ is the momentum of the ith meson $(i = 1,2,3)$ in the rest system of ω^0. Let (r,θ) be the usual polar coordinates in the Dalitz plot. The final amplitudes for reaction (60) and $\omega^0 \to 3\pi^0$ can be represented, respectively, by

$$\mathbf{P} \times A(r,\theta) \quad \text{and} \quad \mathbf{P} \times B(r,\theta),$$

and the Fourier expansions of A and B are given by

$$A(r,\theta) = \alpha_0(r) + \sum_{l=1}^{\infty} [\alpha_l(r) \cos l\theta + \alpha_l'(r) \sin l\theta] \tag{62}$$

and

$$B(r,\theta) = \sum_{l=1}^{\infty} \beta_{3l}(r) \sin 3l\theta. \tag{63}$$

The following statements can be readily established by using essentially the same arguments given in Sec. 3.

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[20] The available 559 events are distributed as

$$\int_0^{\pi/3} N_+(\theta)d\theta : \int_{\pi/3}^{2\pi/3} N_+(\theta)d\theta : \int_{2\pi/3}^{\pi} N_+(\theta)d\theta = 126 : 166 : 267.$$

1. If H_F conserves the total isospin, then

$$A(r,\theta)=A(r,-\theta) \tag{64}$$

and

$$B(r,\theta)=0. \tag{65}$$

Furthermore, since the total isospin I of the 3π system must be in the $I=0$ state,

$$A(r,\theta)=\sum_{l=0}^{\infty}\alpha_{3l}(r)\cos 3l\theta. \tag{66}$$

In this case, there is no C-violation effect in the decay of $\omega^0\rightarrow 3\pi$.

2. If $I\neq 3$, then

$$\beta_{3l}(r)=-3\alpha_{3l}'(r), \tag{67}$$

where l= any positive integer.

3. If the final state interactions between pions are neglected, then by using the CPT invariance and choosing $\alpha_0(r)$ to be real, we find

$$\alpha_l(r)=\text{real},\quad \alpha_l'(r)=\text{pure imaginary},$$

and

$$\beta_{3l}'(r)=\text{pure imaginary}.$$

Thus, if a charge asymmetry is observed in ω^0 decay, it implies that the C-noninvariant interaction H_F exists, that it does not conserve the isospin, and that the final-state pion interactions cannot be neglected.

For a simple phenomenological analysis, one may assume the final 3π system to be in $I=0$ and $I=1$ states only. The simplest form of $A(r,\theta)$ is given by

$$A(r,\theta)=a+be^{i\phi_b}r\sin\theta, \tag{68}$$

where $a\geqq 0$, $b\geqq 0$ and ϕ_b are three real parameters. The relative amplitudes of the $I=0$ state and the $I=1$ state are given, respectively, by a and b. The phase ϕ_b gives the measure of the relevant final-state interactions, since $\phi_b=\pm\pi/2$ if the final-state interactions are neglected. Assuming that H_F contains a $|\Delta\mathbf{I}|=1$ part, the magnitude of the asymmetry parameter b/a may be estimated to be $\sim(Fm_p{}^2)\sim 10^{-2}$.

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Anti-Isobar Production in $\bar{p}n$ Interactions*

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Single-pion production in $\bar{p}n$ interactions has been studied in the BNL 20-in. deuterium-filled bubble chamber at an incident $\bar{p}$ momentum of 1.96 GeV/c. Copious production of the $\frac{3}{2},\frac{3}{2}$ anti-isobar was observed. The differential cross section as a function of momentum transfer has a distribution consistent with that predicted by one-pion exchange (OPE). The experimental cross section of 5.1±0.5 mb, however, is about half that predicted by the OPE model of Ferrari and Selleri.

I. INTRODUCTION

In this paper we present results obtained in a study of single-pion production in antiproton-deuteron interactions at 1.96 GeV/c. The purpose of the study was to investigate production and decay of the doubly charged state of the $\frac{3}{2},\frac{3}{2}$ anti-isobar in the reactions

$$\bar{p}+n\rightarrow \bar{N}^{*--}+p,\quad \bar{N}^{*--}\searrow \bar{p}+\pi^- \tag{1}$$

and to compare this with the similar reaction previously studied in pp interactions

$$p+p\rightarrow N^{*++}+n,\quad N^{*++}\searrow p+\pi^+. \tag{2}$$

Single-pion production in p-p collisions in the 1.5–4.0-GeV/c momentum region has been the subject of several experimental investigations.[1] Qualitative fea-

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