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Application of the Goldberger-Treiman Relation to the Beta Decay of Complex Nuclei*

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The theory of the beta decay of complex nuclei, $N_i \to N_f + e^- + \bar{\nu}_e$, is developed on the basis of a treatment which considers the nuclei involved (N_i and N_f) as "elementary" particles and applies the hypotheses of the conserved polar-vector hadron weak current (CVC) and the partially conserved axial-vector hadron weak current (PCAC) to determine the effective polar-vector and axial-vector weak coupling constants $G_V(N_i \to N_f)$ and $G_A(N_i \to N_f)$; the numerical values of $G_V(N_i \to N_f)$ and $G_A(N_i \to N_f)$ reflect in this treatment the complexity of internal nuclear structure. Using CVC, and supposing that $|N_i\rangle$ and $|N_f\rangle$ are sufficiently pure isospin eigenstates, we can immediately calculate $G_V(N_i \to N_f)$, while PCAC, together with a suitable pion-pole-dominance assumption, implies the Goldberger-Treiman (G-T) relation which expresses $G_A(N_i \to N_f)$ in terms of the pion–initial-nucleus–final-nucleus coupling constant $f_{\pi N_i N_f}$; this coupling constant can be found from a polological analysis of $n + N_f \to p + N_i$ nucleon charge-exchange scattering experiments. Since such experiments are not as yet available, we calculate the values of the $f_{\pi N_i N_f}$ in terms of the known magnetic moments of N_i and N_f by means of a very crude theory, and compare these values with the values of the $f_{\pi N_i N_f}$ calculated by means of the G-T relation from the $G_A(N_i \to N_f)$ deduced from observed beta-decay rates. The agreement is, in general, somewhat better than that found between calculated and observed rates in the customary impulse-approximation theory of beta decay.

I. INTRODUCTION

IN the customary theory of nuclear beta decay: $N_i \to N_f + e^- + \bar{\nu}_e$, the weak-interaction Hamiltonian is taken as that of a collection of mutually isolated physical nucleons while the initial and final nuclear states, $|N_i\rangle$ and $|N_f\rangle$, are described by wave functions Ψ_{N_i} and Ψ_{N_f}, dependent on the position, spin, and isospin of these nucleons. As a consequence, an impulse approximation is employed to relate the transition matrix elements in nuclear and nucleon beta decay; moreover, the calculated matrix elements are in general rather sensitive to the details of the wave functions used. Thus, no very high precision has ever been attained in the prediction of nuclear beta-decay rates and several serious discrepancies still exist between theoretical and experimental ft values (e.g., in ${}_{13}\mathrm{Al}_{12}{}^{25} \to {}_{12}\mathrm{Mg}_{13}{}^{25} + e^+ + \nu_e$); these discrepancies seem too large to be due to a failure of the impulse approximation (i.e., to be due to pion-exchange effects[1]) and probably arise from inadequacies which still afflict even the best available Ψ_{N_i} and Ψ_{N_f}.

In the theory developed in this paper we attempt to avoid the above difficulties by treating the nuclei N_i and N_f which participate in the beta decay as "elementary" particles and by applying the hypothesis of the conserved polar-vector hadron weak current (CVC) and the hypothesis of the partially conserved axial-vector hadron weak current (PCAC) to determine the effective polar-vector and the effective axial-vector weak coupling constants, $G_V(N_i \to N_f)$ and $G_A(N_i \to N_f)$. The coupling constants $G_V(N_i \to N_f)$ and $G_A(N_i \to N_f)$ are characteristic of the $N_i \to N_f$ nuclear beta-decay transition; their numerical values reflect, in the present treatment, the complexity of internal nuclear structure. In spite of this complexity, $G_V(N_i \to N_f)$ and $G_A(N_i \to N_f)$ may be found explicitly in many cases since the CVC hypothesis permits identification of the polar-vector hadron weak current with the isospin current while the PCAC hypothesis, together with a suitable pion-pole-dominance assumption, implies the Goldberger-Treiman (G-T) relation. Thus $G_V(N_i \to N_f)$ is immediately given if $|N_i\rangle$ and $|N_f\rangle$ are sufficiently pure isospin eigenstates while $G_A(N_i \to N_f)$ is proportional to the pion-initial nucleus-final nucleus coupling constant, $f_{\pi N_i N_f}$, which

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[1] J. S. Bell and R. J. Blin-Stoyle, Nucl. Phys. 6, 87 (1958); R. J. Blin-Stoyle, V. Gupta, and H. Primakoff, *ibid.* 11, 444 (1959); R. J. Blin-Stoyle, Phys. Rev. Letters 13, 55 (1964); R. J. Blin-Stoyle and S. Papageorgiou, Nucl. Phys. 64, 1 (1965); R. J. Blin-Stoyle and S. Papageorgiou, Phys. Letters 14, 343 (1965).

can be found, e.g., from a polological analysis of $n+N_f \to p+N_i$ nucleon charge-exchange scattering experiments or can be expressed (as we show below by means of a very crude theory) in terms of the magnetic moments of $|N_i\rangle$ and $|N_f\rangle$.

II. FORMULATION

We recall that neutron beta decay: $n \to p+e^-+\bar{\nu}_e$, is phenomenologically described by the transition matrix element

$$\langle e^-\bar{\nu}_e p|\mathfrak{L}(0)|n\rangle=\frac{G}{\sqrt{2}}[u_e^\dagger\gamma_4\gamma_\alpha(1+\gamma_5)u_{\bar{\nu}}^*]\{\langle p|j_\alpha^{(V)}(0)|n\rangle+\langle p|j_\alpha^{(A)}(0)|n\rangle\},$$

$$\langle p|j_\alpha^{(V)}(0)|n\rangle=\left\{u_p^\dagger\tau_+\gamma_4\left[\gamma_\alpha F_V^{n\to p}(q^2)-\frac{\sigma_{\alpha\beta}q_\beta}{2m_p}F_M^{n\to p}(q^2)\right]u_n\right\},$$

$$\langle p|j_\alpha^{(A)}(0)|n\rangle=\left\{u_p^\dagger\tau_+\gamma_4\left[\gamma_\alpha\gamma_5F_A^{n\to p}(q^2)+\frac{iq_\alpha(m_p+m_n)}{m_\pi^2}\gamma_5F_P^{n\to p}(q^2)\right]u_n\right\}, \quad (1)$$

$$\langle p|\partial j_\alpha^{(A)}(0)/\partial x_\alpha|n\rangle=-iq_\alpha\langle p|j_\alpha^{(A)}(0)|n\rangle=(m_p+m_n)[F_A^{n\to p}(q^2)+(q^2/m_\pi^2)F_P^{n\to p}(q^2)](u_p^\dagger\tau_+\gamma_4\gamma_5u_n)$$
$$\equiv(m_p+m_n)\Phi^{n\to p}(q^2)(u_p^\dagger\tau_+\gamma_4\gamma_5u_n);$$

$$G=1.0\times10^{-5}/m_p^2;\quad q\equiv-(p_e+p_{\bar{\nu}})=(p_p-p_n),$$

where, on the basis of the CVC hypothesis,[2]

$$F_V^{n\to p}(0)\equiv G_V(n\to p)=1-0=1,\quad F_M^{n\to p}(0)=[\mu(p)-1]-[\mu(n)-0]=(2.79-1)-(-1.91-0)=3.70 \quad (2)$$

and, on the basis of the PCAC hypothesis,[2]

$$\Phi^{n\to p}(q^2)=\frac{m_\pi^2a_\pi f_{\pi np}}{m_\pi^2+q^2}+\frac{1}{\pi}\int_{(3m_\pi)^2}^\infty\frac{\text{Im}\Phi^{n\to p}(-m^2)}{m^2+q^2}d(m^2),$$

$$F_P^{n\to p}(q^2)=-\frac{m_\pi^2a_\pi f_{\pi np}}{m_\pi^2+q^2}+\frac{1}{\pi}\int_{(3m_\pi)^2}^\infty\frac{\text{Im}F_P^{n\to p}(-m^2)}{m^2+q^2}d(m^2); \quad (3)$$

$$-m_\pi^2a_\pi f_{\pi np}+\frac{1}{\pi}\int_{(3m_\pi)^2}^\infty\text{Im}F_P^{n\to p}(-m^2)d(m^2)=0$$

so that

$$\Phi^{n\to p}(0)=F_A^{n\to p}(0)\equiv G_A(n\to p)=a_\pi f_{\pi np}+\frac{1}{\pi}\int_{(3m_\pi)^2}^\infty\frac{\text{Im}\Phi^{n\to p}(-m^2)}{m^2}d(m^2)$$

$$=a_\pi f_{\pi np}\left[1+\frac{\frac{1}{\pi}\int_{(3m_\pi)^2}^\infty\text{Im}\Phi^{n\to p}(-m^2)d(m^2)}{\langle m^2\rangle_\Phi{}^{n\to p}a_\pi f_{\pi np}}\right], \quad (4)$$

$$F_P^{n\to p}(0)=-a_\pi f_{\pi np}+\frac{1}{\pi}\int_{(3m_\pi)^2}^\infty\frac{\text{Im}F_P^{n\to p}(-m^2)}{m^2}d(m^2)=-a_\pi f_{\pi np}\left[1-\frac{m_\pi^2}{\langle m^2\rangle_P{}^{n\to p}}\right].$$

In Eqs. (1)–(4), u_e, $u_{\bar{\nu}}$, u_p, and u_n are electron, antineutrino, proton, and neutron spinors; $j_\alpha^{(V)}$ and $j_\alpha^{(A)}$ are polar-vector and axial-vector hadron weak currents; $F_V^{n\to p}(q^2)$, $F_M^{n\to p}(q^2)$, $F_A^{n\to p}(q^2)$, and $F_P^{n\to p}(q^2)$ are polar-vector, weak-magnetism, axial-vector, and induced-pseudoscalar neutron→proton weak form factors; $\mu(p)$ and $\mu(n)$ are proton and neutron magnetic moments (in units of $e/2m_p$); $a_\pi\equiv F_A^{\pi\to\text{vac}}(p_\pi^2=-m_\pi^2)$ is the axial-vector pion → vacuum weak form factor determined numerically from the observed $\pi^+\to\mu^++\nu_\mu$ decay rate as $|a_\pi|=0.95\pm0.01^2$; $f_{\pi np}\equiv f_{\pi np}(p_n^2=-m_n^2, p_p^2=-m_p^2, p_\pi^2=(p_n-p_p)^2=-m_\pi^2)$ is the pion-neutron-proton vertex function evaluated at $p_n^2=-m_n^2$, $p_p^2=-m_p^2$, $p_\pi^2=(p_n-p_p)^2=-m_\pi^2$, i.e., $f_{\pi pn}$ is the pion-neutron-proton coupling constant, given

[2] See, e.g., H. Primakoff, *Proceedings of the International School of Physics "Enrico Fermi," 1964, Course 32: Weak Interactions and High Energy Neutrino Physics* (Academic Press Inc., New York, to be published).

on the basis of dispersion-theoretic analysis of $\pi^{\pm}+p\rightarrow\pi^{\pm}+p$ elastic-scattering experiments[3] or, somewhat less accurately, on the basis of a polological analysis of $n+p\rightarrow p+n$ nucleon charge-exchange scattering experiments,[4] by $f_{\pi np}=\sqrt{2}f_{\pi^0pp}=\sqrt{2}(4\pi)^{1/2}(0.081\pm0.003)^{1/2}=1.43\pm0.03.$[3]

We note that $m_\pi^2/\langle m^2\rangle_P{}^{n\rightarrow p}\lessapprox m_\pi^2/(3m_\pi)^2=0.11$ so that $F_P{}^{n\rightarrow p}(q^2)$ is indeed dominated by the pion-pole term $-m_\pi^2a_\pi f_{\pi np}/(m_\pi^2+q^2)$ for $-m_\pi^2\leq q^2\lessapprox 0$. If we assume that

$$\langle m^2\rangle_\Phi{}^{n\rightarrow p}\approx\langle m^2\rangle_P{}^{n\rightarrow p},\left|\int_{(3m_\pi)^2}^{\infty}\mathrm{Im}\Phi^{n\rightarrow p}(-m^2)d(m^2)\right|\approx\left|\int_{(3m_\pi)^2}^{\infty}\mathrm{Im}F_P{}^{n\rightarrow p}(-m^2)d(m^2)\right| \tag{5}$$

and use Eqs. (3) and (4), we see that a similar pion-pole dominance also characterizes $\Phi^{n\rightarrow p}(q^2)$ and we can write, up to errors $\approx10\%$,

$$G_A(n\rightarrow p)\cong a_\pi f_{\pi np}\cong -F_P{}^{n\rightarrow p}(0). \tag{6}$$

Equation (6) is the Goldberger-Treiman (G-T) relation; since on the basis of the measured ${}_0n_1{}^1$ and ${}_8O_6{}^{14}$ beta-decay rates one obtains $G_A(n\rightarrow p)=1.19\pm0.03$,[5] and since, as mentioned above, $|a_\pi|=0.95\pm0.01$,[2] the value of $f_{\pi np}$ deduced from the first equality in the G-T relation of Eq. (6) is

$$f_{\pi np}=(1.19\pm0.03)/(0.95\pm0.01)=1.25\pm0.04. \tag{7}$$

This value differs by 13% from the above mentioned pion-nucleon elastic scattering value: $f_{\pi np}=1.43\pm0.03$[3]; the relatively small discrepancy is presumably due to the neglect of the contribution of higher mass states in passing from Eq. (4) to Eq. (6). In addition, analysis of the measured muon-capture rates in ${}_1H_0{}^1$ and ${}_2He_1{}^3$ indicates that $-F_p{}^{n\rightarrow p}(0)$ lies between 1.0 and 1.7[6] so that the second equality in the G-T relation of Eq. (6) is also consistent with available experimental information.

We proceed to extend Eqs. (1)–(6) to *nuclear* beta decay: $N_i\rightarrow N_f+e^-+\bar{\nu}_e$. The customary theory assumes

$$\begin{aligned}
&\langle e^-\bar{\nu}_eN_f|\mathfrak{L}(0)|N_i\rangle=(G/\sqrt{2})[u_e{}^\dagger\gamma_4\gamma_\alpha(1+\gamma_5)u_{\bar{\nu}}{}^*]\{\langle N_f|j_\alpha{}^{(V)}(0)|N_i\rangle+\langle N_f|j_\alpha{}^{(A)}(0)|N_i\rangle\},\\
&\langle N_f|j_\alpha{}^{(V)}(0)|N_i\rangle=\langle\Psi_{N_f}(\cdots\mathbf{r}^{(a)},\sigma_3{}^{(a)},\tau_3{}^{(a)},\cdots)|J_\alpha{}^{(V)}|\Psi_{N_i}(\cdots\mathbf{r}^{(a)},\sigma_3{}^{(a)},\tau_3{}^{(a)},\cdots)\rangle,\\
&\langle N_f|j_\alpha{}^{(A)}(0)|N_i\rangle=\langle\Psi_{N_f}(\cdots\mathbf{r}^{(a)},\sigma_3{}^{(a)},\tau_3{}^{(a)},\cdots)|J_\alpha{}^{(A)}|\Psi_{N_i}(\cdots\mathbf{r}^{(a)},\sigma_3{}^{(a)},\tau_3{}^{(a)},\cdots)\rangle,
\end{aligned} \tag{8}$$

with

$$\begin{aligned}
J_\alpha{}^{(V)}&=\sum_{a=1}^{A}\tau_+{}^{(a)}\gamma_4{}^{(a)}\left[\gamma_\alpha{}^{(a)}F_V{}^{n\rightarrow p}(q^2)-\frac{\sigma_{\alpha\beta}{}^{(a)}q_\beta}{2m_p}F_M{}^{n\rightarrow p}(q^2)\right]e^{i\mathbf{q}\cdot\mathbf{r}^{(a)}},\\
J_\alpha{}^{(A)}&=\sum_{a=1}^{A}\tau_+{}^{(a)}\gamma_4{}^{(a)}\left[\gamma_\alpha{}^{(a)}\gamma_5{}^{(a)}F_A{}^{n\rightarrow p}(q^2)+\frac{iq_\alpha(m_p+m_n)}{m_\pi^2}\gamma_5{}^{(a)}F_P{}^{n\rightarrow p}(q^2)\right]e^{i\mathbf{q}\cdot\mathbf{r}^{(a)}},\\
q&\equiv-(p_e+p_{\bar{\nu}}),
\end{aligned} \tag{9}$$

whence, in the "allowed" approximation,

$$J_\alpha{}^{(V)}=\sum_{a=1}^{A}\tau_+{}^{(a)}[\delta_{\alpha4}G_V(n\rightarrow p)],\quad J_\alpha{}^{(A)}=\sum_{a=1}^{A}\tau_+{}^{(a)}[(1-\delta_{\alpha4})i\sigma_\alpha{}^{(a)}G_A(n\rightarrow p)]. \tag{10}$$

In Eq. (8), Ψ_{N_i}, Ψ_{N_f} are wave functions describing the nuclear states $|N_i\rangle$ and $|N_f\rangle$, and $\mathbf{r}^{(a)}$, $\sigma_3{}^{(a)}$, $\tau_3{}^{(a)}$ are position, spin, and isospin coordinates of the ath physical nucleon. The above mentioned impulse approximation corresponds to the representation of $J_\alpha{}^{(V)}$, $J_\alpha{}^{(A)}$ in Eqs. (9) and (10) as a sum of terms each one of which refers to the beta decay of a physical nucleon within the nucleus with a weak-interaction Lagrangian identical with that of an isolated physical nucleon. Actually, pion-exchange terms of the form

$$J_\alpha{}^{(\mathrm{exch})}\approx\left(\frac{f_{\pi^0pp}{}^2}{4\pi}\right)^2\sum_{a=1,b=1}^{A}(\tau_+{}^{(b)}-\tau_+{}^{(a)})\{[\gamma_4{}^{(a)}\gamma_\alpha{}^{(a)}\gamma_5{}^{(a)}]e^{i\mathbf{q}\cdot\mathbf{r}^{(a)}}-[\gamma_4{}^{(b)}\gamma_\alpha{}^{(b)}\gamma_5{}^{(b)}]e^{i\mathbf{q}\cdot\mathbf{r}^{(b)}}\}\frac{e^{-m_\pi|\mathbf{r}^{(a)}-\mathbf{r}^{(b)}|}}{m_\pi|\mathbf{r}^{(a)}-\mathbf{r}^{(b)}|}F_A{}^{n\rightarrow p}(q^2) \tag{11}$$

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[3] See e.g., J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[4] See A. Ashmore, W. H. Range, R. T. Taylor, B. M. Townes, L. Castillejo, and R. F. Peierls, Nucl. Phys. **36**, 258 (1962). The method was originally suggested by G. F. Chew, Phys. Rev. **112**, 1380 (1958) and is rather fully discussed by M. J. Moravcsik in *Dispersion Relations, 1960 Scottish Universities' Summer School* (Oliver and Boyd, Edinburgh, 1961), p. 117.

[5] C. S. Wu, as quoted in A. Halpern, Phys. Rev. Letters **13**, 660 (1964); our $G_A(n\rightarrow p)$ is the negative of the conventionally defined axial-vector neutron $\rightarrow$ proton weak coupling constant.

[6] The G-T value of $-F_P{}^{n\rightarrow p}(0)$ given in Eq. (6): $-F_P{}^{n\rightarrow p}(0)=a_\pi f_{\pi np}=1.36$ corresponds to an effective-for-muon-capture induced-pseudoscalar neutron$\rightarrow$proton weak coupling constant:

$$\left[\frac{m_\mu(m_p+m_n)}{m_\pi^2}\right]F_P{}^{n\rightarrow p}(q^2=0.9m_\mu^2)\cong\left[\frac{m_\mu(m_p+m_n)}{m_\pi^2}\right]\left(\frac{m_\pi^2}{m_\pi^2+0.9m_\mu^2}\right)F_P{}^{n\rightarrow p}(0)=-8.93=7.5[-G_A(n\rightarrow p)].$$

should be adjoined to the $J_\alpha^{(V)}+J_\alpha^{(A)}$ of Eq. (9). It can be shown that in the "allowed" approximation we have[1]

$$\langle\Psi_{N_f}|J_\alpha^{(\text{exch})}|\Psi_{N_i}\rangle\approx\left\{\left(\frac{f_{\pi^0pp}^2}{4\pi}\right)^2 4A\left[m_\pi\left(\frac{0.8}{m_\pi}A^{1/3}\right)\right]^{-3}\right\}\langle\Psi_{N_f}|\sum_{a=1}^{A}\tau_+^{(a)}[(1-\delta_{\alpha 4})i\sigma_\alpha^{(a)}G_A(n\rightarrow p)]|\Psi_{N_i}\rangle \quad (12)$$

so that the impulse approximation should be accurate to something like 10%.

We now set down the basic equations of the theory outlined in the Introduction where the nuclei which participate in the beta decay are treated as "elementary" particles. Confining ourselves for the time being to nuclear beta-decay transitions of the type

$$[N_i\colon (J^{(P)};T)_i=\tfrac{1}{2}^{(\pm)};\tfrac{1}{2}]\rightarrow[N_f\colon (J^{(P)};T)_f=\tfrac{1}{2}^{(\pm)};\tfrac{1}{2}]+e^-+\bar{\nu}_e,$$

we have, on the basis of the validity of the CVC and PCAC hypotheses, and analogously to Eqs. (1)–(4),

$$\begin{aligned}
\langle e^-\bar{\nu}_eN_f|\mathcal{L}(0)|N_i\rangle&=(G/\sqrt{2})[u_e^\dagger\gamma_4\gamma_\alpha(1+\gamma_5)u_{\bar{\nu}}^*]\{\langle N_f|j_\alpha^{(V)}(0)|N_i\rangle+\langle N_f|j_\alpha^{(A)}(0)|N_i\rangle\},\\
\langle N_f|j_\alpha^{(V)}(0)|N_i\rangle&=\{u_{N_f}^\dagger\tau_+\gamma_4[\gamma_\alpha F_V^{N_i\rightarrow N_f}(q^2)-(\sigma_{\alpha\beta}q_\beta/2m_p)F_M^{N_i\rightarrow N_f}(q^2)]u_{N_i}\},\\
\langle N_f|j_\alpha^{(A)}(0)|N_i\rangle&=\{u_{N_f}^\dagger\tau_+\gamma_4[\gamma_\alpha\gamma_5F_A^{N_i\rightarrow N_f}(q^2)+[iq_\alpha(m_{N_f}+m_{N_i})/m_\pi^2]\gamma_5F_P^{N_i\rightarrow N_f}(q^2)]u_{N_i}\},\\
\langle N_f|\partial j_\alpha^{(A)}(0)/\partial x_\alpha|N_i\rangle&=-iq_\alpha\langle N_f|j_\alpha^{(A)}(0)|N_i\rangle\\
&=(m_{N_i}+m_{N_f})[F_A^{N_i\rightarrow N_f}(q^2)+(q^2/m_\pi^2)F_P^{N_i\rightarrow N_f}(q^2)](u_{N_f}^\dagger\tau_+\gamma_4\gamma_5u_{N_i})\\
&\equiv(m_{N_f}+m_{N_i})\Phi^{N_i\rightarrow N_f}(q^2)(u_{N_f}^\dagger\tau_+\gamma_4\gamma_5u_{N_i});\\
G&=(1.0\times10^{-5})/m_p^2;\quad q\equiv-(p_e+p_{\bar{\nu}})=(p_{N_f}-p_{N_i}),
\end{aligned} \quad (13)$$

with

$$F_V^{N_i\rightarrow N_f}(0)\equiv G_V(N_i\rightarrow N_f)=Z(N_f)-Z(N_i)=1,\quad F_M^{N_i\rightarrow N_f}(0)=[\mu(N_f)-Z(N_f)/A]-[\mu(N_i)-Z(N_i)/A] \quad (14)$$

and

$$\begin{aligned}
\Phi^{N_i\rightarrow N_f}(0)=F_A^{N_i\rightarrow N_f}(0)\equiv G_A(N_i\rightarrow N_f)&=a_\pi f_{\pi N_iN_f}+\frac{1}{\pi}\int_{m^2{}_{an}}^\infty\frac{\text{Im}\Phi^{N_i\rightarrow N_f}(-m^2)}{m^2}d(m^2)\\
&=a_\pi f_{\pi np}\left[1+\frac{\frac{1}{\pi}\int_{m^2{}_{an}}^\infty \text{Im}\Phi^{N_i\rightarrow N_f}(-m^2)d(m^2)}{\langle m^2\rangle_\Phi{}^{N_i\rightarrow N_f}a_\pi f_{\pi N_iN_f}}\right],\\
F_P^{N_i\rightarrow N_f}(0)=-a_\pi f_{\pi N_iN_f}+\frac{1}{\pi}\int_{m^2{}_{an}}^\infty\frac{\text{Im}F_P^{N_i\rightarrow N_f}(-m^2)}{m^2}d(m^2)&=-a_\pi f_{\pi N_iN_f}\left[1-\frac{m_\pi^2}{\langle m^2\rangle_P{}^{N_i\rightarrow N_f}}\right].
\end{aligned} \quad (15)$$

In Eqs. (13)–(15), u_{N_f} and u_{N_i} are spinors describing the motion as a whole of the final nucleus and the initial nucleus; $F_V^{N_i\rightarrow N_f}(q^2)$, $F_M^{N_i\rightarrow N_f}(q^2)$, $F_A^{N_i\rightarrow N_f}(q^2)$, and $F_P^{N_i\rightarrow N_f}(q^2)$ are polar-vector, weak-magnetism, axial-vector, and induced-pseudoscalar $N_i\rightarrow N_f$ weak form factors; $\mu(N_f)$ and $\mu(N_i)$ are magnetic moments of the final nucleus and the initial nucleus (again in units of $e/2m_p$);

$$f_{\pi N_iN_f}\equiv f_{\pi N_iN_f}(p_{N_i}^2=-m_{N_i}^2,\ p_{N_f}^2=-m_{N_f}^2,\ p_\pi^2=(p_{N_i}-p_{N_f})^2=-m_\pi^2)$$

is the pion–initial-nucleus–final-nucleus vertex function evaluated at $p_{N_i}^2=-m_{N_i}^2$, $p_{N_f}^2=-m_{N_f}^2$, $p_\pi^2=(p_{N_i}-p_{N_f})^2=-m_\pi^2$, i.e., $f_{\pi N_iN_f}$ is the pion–initial-nucleus–final-nucleus coupling constant; $m^2{}_{an}$ is the anomalous threshold squared mass value associated with the possibility of the process $({}_ZN_{A-Z}{}^A)_i\rightarrow({}_ZN_{A-Z-1}{}^{A-1})+n\rightarrow({}_ZN_{A-Z-1}{}^{A-1})+p+e^-+\bar{\nu}_e\rightarrow({}_{Z+1}N_{A-Z-1}{}^A)_f+e^-+\bar{\nu}_e$ and is given by formula $m_{an}{}^2=[8A/(A-1)]m_p\epsilon\cong(1.7m_\pi)^2$ where $\epsilon\cong8$ MeV$=0.057\ m_\pi$ is the binding energy of a nucleon to the nucleus.[7] On the basis of the impulse approximation of Eqs. (8), (9) we can then write an equation connecting $F^{N_i\rightarrow N_f}(q^2)$ with $F^{n\rightarrow p}(q^2)$

$$\begin{aligned}
&\frac{G}{\sqrt{2}}[u_e^\dagger\gamma_4\gamma_\alpha(1+\gamma_5)u_{\bar{\nu}}^*]\left\{iq_\alpha\frac{(m_{N_i}+m_{N_f})}{m_\pi^2}F_P^{N_i\rightarrow N_f}(q^2)\right\}[u_{N_f}^\dagger\tau_+\gamma_4\gamma_5u_{N_i}]\\
&\qquad\cong\frac{G}{\sqrt{2}}[u_e^\dagger\gamma_4\gamma_\alpha(1+\gamma_5)u_{\bar{\nu}}^*]\left\{iq_\alpha\frac{(m_n+m_p)}{m_\pi^2}F_P^{n\rightarrow p}(q^2)\right\}\langle\Psi_{N_f}|\sum_{a=1}^{A}\tau_+^{(a)}\gamma_4^{(a)}\gamma_5^{(a)}e^{i\mathbf{q}\cdot\mathbf{r}^{(a)}}|\Psi_{N_i}\rangle,
\end{aligned} \quad (16)$$

[7] See R. Karplus, C. M. Sommerfield, and E. H. Wichmann, Phys. Rev. **111**, 1187 (1958).

whence, using also Eqs. (15) and (4),

$$-a_\pi f_{\pi N_i N_f}+\frac{1}{\pi}\int_{m^2{}_{an}}^{\infty}\frac{\mathrm{Im}F_P{}^{N_i\to N_f}(-m^2)}{m^2}d(m^2)\cong -a_\pi f_{\pi np}\left\{\left(\frac{m_n+m_p}{m_{N_i}+m_{N_f}}\right)\left[\frac{\langle\Psi_{N_f}|\sum_{a=1}^{A}\tau_+{}^{(a)}\gamma_4{}^{(a)}\gamma_5{}^{(a)}|\Psi_{N_i}\rangle}{(u_{N_f}{}^\dagger\tau_+\gamma_4\gamma_5 u_{N_i})}\right]\right\}$$

$$+\frac{1}{\pi}\int_{(3m_\pi)^2}^{\infty}\frac{\mathrm{Im}F_P{}^{n\to p}(-m^2)}{m^2}d(m^2)\left\{\left(\frac{m_n+m_p}{m_{N_i}+m_{N_f}}\right)\left[\frac{\langle\Psi_{N_f}|\sum_{a=1}^{A}\tau_+{}^{(a)}\gamma_4{}^{(a)}\gamma_5{}^{(a)}|\Psi_{N_i}\rangle}{(u_{N_f}{}^\dagger\tau_+\gamma_4\gamma_5 u_{N_i})}\right]\right\}. \quad (17)$$

Clearly, a similar equation connects $\Phi^{N_i\to N_f}$ with $\Phi^{n\to p}$. Equation (17) shows that the contribution of the pion-pole term and that of the higher-mass cut term are multiplied by the *same* factor in passing from the $n\to p$ to the $N_i\to N_f$ case so that the extent of pion-pole dominance should not be appreciably different in these two cases. Thus, the pion-pole-dominance assumption for $\Phi^{N_i\to N_f}(q^2)$ and $F_P{}^{N_i\to N_f}(q^2)$ may be expected to hold about as well as for $\Phi^{n\to p}(q^2)$ and $F_P{}^{n\to p}(q^2)$ so that, analogously to Eq. (6), we have the Goldberger-Treiman relation

$$G_A(N_i\to N_f)\cong a_\pi f_{\pi N_i N_f}\cong -F_P{}^{N_i\to N_f}(0). \quad (18)$$

Equation (18) is fundamental in what follows.

We close the present section by appending formulas for ft values in the "allowed" approximation for nuclear beta-decay transitions of the type $[N_i\colon (J^{(P)};T)_i=\frac{1}{2}{}^{(\pm)};\frac{1}{2}]\to[N_f;(J^{(P)};T)_f=\frac{1}{2}{}^{(\pm)};\frac{1}{2}]+e^-+\bar\nu_e$. Thus, using Eqs. (13)–(15), we can write

$$[(ft)_{N_i\to N_f}]^{-1}\left(\frac{2\pi^3\ln 2}{G^2}\right)=[G_V(N_i\to N_f)]^2\{\sum_{M_f=\pm\frac{1}{2}}|(u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+u_{N_i;\cdots M_i\cdots})|^2\}$$
$$+[G_A(N_i\to N_f)]^2\{\sum_{M_f=\pm\frac{1}{2}}|(u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+\boldsymbol{\sigma}u_{N_i;\cdots M_i\cdots})|^2\}$$
$$=1\times1+[G_A(N_i\to N_f)]^2\times3$$
$$=1+(1.19)^2\left[\frac{G_A(N_i\to N_f)}{G_A(n\to p)}\right]^2\times3 \quad (19)$$

so that, expressing $G_A(N_i\to N_f)/G_A(n\to p)$ via the G-T relations of Eqs. (6) and (18),

$$G_A(N_i\to N_f)/G_A(n\to p)\cong f_{\pi N_i N_f}/f_{\pi np} \quad (20)$$

and substituting into Eq. (19),

$$[(ft)_{N_i\to N_f}]^{-1}(2\pi^3\ln 2/G^2)=1\times1+[G_A(n\to p)]^2(f_{\pi N_i N_f}/f_{\pi np})^2\times3$$
$$=1+(1.19)^2(f_{\pi N_i N_f}/f_{\pi np})^2\times3. \quad (21)$$

On the other hand, on the basis of the impulse approximation of Eqs. (8)–(10) together with the pion-exchange correction of Eq. (12), we have

$$\frac{G_A(N_i\to N_f)}{G_A(n\to p)}=\frac{(1+\xi)\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+{}^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle}{(u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+\boldsymbol{\sigma}u_{N_i;\cdots M_i\cdots})};$$

$$\left[\frac{G_A(N_i\to N_f)}{G_A(n\to p)}\right]^2=\frac{(1+\xi)^2\sum_{M_f=\pm\frac{1}{2}}|\langle\Psi_{N_f,\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+{}^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle|^2}{\sum_{M_f=\pm\frac{1}{2}}|(u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+\boldsymbol{\sigma}u_{N_i;\cdots M_i\cdots})|^2}, \quad (22)$$
$$=\tfrac{1}{3}(1+\xi)^2\sum_{M_f=\pm\frac{1}{2}}|\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+{}^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle|^2,$$

$$(1+\xi)^2\approx\left\{1+\left(\frac{f_{\pi^0pp}}{4\pi}\right)^2\frac{4A}{[m_\pi(0.8A^{1/3}/m_\pi)]^3}\right\}^2=1.10,$$

whence, substituting into Eq. (19),

$$[(ft)_{N_i\to N_f}]^{-1}\left(\frac{2\pi^3 \ln 2}{G^2}\right)=1\times 1+[G_A(n\to p)]^2(1+\xi)^2\{\sum_{M_f=\pm\frac{1}{2}}|\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+{}^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle|^2\}$$

$$=1+(1.19)^2(1+\xi)^2\{\sum_{M_f=\pm\frac{1}{2}}|\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+{}^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle|^2\}. \tag{23}$$

Finally, combination of Eq. (20) with Eq. (22) yields

$$\frac{f_{\pi N_i N_f}}{f_{\pi n p}}=\frac{(1+\xi)\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+{}^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle}{(u_{N_f;\cdots M_f\cdots}{}^{\dagger}\tau_+\boldsymbol{\sigma}u_{N_i;\cdots M_i\cdots})}, \tag{24}$$

which is consistent with an impulse-approximation expression for the transition matrix element of nuclear pion emission

$$[N_i\colon (J^{(P)};T)_i=\tfrac{1}{2}{}^{(\pm)};\tfrac{1}{2}]\to[N_f\colon (J^{(P)};T)_f=\tfrac{1}{2}{}^{(\pm)};\tfrac{1}{2}]+\pi^-$$

with the pion-exchange correction factor $(1+\xi)$ acting to renormalize the πnp vertex.

III. ESTIMATES FOR THE RATIO $(f_{\pi N_i N_f}/f_{\pi np})^2$

Values of ft in the "allowed" approximation for nuclear beta-decay transitions of the type

$$[N_i\colon (J^{(P)};T)_i=\tfrac{1}{2}{}^{(\pm)};\tfrac{1}{2}]\to[N_f\colon (J^{(P)};T)_f=\tfrac{1}{2}{}^{(\pm)};\tfrac{1}{2}]+e^-+\bar{\nu}_e \quad (\text{e.g., } {}_1\text{H}_2{}^3\to {}_2\text{He}_1{}^3+e^-+\bar{\nu}_e)$$

are, as we have seen in the last section, calculable from Eqs. (19)–(21) which, for purposes of numerical work, can be conveniently written as[8]

$$[(ft)_{N_i\to N_f}]^{-1}=[(ft)_{n\to p}]^{-1}\frac{1+(1.19)^2[G_A(N_i\to N_f)/G_A(n\to p)]^2\times 3}{1+(1.19)^2\times 3}$$

$$=[(ft)_{n\to p}]^{-1}\frac{1+(1.19)^2(f_{\pi N_i N_f}/f_{\pi np})^2\times 3}{1+(1.19)^2\times 3}; \tag{25}$$

$$(ft)_{n\to p}=1180\ \text{sec}^{-1}.$$

With this equation, and with experimental values of $(ft)_{N_i\to N_f}$, we can obtain $(f_{\pi N_i N_f}/f_{\pi np})^2=[G_A(N_i\to N_f)/G_A(n\to p)]^2$ and compare these "Goldberger-Treiman experimental" values of $(f_{\pi N_i N_f}/f_{\pi np})^2$ with values of $(f_{\pi N_i N_f}/f_{\pi np})^2$ deduced from a polological analysis of $n+N_f\to p+N_i$ nucleon charge-exchange scattering data or expressed, by means of a very crude theory, in terms of the magnetic moments of N_i and N_f (see below). Before embarking on such a comparison we note that a treatment of nuclear beta-decay transitions of the type

$$[N_i\colon(J^{(P)};T)_i=\tfrac{3}{2}{}^{(\pm)},\tfrac{5}{2}{}^{(\pm)},\tfrac{7}{2}{}^{(\pm)},\cdots;\tfrac{1}{2}]\to$$
$$[N_f\colon (J^{(P)};T)_f=\tfrac{3}{2}{}^{(\pm)},\tfrac{5}{2}{}^{(\pm)},\tfrac{7}{2}{}^{(\pm)},\cdots;\tfrac{1}{2}]+e^-+\bar{\nu}_e \quad (\text{e.g., } {}_6\text{C}_5{}^{11}\to {}_5\text{B}_6{}^{11}+e^++\nu_e),$$

wholly analogous to that given in Eqs. (13)–(24) for $(J^{(P)})_i=(J^{(P)})_f=\frac{1}{2}{}^{(\pm)}$, yields (see Appendix I)

$$[(ft)_{N_i\to N_f}]^{-1}=[(ft)_{n\to p}]^{-1}\frac{1+(1.19)^2[G_A(N_i\to N_f)/G_A(n\to p)]^2\times(J+1)/J}{1+(1.19)^2\times 3}$$

$$=[(ft)_{n\to p}]^{-1}\frac{1+(1.19)^2(f_{\pi N_i N_f}/f_{\pi np})^2\times(J+1)/J}{1+(1.19)^2\times 3}; \tag{26}$$

$$(ft)_{n\to p}=1180\ \text{sec}^{-1}$$

[8] A. N. Sosnovskii, P. E. Spivak, I. A. Prokofiev, I. E. Kutikov, and I. P. Dobrinin, Zh. Eksperim. i Teor. Fiz. 35, 1059 (1958) [English transl.: Soviet Phys.—JETP 8, 739 (1959)].

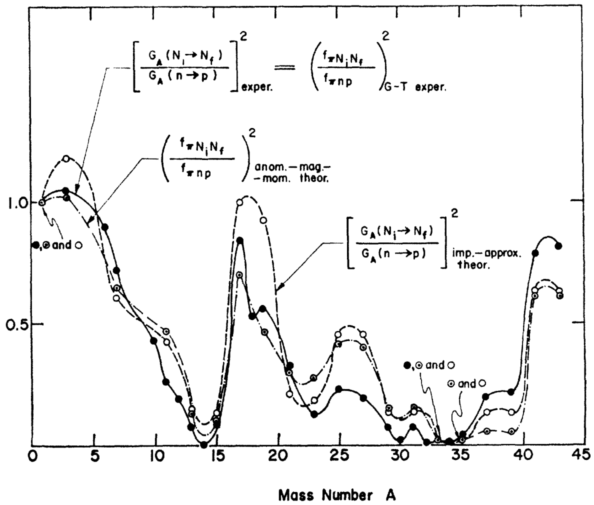

FIG. 1. Comparison of theoretical and experimental values of $[G_A(N_i\rightarrow N_f)/G_A(n\rightarrow p)]^2$.

which holds for $J=\frac{3}{2},\frac{5}{2},\frac{7}{2},\cdots$, and, in fact, reduces to Eq. (25) for $J=\frac{1}{2}$. Equation (26) yields "G-T experimental" values of

$$\left(\frac{f_{\pi N_iN_f}}{f_{\pi np}}\right)^2=\left[\frac{G_A(N_i\rightarrow N_f)}{G_A(n\rightarrow p)}\right]^2 \tag{27}$$

for all nuclear beta-decay transitions of the type

$$[N_i\colon (J^{(P)};T)_i=\tfrac{1}{2}^{(\pm)},\tfrac{3}{2}^{(\pm)},\tfrac{5}{2}^{(\pm)},\tfrac{7}{2}^{(\pm)},\cdots;\tfrac{1}{2}]\rightarrow[N_f\colon (J^{(P)};P)_f=\tfrac{1}{2}^{(\pm)},\tfrac{3}{2}^{(\pm)},\tfrac{5}{2}^{(\pm)},\tfrac{7}{2}^{(\pm)},\cdots;\tfrac{1}{2}]+e^-+\bar{\nu}_e;$$

the results are shown in the fourth column of Table I and in the solid curve of Fig. 1 and exhibit a strikingly non-monotonic dependence of $(f_{\pi N_iN_f}/f_{\pi np})^2$ on the mass number A of N_i and N_f.

We now describe an extremely crude theoretical derivation of these values of $(f_{\pi N_iN_j}/f_{\pi np})^2$—our derivation is in the spirit of the semiclassical meson-theoretic treatment of the isovector anomalous magnetic moment of the isobaric doublet pair: proton and neutron. On the basis of such a treatment we can write,[9]

$$|[\mu(p)-1]-[\mu(n)-0]|=kf^2_{\pi np} \tag{28}$$

where k is a numerical constant and $|(\mu(p)-1)-(\mu(n)-0)|=3.70$ [Eq. (2)]. In a similar way we can set down an expression for the isovector anomalous magnetic moment of the odd-A isobaric doublet pair: N_i and N_f,

$$\left|\left(\mu(N_f)-\frac{Z(N_f)}{A}\right)-\left(\mu(N_i)-\frac{Z(N_i)}{A}\right)\right|=kf^2_{\pi N_iN_f}g(A);\quad g(1)=1\,, \tag{29}$$

where $g(A)$ is a more or less smoothly varying function of A. We have been unable to devise a convincing *a priori* specification of $g(A)$ and make the *a posteriori* choice: $g(A)=A^{1/3}$ in order to obtain a good over-all fit to the experimental values of $(ft)_{N_i\rightarrow N_f}$. Equations (28) and (29) yield

$$\left(\frac{f_{\pi N_iN_f}}{f_{\pi np}}\right)^2=\frac{|[\mu(N_f)-Z(N_f)/A]-[\mu(N_i)-Z(N_i)/A]|}{|(\mu(p)-1)-(\mu(n)-0)|}\frac{1}{g(A)}=\frac{|[\mu(N_f)-Z(N_f)/A]-[\mu(N_i)-Z(N_i)/A]|}{3.70A^{1/3}} \tag{30}$$

and this equation, together with experimental values of $\mu(N_f)$ and $\mu(N_i)$, yields "anomalous-magnetic-moment theoretical" values of $(f_{\pi N_iN_f}/f_{\pi np})^2$ shown in the fifth column of Table I and in the dash-dotted curve of Fig. 1—the overall agreement between these values of $(f_{\pi N_iN_f}/f_{\pi np})^2{}_{\text{anom-mag-mom theor}}$ and the corresponding values of $(f_{\pi N_iN_f}/f_{\pi np})^2{}_{\text{G-T exper}}$ (fourth column of Table I and solid curve of Fig. 1) lends some confidence to the calculation of $(f_{\pi N_iN_f}/f_{\pi np})^2$ from $|[\mu(N_f)-Z(N_f)/A]-[\mu(N_i)-Z(N_i)/A]|\cdot|(\mu(p)-1)-(\mu(n)-0)|^{-1}\cdot A^{-1/3}$ [Eq. (30)] and to the G-T identification of $(f_{\pi N_iN_f}/f_{\pi np})^2$ with $[G_A(N_i\rightarrow N_f)/G_A(n\rightarrow p)]^2$ [Eq. (27)]. It is also of interest to calculate $[G_A(N_i\rightarrow N_f)/G_A(n\rightarrow p)]^2$ from the impulse-approximation based Eq. (22) (generalized to any half-integral J) using appropriate nuclear models to specify $\Psi_{N_i;\cdots M_i\cdots}$ and $\Psi_{N_f;\cdots M_f\cdots}$ (see Appendix II); these values

[9] See e.g., J. D. Jackson, *The Physics of Elementary Particles* (Princeton University Press, Princeton, 1958), p. 44.

TABLE I. Comparison of theory with experiment.

Transition	$J_i \to J_f$	$(ft)_{\text{exper}}$	$\left[\frac{G_A(N_i\to N_f)}{G_A(n\to p)}\right]^2_{\text{exper}} = \left(\frac{f_{\pi N_i N_f}}{f_{\pi np}}\right)^2_{\text{G-T exper}}$	$\left(\frac{f_{\pi N_i N_f}}{f_{\pi np}}\right)^{2\,a}_{\text{anom-mag-mom theor}}$	$\left[\frac{G_A(N_i\to N_f)}{G_A(n\to p)}\right]^{2\,a}_{\text{imp-approx theor}}$
$_0n_1^1 \to {}_1p_0^1$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	1180 ± 35	1	1	1
$_1H_2^3 \to {}_2He_1^3$	$\frac{1}{2}\to\frac{1}{2}$	1137 ± 20	1.048	1.020	1.180, 0.982[b]
$_2He_4^6 \to {}_3Li_3^6$	$0\to 1$	810 ± 30	0.902	...	...
$_4Be_3^7 \to {}_3Li_4^7$	$\frac{3}{2}\to\frac{3}{2}$	2300 ± 80	0.719	0.646	0.605
$_6C_4^{10} \to {}_5B_5^{10}$ *	$0\to 1$	1700 ± 150	0.43	...	...
$_6C_5^{11} \to {}_5B_6^{11}$	$\frac{3}{2}\to\frac{3}{2}$	3840 ± 70	0.261	0.462	0.425
$_5B_7^{12} \to {}_6C_6^{12}$	$1\to 0$	$\approx 10^{4.1}$	0.19	...	...
$_7N_6^{13} \to {}_6C_7^{13}$	$\frac{1}{2}\to\frac{1}{2}$	4700 ± 80	0.075	0.126	0.144
$_6C_8^{14} \to {}_7N_7^{14}$	$0\to 1$	$\approx 10^{9.0}$	7.3×10^{-7}	...	...
$_8O_6^{14} \to {}_7N_7^{14}$	$0\to 1$	$\approx 10^{7.6}$	1.8×10^{-5}	...	...
$_8O_7^{15} \to {}_7N_8^{15}$	$\frac{1}{2}\to\frac{1}{2}$	4475 ± 30	0.091	0.103	0.138
$_9F_8^{17} \to {}_8O_9^{17}$	$\frac{5}{2}\to\frac{5}{2}$	2330 ± 80	0.837	0.679	0.995, 0.995[c]
$_9F_9^{18} \to {}_8O_{10}^{18}$	$1\to 0$	4170 ± 160	0.525	...	...
$_{10}Ne_9^{19} \to {}_9F_{10}^{19}$	$\frac{1}{2}\to\frac{1}{2}$	1840 ± 50	0.558	0.461	0.922, 0.693[c]
$_{11}Na_{10}^{21} \to {}_{10}Ne_{11}^{21}$	$\frac{3}{2}\to\frac{3}{2}$	3500 ± 200	0.326	0.294	0.208, 0.246[c]
$_{12}Mg_{11}^{23} \to {}_{11}Na_{12}^{23}$	$\frac{3}{2}\to\frac{3}{2}$	4780 ± 150	0.125	0.277	0.185, 0.204[c]
$_{13}Al_{12}^{25} \to {}_{12}Mg_{13}^{25}$	$\frac{5}{2}\to\frac{5}{2}$	4280 ± 350	0.226	0.412	0.450, 0.507[c]
$_{14}Si_{13}^{27} \to {}_{13}Al_{14}^{27}$	$\frac{5}{2}\to\frac{5}{2}$	4500 ± 100	0.191	0.400	0.450, 0.507[c]
$_{14}Si_{18}^{32} \to {}_{15}P_{17}^{32}$	$0\to 1$	$\approx 10^{7.7}$	1.5×10^{-5}	...	...
$_{15}P_{14}^{29} \to {}_{14}Si_{15}^{29}$	$\frac{1}{2}\to\frac{1}{2}$	4750 ± 200	0.072	0.145	0.130, 0.067[c]
$_{15}P_{15}^{30} \to {}_{14}Si_{16}^{30}$	$1\to 0$	$\approx 10^{5.2}$	0.014	...	...
$_{16}S_{15}^{31} \to {}_{15}P_{16}^{31}$	$\frac{1}{2}\to\frac{1}{2}$	4820 ± 250	0.068	0.142	0.130, 0.067[c]
$_{17}Cl_{16}^{33} \to {}_{16}S_{17}^{33}$	$\frac{2}{3}\to\frac{2}{3}$	6000 ± 500	0.014	0.018	0.013, 0.021[c]
$_{15}P_{19}^{34} \to {}_{16}S_{18}^{34}$	$1\to 0$	$\approx 10^{5.2}$	0.014	...	...
$_{18}A_{17}^{35} \to {}_{17}Cl_{18}^{35}$	$\frac{3}{2}\to\frac{3}{2}$	5680 ± 400	0.039	0.018	0.013, 0.021[c]
$_{19}K_{18}^{37} \to {}_{18}A_{19}^{37}$	$\frac{3}{2}\to\frac{3}{2}$	4250 ± 500	0.195	0.052	0.128, 0.217[c]
$_{20}Ca_{19}^{39} \to {}_{19}K_{20}^{39}$	$\frac{3}{2}\to\frac{3}{2}$	4150 ± 300	0.209	0.051	0.128, 0.228[c]
$_{21}Sc_{20}^{41} \to {}_{20}Ca_{21}^{41}$	$\frac{7}{2}\to\frac{7}{2}$	2560 ± 160	0.780	0.605	0.621
$_{22}Ti_{21}^{43} \to {}_{21}Sc_{22}^{43}$	$\frac{7}{2}\to\frac{7}{2}$	2500 ± 300	0.812	0.605	0.621

[a] In connection with the calculation of the $[G_A(N_i\to N_f)/G_A(n\to p)]^2_{\text{imp-approx theor}}$ from Eqs. (A19) and (A18) and of $(f_{\pi N_i N_f}/f_{\pi np})^2_{\text{anom-mag-mom theor}}$ from Eqs. (30) or (A20), it is to be noted that $\mu(N_i)$ has been measured only for $_1H_2^3$, $_6C_5^{11}$, $_7N_6^{13}$, $_8O_7^{15}$, $_{10}Ne_9^{19}$, $_{11}Na_{10}^{21}$ [see Table I of O. Ames, E. A. Phillips, and S. S. Glickstein, Phys. Rev. **137**, B1157 (1965)] and $_{18}A_{17}^{35}$ [see F. P. Calaprice, E. D. Commins, and D. A. Dobson, Phys. Rev. **137**, B1453 (1965)]. For other N_i, e.g., $N_i = {}_{17}Cl_{16}^{33}$, we take $\mu({}_{17}Cl_{16}^{33}) = \mu({}_{17}Cl_{18}^{35})$ and this last has been measured since $_{17}Cl_{18}^{35}$ is stable.

[b] Calculated from Eq. (A12) with Ψ_{N_f}, Ψ_{N_i} characterized by 4% D state and 2% S' state, or by 6% D state and 1% S' state, and with $(1+\xi)^2=1.10$. [See, e.g., R. J. Blin-Stoyle, Phys. Rev. Letters **13**, 55 (1964).]

[c] Calculated from Eq. (A12) with deformed-core model Ψ_{N_f}, Ψ_{N_i} and with $(1+\xi)^2=1$. [See C. W. Kim, Nucl. Phys. **49**, 383 (1963) and also J. C. Hardy and B. Margolis, Phys. Letters **15**, 276 (1965).]

of $[G_A(N_i\to N_f)/G_A(n\to p)]^2{}_{\text{imp-approx theor}}$ [Eqs. (A19) and (A18) below] are shown in the sixth column of Table I and in the dashed curve of Fig. 1 and agree no better (in fact, somewhat worse) with the $[G_A(N_i\to N_f)/G_A(n\to p)]^2{}_{\text{exper}}$ (fourth column of Table I and solid curve of Fig. 1) than do the $(f_{\pi N_iN_f}/f_{\pi np})^2{}_{\text{anom-mag-mom theor}}$ (fifth column of Table I and dash-dotted curve of Fig. 1) with the $(f_{\pi N_iN_f}/f_{\pi np})^2{}_{\text{G-T exper}}$ (fourth column of Table I and solid curve of Fig. 1).

We proceed to discuss nuclear beta-decay transitions of the type

$$[N_i\colon (J^{(P)};T)_i=0^{(+)};1]\to[N_f\colon (J^{(P)};T)_f=1^{(+)};0]+e^-+\bar\nu_e\quad (\text{e.g., } {}_2\text{He}_4{}^6\to{}_3\text{Li}_3{}^6+e^-+\bar\nu_e);$$

the nuclei N_i, N_f are here again treated as elementary particles. We then have, on the basis of the CVC and PCAC hypotheses, and with neglect of certain relatively small terms,

$$\langle e^-\bar\nu_eN_f|\mathcal{L}(0)|N_i\rangle=(G/\sqrt2)[u_e{}^\dagger\gamma_4\gamma_\alpha(1+\gamma_5)u_{\bar\nu}{}^*]\{\langle N_f|j_\alpha{}^{(V)}(0)|N_i\rangle+\langle N_f|j_\alpha{}^{(A)}(0)|N_i\rangle\},$$
$$\langle N_f|j_\alpha{}^{(V)}(0)|N_i\rangle=-\{u_{N_f}{}^\dagger[\epsilon_{\alpha\beta\gamma}S_\gamma(q_\beta/2m_p)F_M{}^{N_i\to N_f}(q^2)]u_{N_i}\},$$
$$\langle N_f|j_\alpha{}^{(A)}(0)|N_i\rangle=\{u_{N_f}{}^\dagger[iS_\alpha F_A{}^{N_i\to N_f}(q^2)+(iq_\alpha(S_\beta q_\beta)/m_\pi{}^2)F_P{}^{N_i\to N_f}(q^2)]u_{N_i}\},$$
$$\begin{aligned}\langle N_f|\partial j_\alpha{}^{(A)}(0)/\partial x_\alpha|N_i\rangle&=-iq_\alpha\langle N_f|j_\alpha{}^{(A)}(0)|N_i\rangle\\&=[F_A{}^{N_i\to N_f}(q^2)+(q^2/m_\pi{}^2)F_P{}^{N_i\to N_f}(q^2)]\cdot[u_{N_f}{}^\dagger(S_\beta q_\beta)u_{N_i}]\equiv\Phi^{N_i\to N_f}(q^2)[u_f{}^\dagger(S_\beta q_\beta)u_{N_i}];\\G&=(1.0\times10^{-5})/m_p{}^2;\quad q\equiv-(p_e+p_{\bar\nu})=(p_{N_f}-p_{N_i}),\end{aligned}\tag{31}$$

with

$$F_M{}^{N_i\to N_f}(0)=\sqrt2\mu([0^{(+)};1]\to[1^{(+)};0])$$
$$\begin{aligned}\Phi^{N_i\to N_f}(0)=F_A{}^{N_i\to N_f}(0)\equiv G_A(N_i\to N_f)&=a_\pi f_{\pi N_iN_f}+\frac1\pi\int_{m^2{}_{an}}^\infty\frac{\text{Im}\Phi^{N_i\to N_f}(-m^2)}{m^2}d(m^2)\\&=a_\pi f_{\pi N_iN_f}\left\{1+\frac{\dfrac1\pi\displaystyle\int_{m^2{}_{an}}^\infty\text{Im}\Phi^{N_i\to N_f}(-m^2)d(m^2)}{\langle m^2\rangle_\Phi{}^{N_i\to N_f}a_\pi f_{\pi N_iN_f}}\right\},\end{aligned}\tag{32}$$
$$F_P{}^{N_i\to N_f}(0)=-a_\pi f_{\pi N_iN_f}+\frac1\pi\int_{m^2{}_{an}}^\infty\frac{\text{Im}F_P{}^{N_i\to N_f}(-m^2)}{m^2}d(m^2)=-a_\pi f_{\pi N_iN_f}\left\{1-\frac{m_\pi{}^2}{\langle m^2\rangle_P{}^{N_i\to N_f}}\right\}.$$

In Eqs. (31) and (32), u_{N_f} and u_{N_i} are spinors describing the motion as a whole of the final (spin-1) nucleus N_f and the initial (spin-0) nucleus N_i; $(u^\dagger{}_{N_f;\cdots M_f=1,0,-1\cdots}S_\alpha u_{N_i;\cdots M_i=0\cdots})$ is to be understood as $[S(S+1)]^{1/2}(\xi_\alpha(M_f))^*$ where $\xi_\alpha(M_f)$ is a spin-1-type polarization four-vector orthogonal to $(p_{N_f})_\alpha$ and $S=1$, $F_A{}^{N_i\to N_f}(q^2)$, and $F_P{}^{N_i\to N_f}(q^2)$ are weak-magnetism, axial-vector, and induced-pseudoscalar $N_i\to N_f$ weak form factors; $u([0^{(+)};1]\to[1^{(+)};0])$ is the transition magnetic moment to the ground state of N_f from an excited state of N_f with the same quantum numbers (except for T_3) as the ground state of N_i; as before, $f_{\pi N_iN_f}$ is the pion–initial-nucleus–final-nucleus coupling constant. Assuming further that the pion-pole-dominance assumption is also valid in this case [see the analogous discussion after Eqs. (16) and (17) and also Eqs. (18) and (20)] we have the Goldberger-Treiman relation

$$G_A(N_i\to N_f)\cong a_\pi f_{\pi N_iN_f}\cong-F_P{}^{N_i\to N_f}(0),\quad G_A(N_i\to N_f)/G_A(n\to p)\cong f_{\pi N_iN_f}/f_{\pi np},\tag{33}$$

wholly analogous to Eq. (18). From Eq. (33) we can calculate the ft values in the "allowed" approximation for nuclear beta-decay transitions of the type $[N_i\colon (J^{(P)};T)_i=0^{(+)};1]\to[N_f\colon (J^{(P)};T)_f=1^{(+)};0]+e^-+\bar\nu_e$, viz.,

$$\begin{aligned}[(ft)_{N_i\to N_f}]^{-1}&=[(ft)_{n\to p}]^{-1}\frac{(1.19)^2[G_A(N_i\to N_f)/G_A(n\to p)]^2\times6}{1+(1.19)^2\times3}\\&=[(ft)_{n\to p}]^{-1}\frac{(1.19)^2(f_{\pi N_iN_f}/f_{\pi np})^2\times6}{1+(1.19)^2\times3},\\(ft)_{n\to p}&=1180\text{ sec}^{-1}.\end{aligned}\tag{34}$$

Similarly, the ft values in the "allowed" approximation for nuclear beta-decay transitions of the type

$$[N_i\colon (J^{(P)};T)_i=1^{(+)};0]\to[N_f\colon (J^{(P)};T)_f=0^{(+)};1]+e^++\nu_e\quad (\text{e.g., } {}_9\text{F}_9{}^{18}\to{}_8\text{O}_{10}{}^{18}+e^++\nu_e)$$

and

$$[N_i\colon (J^{(P)};T)_i=1^{(+)};1]\to[N_f\colon (J^{(P)};T)_f=0^{(+)};0]+e^-+\bar{\nu}_e \quad (\text{e.g., } {}_5B_7{}^{12}\to {}_6C_6{}^{12}+e^-+\bar{\nu}_e)$$

are

$$[(ft)_{N_i\to N_f}]^{-1}=[(ft)_{n\to p}]^{-1}\frac{(1.19)^2[G_A(N_i\to N_f)/G_A(n\to p)]^2\times 2}{1+(1.19)^2\times 3}$$
$$=[(ft)_{n\to p}]^{-1}\frac{(1.19)^2(f_{\pi N_iN_f}/f_{\pi np})^2\times 2}{1+(1.19)^2\times 3}; \tag{35}$$
$$(ft)_{n\to p}=1180\text{ sec}^{-1}.$$

Use of Eqs. (34) and (35) and of experimental values of $(ft)_{N_i\to N_f}$ permits calculation of "G-T experimental" values of $(f_{\pi N_iN_f}/f_{\pi np})^2=[G_A(N_i\to N_f)/G_A(n\to p)]^2$ for even-A nuclei and these values are included in the fourth column of Table I and in the solid curve of Fig. 1—it is seen that $(f_{\pi N_iN_f}/f_{\pi np})^2{}_{\text{G-T exper}}$ has the same general (strikingly nonmonotonic) dependence on A for even A as for odd A. On the other hand, particularly in the cases ${}_6C_8{}^{14}\to {}_7N_7{}^{14}+e^-+\bar{\nu}_e$, ${}_8O_6{}^{14}\to {}_7N_7{}^{14}+e^++\nu_e$, and ${}_{14}Si_{18}{}^{32}\to {}_{15}P_{17}{}^{32}+e^-+\bar{\nu}_e$, $[G_A(N_i\to N_f)/G_A(n\to p)]^2{}_{\text{exper}}=(f_{\pi N_iN_f}/f_{\pi np})^2{}_{\text{G-T exper}}$ is very small and it may be doubted that the corresponding $\Phi^{N_i\to N_f}(q^2)$ and $F_P{}^{N_i\to N_f}(q^2)$ are indeed dominated by a pion pole with residue proportional to $f_{\pi N_iN_f}$ [see however the argument after Eqs. (16) and (17)].

What can we say about a theoretical derivation of the values of $f_{\pi N_iN_f}/f_{\pi np}$ for the even-A nuclei? It is clear that a treatment analogous to that described in Eqs. (29) and (30) for the odd-A nuclei cannot be given in the even-A case if only because one of the two nuclei involved has zero spin and therefore zero magnetic moment. Thus the $f_{\pi N_iN_f}/f_{\pi np}$ for even A can only be deduced from a polological analysis of $n+N_f\to p+N_i$ nucleon charge-exchange scattering experiments (e.g., $n+{}_3Li_3{}^6\to p+{}_2He_4{}^6$ or $n+{}_6C_6{}^{12}\to p+{}_5B_7{}^{12}$). In the absence of such experiments, our sole recourse is an estimate of $f_{\pi N_iN_f}/f_{\pi np}$ on the basis of the impulse approximation [see the analogous Eq. (24)]

$$\frac{f_{\pi N_iN_f}}{f_{\pi np}}=\frac{(1+\xi)\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+{}^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle}{(u_{N_f;\cdots M_f\cdots}{}^\dagger \mathbf{S} u_{N_i;\cdots M_i\cdots})}. \tag{36}$$

However, Eq. (36) yields no new information since, together with Eq. (33), it merely gives the usual impulse-approximation expression for $G_A(N_i\to N_f)/G_A(n\to p)$, viz. [see the analogous Eq. (22)]

$$\frac{G_A(N_i\to N_f)}{G_A(n\to p)}=\frac{(1+\xi)\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+{}^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle}{(u_{N_f;\cdots M_f\cdots}{}^\dagger \mathbf{S} u_{N_i;\cdots M_i\cdots})}. \tag{37}$$

We conclude this Section by giving a brief discussion of nuclear beta-decay transitions of the type

$$[N_i\colon (J^{(P)};T)_i=\tfrac{1}{2}^{(\pm)};T]\to[N_f\colon (J^{(P)};T)_f=\tfrac{1}{2}^{(\mp)};T-1]+e^-+\bar{\nu}_e$$
$$(\text{e.g., } {}_{48}Cd_{67}{}^{115}(\tfrac{1}{2}^{(+)};19/2)\to {}_{49}In_{66}{}^{115}(\tfrac{1}{2}^{(-)};17/2)+e^-+\bar{\nu}_e);$$

in contradiction to the cases previously treated, this last type of transition is not "allowed" but rather "parity-forbidden." Analogous to Eqs. (13) and (14) we then have, using the CVC and PCAC hypotheses and with the same notation as before,

$$\langle e^-\bar{\nu}_eN_f|\mathfrak{L}(0)|N_i\rangle=(G/\sqrt{2})[u_e{}^\dagger\gamma_4\gamma_\alpha(1+\gamma_5)u_{\bar{\nu}}{}^*]\{\langle N_f|j_\alpha{}^{(V)}(0)|N_i\rangle+\langle N_f|j_\alpha{}^{(A)}(0)|N_i\rangle\},$$

$$\langle N_f|j_\alpha{}^{(V)}(0)|N_i\rangle=\left\{u_{N_f}{}^\dagger\gamma_4\left(\left[\gamma_\alpha\gamma_5-\frac{iq_\alpha}{q^2}(m_{N_i}+m_{N_f})\gamma_5\right]F_V{}^{N_i\to N_f}(q^2)-\frac{\sigma_{\alpha\beta}q_\beta\gamma_5}{2m_p}F_M{}^{N_i\to N_f}(q^2)\right)u_{N_i}\right\},$$

$$\lim_{q^2\to 0}[F_V{}^{N_i\to N_f}(q^2)/q^2]=\text{finite constant};$$

$$\langle N_f|j_\alpha{}^{(A)}(0)|N_i\rangle=\left\{u_{N_f}{}^\dagger\gamma_4\left[\gamma_\alpha F_A{}^{N_i\to N_f}(q^2)-\frac{\sigma_{\alpha\beta}q_\beta}{2m_p}F_E{}^{N_i\to N_f}(q^2)+\frac{iq_\alpha(m_{N_f}-m_{N_i})}{m_\pi{}^2}F_P{}^{N_i\to N_f}(q^2)\right]u_{N_i}\right\},$$

$$\langle N_f|\partial j_\alpha^{(A)}(0)/\partial x_\alpha|N_i\rangle = -iq_\alpha\langle N_f|j_\alpha^{(A)}(0)|N_i\rangle$$
$$= (m_{N_f}-m_{N_i})[F_A{}^{N_i\to N_f}(q^2)+(q^2/m_\pi{}^2)F_P{}^{N_i\to N_f}(q^2)](u_{N_f}{}^\dagger\gamma_4 u_{N_i})$$
$$\equiv (m_{N_f}-m_{N_i})\Phi^{N_i\to N_f}(q^2)(u_{N_f}{}^\dagger\gamma_4 u_{N_i}); \tag{38}$$

$$G=\frac{1.0\times10^{-5}}{m_p{}^2};\quad q=-(p_e+p_{\bar\nu})=(p_{N_f}-p_{N_i});$$

$$\Phi^{N_i\to N_f}(0)=F_A{}^{N_i\to N_f}(0)\equiv G_A(N_i\to N_f)=\left(\frac{m_{N_f}+m_{N_i}}{m_{N_f}-m_{N_i}}\right)a_\pi f_{\pi N_i N_f}+\frac{1}{\pi}\int_{m^2_{an}}^\infty\frac{\mathrm{Im}\Phi^{N_i\to N_f}(-m^2)}{m^2}d(m^2)$$
$$=\left(\frac{m_{N_f}+m_{N_i}}{m_{N_f}-m_{N_i}}\right)a_\pi f_{\pi N_i N_f}\left\{1+\frac{\frac{1}{\pi}\int_{m^2_{an}}^\infty \mathrm{Im}\Phi^{N_i\to N_f}(-m^2)d(m^2)}{\langle m^2\rangle_\Phi{}^{N_i\to N_f}[(m_{N_f}+m_{N_i})/(m_{N_f}-m_{N_i})]a_\pi f_{\pi N_i N_f}}\right\},$$

$$F_P{}^{N_i\to N_f}(0)=-\left(\frac{m_{N_f}+m_{N_i}}{m_{N_f}-m_{N_i}}\right)a_\pi f_{\pi N_i N_f}+\frac{1}{\pi}\int_{m^2_{an}}^\infty\frac{\mathrm{Im}F_P{}^{N_i\to N_f}(-m^2)}{m^2}d(m^2)$$
$$=-\left(\frac{m_{N_f}+m_{N_i}}{m_{N_f}-m_{N_i}}\right)a_\pi f_{\pi N_i N_f}\left[1-\frac{m_\pi{}^2}{\langle m^2\rangle_P{}^{N_i\to N_f}}\right],$$

whence, postulating also pion-pole dominance [see the analogous discussion after Eqs. (16) and (17)], we have the Goldberger-Treiman relation

$$G_A(N_i\to N_f)\cong\left(\frac{m_{N_f}+m_{N_i}}{m_{N_f}-m_{N_i}}\right)a_\pi f_{\pi N_i N_f}\cong -F_P{}^{N_i\to N_f}(0),\quad \frac{G_A(N_i\to N_f)}{G_A(n\to p)}\cong\left(\frac{m_{N_f}+m_{N_i}}{m_{N_f}-m_{N_i}}\right)\frac{f_{\pi N_i N_f}}{f_{\pi np}}. \tag{39}$$

Here, however, $f_{\pi N_i N_f}$ is a scalar-type, rather than the previously used pseudoscalar-type, pion–initial-nucleus–final-nucleus coupling constant, i.e., $f_{\pi N_i N_f}$ is here defined via the vertex function

$$[(m_{N_f}+m_{N_i})/m_\pi]f_{\pi N_i N_f}(p_{N_i}{}^2,p_{N_f}{}^2,p_\pi{}^2)(u_{N_f}{}^\dagger\gamma_4 u_{N_i})$$

rather than via the previously used vertex function

$$[(m_{N_f}+m_{N_i})/m_\pi]f_{\pi N_i N_f}(p_{N_i}{}^2,p_{N_f}{}^2,p_\pi{}^2)(u_{N_f}{}^\dagger\gamma_4\gamma_5 u_{N_i});$$

$f_{\pi N_i N_f}/f_{\pi np}$ is deducible on the basis of a polological analysis of $n+N_f\to p+N_i$ nucleon charge—exchange scattering experiments (e.g., $n+{}_{49}\mathrm{In}_{66}{}^{115}\to p+{}_{48}\mathrm{Cd}_{67}{}^{115}$). In the absence of such experiments our sole recourse is again an estimate of $f_{\pi N_i N_f}/f_{\pi np}$ on the basis of the impulse approximation [see the analogous Eq. (36)]:

$$\frac{[(m_{N_f}+m_{N_i})/m_\pi]f_{\pi N_i N_f}}{[(m_p+m_n)/m_\pi]f_{\pi np}}=\frac{(1+\xi)\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^A\tau_+{}^{(a)}\gamma_4{}^{(a)}\gamma_5{}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle}{(u_{N_f;\cdots M_f\cdots}{}^\dagger\gamma_4 u_{N_i;\cdots M_i\cdots})}. \tag{40}$$

However, and just as before, Eq. (40) yields no new information since, together with Eq. (39), it merely gives the usual impulse-approximation expression for $G_A(N_i\to N_f)/G_A(n\to p)$, viz. [see the analogous Eq. (37)]

$$\frac{G_A(N_i\to N_f)}{G_A(n\to p)}=\left(\frac{m_p+m_n}{m_{N_f}-m_{N_i}}\right)\frac{(1+\xi)\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^A\tau_+{}^{(a)}\gamma_4{}^{(a)}\gamma_5{}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle}{(u_{N_f;\cdots M_f\cdots}{}^\dagger\gamma_4 u_{N_i,\cdots M_i\cdots})}$$
$$\cong\left(\frac{m_p+m_n}{m_{N_f}-m_{N_i}}\right)\frac{(1+\xi)[|q_4|/(m_p+m_n)]\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^A\tau_+{}^{(a)}\gamma_4{}^{(a)}\gamma_4{}^{(a)}\gamma_5{}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle}{(u_{N_f;\cdots M_f\cdots}{}^\dagger\gamma_4 u_{N_i;\cdots M_i\cdots})}$$
$$\cong\frac{(1+\xi)\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^A\tau_+{}^{(a)}\gamma_5{}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle}{(u_{N_f;\cdots M_f\cdots}{}^\dagger u_{N_i;\cdots M_i\cdots})}. \tag{41}$$

Equations (20), (24), and (22), Eqs. (33), (36), and (37), and Eqs. (39), (40), and (41) show that the Goldberger-Treiman relation together with the impulse-approximation expression for $f_{\pi N_i N_f}/f_{\pi n p}$ leads in all cases to the impulse-approximation expression for $G_A(N_i \to N_f)/G_A(n \to p)$. The essential reason for this consistency of the G-T relation with the use of impulse-approximation expressions for *both* $G_A(N_i \to N_f)/G_A(n \to p)$ and $f_{\pi N_i N_f}/f_{\pi n p}$ can be seen particularly clearly if we cast the PCAC hypothesis together with the pion-pole-dominance assumption into the form[10]

$$\partial j_\alpha^{(A)}(x)/\partial x_\alpha = C_\pi \varphi^{(\pi)}(x) + \cdots = C_\pi[-(\partial/\partial x_\alpha)(\partial/\partial x_\alpha) + m_\pi^2]^{-1} j^{(\pi)}(x) + \cdots, \tag{42}$$

where C_π is a constant, $\varphi^{(\pi)}(x)$ is the pion-field operator which destroys a physical π^- (and creates a physical π^+), $j^{(\pi)}(x) = [-(\partial/\partial x_\alpha)(\partial/\partial x_\alpha) + m_\pi^2]\varphi^{(\pi)}(x)$ is the pion-field source-density operator, and the terms in $\cdots$, which are associated with higher mass $J_T^{PG} = 0_1^{--}$ meson-field operators, are supposed to give relatively small contributions for processes with hadron momentum transfers q^2 in the range $-m_\pi^2 \leqq q^2 \lessapprox 0$. Equation (42) yields, using also Eq. (1) and, e.g., Eq. (13),

$$\begin{aligned}\langle \text{vac} | \partial j_\alpha^{(A)}(0)/\partial x_\alpha | \pi^- \rangle &= i(p_\pi)_\alpha [[1/(2E_\pi)^{1/2}] i(p_\pi)_\alpha m_\pi F_A^{\pi \to \text{vac}}(p_\pi^2)]_{p_\pi^2 = -m_\pi^2} \\ &= [m_\pi^3 a_\pi/(2E_\pi)^{1/2}] \cong \langle \text{vac} | C_\pi \varphi^{(\pi)}(0) | \pi^- \rangle = [C_\pi/(2E_\pi)^{1/2}]; \\ & m_\pi^3 a_\pi \cong C_\pi\end{aligned} \tag{43}$$

$$\begin{aligned}\langle p | \frac{\partial j_\alpha^{(A)}(0)}{\partial x_\alpha} | n \rangle &= (m_p + m_n) \Phi^{n \to p}(q^2)(u_p^\dagger \tau_+ \gamma_4 \gamma_5 u_n) \cong \langle p | C_\pi \left[-\frac{\partial}{\partial x_\alpha}\frac{\partial}{\partial x_\alpha} + m_\pi^2 \right]^{-1} j^{(\pi)}(0) | n \rangle \\ &= \frac{C_\pi}{q^2 + m_\pi^2}\left[\left(\frac{m_p + m_n}{m_\pi}\right) f_{\pi n p}(-m_n^2, -m_p^2, p_\pi^2 = (p_n - p_p)^2 = q^2)\right](u_p^\dagger \tau_+ \gamma_4 \gamma_5 u_n); \\ \Phi^{n \to p}(q^2) &\cong \frac{m_\pi^2}{m_\pi^2 + q^2} a_\pi f_{\pi n p}\left[\frac{f_{\pi n p}(-m_n^2, -m_p^2, q^2)}{f_{\pi n p}(-m_n^2, -m_p^2, -m_\pi^2)}\right] \equiv \frac{m_\pi^2}{m_\pi^2 + q^2} a_\pi f_{\pi n p} K_{\pi n p}(q^2); \\ \Phi^{n \to p}(0) &= F_A^{n \to p}(0) \equiv G_A(n \to p) \cong a_\pi f_{\pi n p} K_{\pi n p}(0).\end{aligned} \tag{44}$$

$$\begin{aligned}\langle N_f | \frac{\partial j_\alpha^{(A)}(0)}{\partial x_\alpha} | N_i \rangle &= (m_{N_f} + m_{N_i}) \Phi^{N_i \to N_f}(q^2)(u_{N_f}^\dagger \tau_+ \gamma_4 \gamma_5 u_{N_i}) \cong \langle N_f | C_\pi \left[-\frac{\partial}{\partial x_\alpha}\frac{\partial}{\partial x_\alpha} + m_\pi^2 \right]^{-1} j^{(\pi)}(0) | N_i \rangle \\ &= \frac{C_\pi}{q^2 + m_\pi^2}\left[\left(\frac{m_{N_f} + m_{N_i}}{m_\pi}\right) f_{\pi N_i N_f}(-m_{N_i}^2, -m_{N_f}^2, p_\pi^2 = (p_{N_i} - p_{N_f})^2 = q^2)\right](u_{N_f}^\dagger \tau_+ \gamma_4 \gamma_5 u_{N_i}); \\ \Phi^{N_i \to N_f}(q^2) &\cong \frac{m_\pi^2}{m_\pi^2 + q^2} a_\pi f_{\pi N_i N_f}\left[\frac{f_{\pi N_i N_f}(-m_{N_i}^2, -m_{N_f}^2, q^2)}{f_{\pi N_i N_f}(-m_{N_i}^2, -m_{N_f}^2, -m_\pi^2)}\right] \equiv \frac{m_\pi^2}{m_\pi^2 + q^2} a_\pi f_{\pi N_i N_f} K_{\pi N_i N_f}(q^2); \\ \Phi^{N_i \to N_f}(0) &= F_A^{N_i \to N_f}(0) \equiv G_A(N_i \to N_f) \cong a_\pi f_{\pi N_i N_f} K_{\pi N_i N_f}(0),\end{aligned} \tag{45}$$

where the G-T relations in Eqs. (44) and (45) differ from those in Eqs. (6) and (18) by the presence of the neutron $\to$ proton and initial-nucleus $\to$ final-nucleus pionic form factors $K_{\pi n p}(q^2)$ and $K_{\pi N_i N_f}(q^2)$ evaluated at $q^2 = 0$ i.e., evaluated at zero virtual pion mass [by definition $K_{\pi n p}(-m_\pi^2) = K_{\pi N_i N_f}(-m_\pi^2) = 1$]. Thus, if the G-T relation in Eq. (44) is exactly correct, $K_{\pi n p}(0) = (G_A(n \to p))(a_\pi f_{\pi n p})^{-1} = [1.19/(0.95)(1.43)] = 0.87$ [see Eqs. (6), (7) *et seq.*]; on the other hand, nothing is known about the numerical value of $K_{\pi N_i N_f}(0)$.

The G-T relation in Eq. (45) essentially consists of an equality between the $N_i \to N_f$ matrix element of $\partial j_\alpha^{(A)}(0)/\partial x_\alpha$ which gives $G_A(N_i \to N_f)$ and the $N_i \to N_f$ matrix element of $C_\pi[-(\partial/\partial x_\alpha)(\partial/\partial x_\alpha) + m_\pi^2]^{-1} j^{(\pi)}(0)$ which gives $a_\pi f_{\pi N_i N_f} K_{\pi N_i N_f}(0)$; this equality is not appreciably perturbed if *each* of the two matrix elements is evaluated in impulse approximation. The last remark establishes the consistency in question.

IV. DISCUSSION

We now discuss in a little more detail what appears to us as the most promising experimentally based method for the determination of

$$f_{\pi N_i N_f} \equiv f_{\pi N_i N_f}(p_{N_i}^2 = -m_{N_i}^2, p_{N_f}^2 = -m_{N_f}^2, p_\pi^2 = (p_{N_i} - p_{N_f})^2 = -m_\pi^2);$$

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[10] See M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, *ibid.* **17**, 757 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); S. L. Adler, Phys. Rev. **137**, B1022 (1965).

this pion-initial-nucleus-final-nucleus coupling constant enters into the G-T relation of Eq. (18). As already mentioned, $f_{\pi N_i N_f}$ can be found from a polological analysis of $n+N_f \to p+N_i$ nucleon charge-exchange scattering experiments. Thus, in the case that $(J^{(P)};T)_f=(J^{(P)};T)_i=(\frac{1}{2}^{(\pm)};\frac{1}{2})$—e.g., $N_f={}_2\mathrm{He}_1{}^3$, $N_i={}_1H_2{}^3$, the differential cross section for $n+N_f \to p+N_i$ nucleon charge-exchange scattering can be written as

$$\frac{d\sigma(\cos\theta_{np},E)}{d\Omega}=|A_{\text{pion-exch}}(\cos\theta_{np},E)+A_{\text{multipion-exch, etc.}}(\cos\theta_{np},E)|^2$$

$$A_{\text{pion-exch}}(\cos\theta_{np},E)$$

$$=\frac{[(2m_n/m_\pi)f_{\pi np}(-m_n{}^2,-m_p{}^2,q^2)(\frac{1}{2}q^2)^{1/2}][(2m_N/m_\pi)f_{\pi N_iN_f}(-m_{N_i}{}^2,-m_{N_f}{}^2,q^2)(\frac{1}{2}q^2)^{1/2}](1/4\pi E)}{q^2+m_\pi{}^2}$$

$$=\frac{[(2m_n/m_\pi)(1/\sqrt{2})(4\pi)^{-1/2}f_{\pi np}(-m_n{}^2,-m_p{}^2,2|\mathbf{p}_n|^2(1-\cos\theta_{np}))]\times[(2m_N/m_\pi)(1/\sqrt{2})(4\pi)^{-1/2}f_{\pi N_iN_f}(-m_{N_i}{}^2,-m_{N_f}{}^2,2|\mathbf{p}_n|^2(1-\cos\theta_{np}))]}{[(1+m_\pi{}^2/2|\mathbf{p}_n|^2)-\cos\theta_{np}][(1/E)(1-\cos\theta_{np})]^{-1}},\quad(46)$$

$$\lim_{\cos\theta_{np}\to\left(1+\frac{m_\pi{}^2}{2|\mathbf{p}_n|^2}\right)}\left\{\left[\left(1+\frac{m_\pi{}^2}{2|\mathbf{p}_n|^2}\right)-\cos\theta_{np}\right]A_{\text{multipion-exch, etc.}}(\cos\theta_{np},E)\right\}=0;$$

$$m_p\cong m_n,\quad m_{N_i}\cong m_{N_f}\equiv m_N;$$

$$q^2\equiv(p_{N_i}-p_{N_f})^2=p_\pi{}^2=(p_n-p_p)^2=2|\mathbf{p}_n|^2(1-\cos\theta_{np}),\quad \theta_{np}\equiv\cos^{-1}((\mathbf{p}_n\cdot\mathbf{p}_p)/|\mathbf{p}_n||\mathbf{p}_p|);$$

$$E\equiv[-(p_n+p_{N_f})^2]^{1/2}=E_n+E_{n_f}=(|\mathbf{p}_n|^2+m_n{}^2)^{1/2}+(|\mathbf{p}_n|^2+m_N{}^2)^{1/2};$$

where $\mathbf{p}_n$ and $\mathbf{p}_p$ are, respectively, the neutron and proton center-of-mass momenta and where the pole in $A_{\text{pion-exch}}(\cos\theta_{np},E)$ associated with the exchange of the virtual (charged) pion occurs at an unphysical value of the cosine of the scattering angle, viz.: $\cos\theta_{np}=(1+m_\pi{}^2/2|\mathbf{p}_n|^2)$; as a numerical example, $(1+m_\pi{}^2/2|\mathbf{p}_n|^2)=1.06$ for a neutron with laboratory kinetic energy of 150 MeV incident on ${}_2\mathrm{He}_1{}^3$. Equation (46) yields

$$\lim_{\cos\theta_{np}\to\left(1+\frac{m_\pi{}^2}{2|\mathbf{p}_n|^2}\right)}\left\{\left[\left(1+\frac{m_\pi{}^2}{2|\mathbf{p}_n|^2}\right)-\cos\theta_{np}\right]A_{\text{pion-exch}}(\cos\theta_{np},E)\right\}$$
$$=\left[\left(\frac{2m_n}{m_\pi}\right)\left(\frac{f_{\pi np}}{\sqrt{2}(4\pi)^{1/2}}\right)\right]\left[\left(\frac{2m_N}{m_\pi}\right)\left(\frac{f_{\pi N_iN_f}}{\sqrt{2}(4\pi)^{1/2}}\right)\right]\left[\frac{(-m_\pi{}^2/2|\mathbf{p}_n|^2)}{E}\right],\quad(47)$$

which, supposing $f_{\pi np}$ known, determines $f_{\pi N_iN_f}$. In this connection it should however be noted that $f_{\pi N_iN_f}[-m_{N_i}{}^2,-m_{N_f}{}^2,2|\mathbf{p}_n|^2(1-\cos\theta_{np})]$ varies more rapidly with $\cos\theta_{np}$ in the physical region than does $f_{\pi np}[-m_n{}^2,-m_p{}^2,2|\mathbf{p}_n|^2(1-\cos\theta_{np})]$ because of the relatively large size of a nucleus compared to a nucleon; in addition, $A_{\text{pion-exch}}(\cos\theta_{np},E)=0$ at $\cos\theta_{np}=1$ because of the $(1-\cos\theta_{np})$ factor. Unfortunately, each of these circumstances, as well as the necessary multiplication of the above expression for $A_{\text{pion-exch}}(\cos\theta_{np},E)$ by

$$\exp\left\{-\left[\frac{Z(N_i)}{137}\frac{E_n}{|\mathbf{p}_n|}\left(\tan^{-1}\left(\frac{2|\mathbf{p}_n|}{m_\pi}\right)+i\ln\frac{(m_\pi{}^4+4m_\pi{}^2|\mathbf{p}_n|^2)^{1/2}}{m_\pi{}^2+q^2}\right)\right]\right\}\quad(48)$$

to include the effect of the final-state p-N_i Coulomb interaction, will tend to make the isolation of the pion-pole contribution to $d\sigma(\cos\theta_{np},E)/d\Omega$ more difficult.[11]

An extrapolation of $A_{\text{pion-exch}}(\cos\theta_{np},E)$ to $\cos\theta_{np}=(1+m_\pi{}^2/2|\mathbf{p}_n|^2)$ has, in effect, been carried out in the case $N_f=p$, $N_i=n$[4] and gives a value of $f_{\pi np}$ somewhat less precise than, but consistent with, the value of $f_{\pi np}$ obtained

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[11] It should be mentioned that $|f_{\pi N_iN_f}|=\sqrt{2}|f_{\pi^0N_fN_f}|=\sqrt{2}|f_{\pi^0N_iN_i}|$ so that we can also obtain $|f_{\pi N_iN_f}|$ from a determination of $|f_{\pi^0N_fN_f}|$ on the basis of a polological analysis of $n+N_f\to n+N_f$ neutron elastic-scattering experiments (any polological analysis of $p+N_f\to p+N_f$ proton elastic-scattering experiments involves additional complications due to the presence of a relatively large Coulomb term in the scattering amplitude at small scattering angles). However, the small scattering angle $n+N_f\to n+N_f$ elastic scattering amplitude, in contradistinction to the small scattering angle $n+N_f\to p+N_i$ charge-exchange scattering amplitude, necessarily contains an imaginary part associated with the possibility of various inelastic processes (optical theorem) and this imaginary part will help mask the pion-exchange pole term in the real part.

from an analysis of $\pi^{\pm}+p\to\pi^{\pm}+p$ elastic-scattering experiments, viz.: $\frac{1}{2}f_{\pi np}^2/4\pi=0.079\pm0.006$ versus 0.081 ± 0.003; we should also mention that any determination of $f_{\pi N_iN_f}$ from a dispersion-theoretic analysis of $\pi^{\pm}+N_f\to\pi^{\pm}+N_f$ elastic-scattering experiments would be very considerably complicated by the presence of $\pi^-+N_f\to N_{i'}$ pole terms in the forward $\pi^{\pm}+N_f\to\pi^{\pm}+N_f$ elastic-scattering amplitude—here the $N_{i'}$ are the various bound and unbound excited states of the nucleus whose ground state is N_i. To our best knowledge, no experimental study of $n+N_f\to p+N_i$ nucleon charge-exchange scattering from the point of view of determination of the $f_{\pi N_iN_f}$ has ever been undertaken and we would like to take this opportunity to advocate such a study; it is important to note in this connection that $E\approx m_N$ for all practical $|\mathbf{p}_n|$ so that, if $f_{\pi N_iN_f}\approx f_{\pi np}$, the right side of Eq. (47) is of the same order in the N_f, N_i case as in the p, n case. If $f_{\pi N_iN_f}\ll f_{\pi np}$, as one anticipates on the basis of the fourth column of Table I, e.g., for ${}_6C_7{}^{13}$, $N_i={}_7N_6{}^{13}$, $A_{\text{pion-exch}}(\cos\theta_{np},E)$ will be small compared to $A_{\text{multipion-exch, etc.}}(\cos\theta_{np};E)$ for all physical values of $\cos\theta_{np}$ and the determination of $f_{\pi N_iN_f}$ from Eq. (47) will become extremely difficult in practice. In general, it is of course clear that any experimentally based determination of those $f_{\pi N_iN_f}$ which are small compared to $f_{\pi np}$ is bound to be a formidable task but this gloomy circumstance should not deter efforts to perform experiments from which the larger $f_{\pi N_iN_f}$ can conceivably be deduced.

APPENDIX I

In this Appendix we wish to establish the analog of Eqs. (13)–(24) for nuclear beta-decay transitions of the type

$$[N_i:(J^{(P)};T)_i=\tfrac{3}{2}^{(\pm)},\tfrac{5}{2}^{(\pm)},\tfrac{7}{2}^{(\pm)},\cdots;\tfrac{1}{2}]\to[N_f:(J^{(P)};T)_f=\tfrac{3}{2}^{(\pm)},\tfrac{5}{2}^{(\pm)},\tfrac{7}{2}^{(\pm)},\cdots;\tfrac{1}{2}]+e^-+\bar{\nu}_e$$

and, in particular, to justify Eq. (26). We have, analogously to Eq. (13), and in the "allowed" approximation,

$$\langle N_f;\cdots M_f\cdots|j_\alpha^{(A)}(0)|N_i;\cdots M_i\cdots\rangle=[u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+(i\sigma_\alpha)u_{N_i;\cdots M_i\cdots}](1-\delta_{\alpha4})G_A(N_i\to N_f)$$
$$+\{\text{terms which vanish in the limit of small-momentum transfer } q=(p_{N_f}-p_{N_i})\},\quad \text{(A1)}$$

where $u_{N_i;\cdots M_i\cdots}$ and $u_{N_f;\cdots M_f\cdots}$ are spinors which describe the initial and final nuclei as "elementary" particles with spin and spin projection J_i, M_i and $J_f=J_i\equiv J$, M_f, while $i\sigma_1$, $i\sigma_2$, $i\sigma_3\equiv\gamma_4\gamma_1\gamma_5$, $\gamma_4\gamma_2\gamma_5$, $\gamma_4\gamma_3\gamma_5$ are spin-$\frac{1}{2}$ angular-momentum operators which work on certain factors within $u_{N_i;\cdots M_i\cdots}$ and $u_{N_f;\cdots M_f\cdots}$ [see Eq. (A3) below]; it is easy to write explicit expressions for $u_{N_i;\cdots M_i=J\cdots}$ and $u_{N_f;\cdots M_f=J\cdots}$, viz:

$$\begin{gathered}u_{N_i;\cdots M_i=J\cdots}=v_{N_i}(\tau)\xi_{+1}(1)\xi_{+1}(2)\cdots\xi_{+1}(J-\tfrac{1}{2})\chi_{+1/2},\\ u_{N_f;\cdots M_f=J\cdots}=v_{N_f}(\tau)\xi_{+1}(1)\xi_{+1}(2)\cdots\xi_{+1}(J-\tfrac{1}{2})\chi_{+1/2},\\ (v_{N_f}{}^\dagger(\tau)\tau_+v_{N_i}(\tau))=1\end{gathered}\quad \text{(A2)}$$

with $\chi_{\pm1/2}$ spin-$\frac{1}{2}$ type wave functions appropriate to spin-projections $\pm\frac{1}{2}$ so that

$$\sigma_3\chi_{\pm1/2}=\pm\chi_{\pm1/2},\quad \sigma_1\chi_{\pm1/2}=\chi_{\mp1/2},\quad \sigma_2\chi_{\pm1/2}=\pm i\chi_{\mp1/2}\quad \text{(A3)}$$

and $\xi_{+1}(i)$ spin-1-type wave functions, i.e., spin-1-type polarization three-vectors, appropriate to spin projection $+1$. Thus

$$\begin{aligned}&(u_{N_f;\cdots M_f=J\cdots}{}^\dagger\tau_+\sigma_\alpha u_{N_i;\cdots M_i=J\cdots})\\&=(v_{N_f}{}^\dagger(\tau)\tau_+v_{N_i}(\tau))(\xi_{+1}{}^\dagger(1)\cdot\xi_{+1}(1))\cdots(\xi_{+1}{}^\dagger(J-\tfrac{1}{2})\cdot\xi_{+1}(J-\tfrac{1}{2}))(\chi_{+1/2}{}^\dagger\sigma_\alpha\chi_{+1/2})=1\cdot1\cdots1\cdot\delta_{\alpha,3}=\delta_{\alpha,3},\end{aligned}\quad \text{(A4)}$$

and it only remains to relate $(u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+\sigma_\alpha u_{N_i;\cdots M_i\cdots})$ for *any* M_f, M_i to the just evaluated $(u_{N_f;\cdots M_f=J\cdots}{}^\dagger\tau_+\sigma_\alpha u_{N_i;\cdots M_i=J\cdots})$. This can however be very simply done since we wish to calculate [see the analogous Eqs. (19) and (25)]

$$\sum_{\alpha=1,2,3}\sum_{M_f=-J,\cdots,+J}|(u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+\sigma_\alpha u_{N_i;\cdots M_i\cdots})|^2\quad \text{(A5)}$$

and this is given by[12]

$$\sum_{\alpha=1,2,3}\sum_{M_f=-J,\cdots,+J}|(u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+\sigma_\alpha u_{N_i;\cdots M_i\cdots})|^2$$
$$=[(J+1)/J]|(u_{N_f;\cdots M_f=J\cdots}{}^\dagger\tau_+\sigma_3 u_{N_i;\cdots M_i=J\cdots})|^2=[(J+1)/J].\quad \text{(A6)}$$

[12] See e.g., E. Feenberg and G. E. Pake, *Notes On The Quantum Theory of Angular Momentum* (Addison-Wesley, Cambridge, Massachusetts, 1953), p. 50.

Equations (A6) and (A1) and the fact that CVC again implies $F_V^{N_i\to N_f}(0)\equiv G_V(N_i\to N_f)=1$ yield the desired Eq. (26).

As a more explicit version of Eqs. (A1)–(A6) consider the case of $J=\frac{3}{2}$. Here, analogously to Eq. (13),

$$\langle N_f;\cdots M_f\cdots|j_\alpha^{(A)}(0)|N_i;\cdots M_i\cdots\rangle$$

$$=(u_{N_f;\cdots M_f\cdots}{}^\dagger)_\mu\tau_+\gamma_4\Bigg\{\Bigg[\gamma_\alpha\gamma_5F_A{}^{N_i\to N_f}(q^2)+\frac{iq_\alpha(m_{N_f}+m_{N_i})}{m_\pi{}^2}\gamma_5F_P{}^{N_i\to N_f}(q^2)\Bigg]\delta_{\mu\nu}$$

$$+\Bigg[\gamma_\alpha\gamma_5F_{A'}{}^{N_i\to N_f}(q^2)+\frac{iq_\alpha(m_{N_f}+m_{N_i})}{m_\pi{}^2}\gamma_5F_{P'}{}^{N_i\to N_f}(q^2)\Bigg]\frac{q_\mu q_\nu}{m_\pi{}^2}$$

$$+\frac{\gamma_5q_\beta}{m_\pi}[\delta_{\mu\alpha}\delta_{\beta\nu}F_{P''}{}^{N_i\to N_f}(q^2)+\delta_{\mu\beta}\delta_{\alpha\nu}F_{P'''}{}^{N_i\to N_f}(q^2)]\Bigg\}(u_{N_i;\cdots M_i\cdots})_\nu,\quad\text{(A7)}$$

where $(u_{\cdots M\cdots})_\nu$ is a spin-$\frac{3}{2}$-type wave function[13] satisfying the supplementary conditions

$$\gamma_\mu(u_{\cdots M\cdots})_\mu=0,\quad p_\mu(u_{\cdots M\cdots})_\mu=0,\quad\text{(A8)}$$

and representable as

$$(u_{\cdots M\cdots})_\mu=v(\tau)\sum_{\sigma=-\frac{1}{2},+\frac{1}{2}}(\xi_{M-\sigma})_\mu\chi_\sigma\langle1,M-\sigma;\tfrac{1}{2},\sigma|\tfrac{3}{2},M\rangle\quad\text{(A9)}$$

with $(\xi_{M-\sigma})_\mu=\{\boldsymbol{\xi}_{M-\sigma},(\xi_{M-\sigma})_4\}$ a spin-1-type polarization four-vector appropriate to spin projection $M-\sigma$, χ_σ a spin-$\frac{1}{2}$-type wave function appropriate to spin projection σ and $\langle1,M-\sigma;\frac{1}{2},\sigma|\frac{3}{2},M\rangle$ a vector-angular-momentum addition coefficient appropriate to $1+1/2=3/2$; $(M-\sigma)+\sigma=M$. In the "allowed" approximation, Eqs. (A7), (A8), and (A9) yield

$$\langle N_f;\cdots M_f\cdots|j_\alpha^{(A)}(0)|N_i;\cdots M_i\cdots\rangle$$

$$=(u_{N_f;\cdots M_f\cdots}{}^\dagger)_\mu\tau_+(i\sigma_\alpha)(u_{N_i;\cdots M_i\cdots})_\mu(1-\delta_{\mu4})(1-\delta_{\alpha4})G_A(N_i\to N_f)$$

$$+\{\text{terms which vanish in the limit of small-momentum transfer } q=(p_{N_f}-p_{N_i})\}$$

$$=i(v_{N_f}{}^\dagger(\tau)\tau_+v_{N_i}(\tau))\sum_{\sigma=-\frac{1}{2},+\frac{1}{2}}\sum_{\sigma'=-\frac{1}{2},+\frac{1}{2}}(\boldsymbol{\xi}_{M_f-\sigma}\cdot\boldsymbol{\xi}_{M_i-\sigma'})(\chi_\sigma{}^\dagger\sigma_\alpha\chi_{\sigma'})$$

$$\times\langle1,M_f-\sigma;\tfrac{1}{2},\sigma|\tfrac{3}{2},M_f\rangle^*\langle1,M_i-\sigma';\tfrac{1}{2},\sigma'|\tfrac{3}{2},M_i\rangle(1-\delta_{\alpha4})G_A(N_i\to N_f)+\{\cdots\}$$

$$=i\sum_{\sigma=-\frac{1}{2},+\frac{1}{2}}(\chi_\sigma{}^\dagger\sigma_\alpha\chi_{M_i-M_f+\sigma})\langle1,M_f-\sigma;\tfrac{1}{2},\sigma|\tfrac{3}{2},M_f\rangle^*\langle1,M_f-\sigma;\tfrac{1}{2},M_i-M_f+\sigma|\tfrac{3}{2},M_i\rangle$$

$$\times(1-\delta_{\alpha4})G_A(N_i\to N_f)+\{\cdots\}\quad\text{(A10)}$$

so that, for $M_f=M_i=J=\frac{3}{2}$,

$$(u_{N_f;\cdots M_f=\frac{3}{2}\cdots}{}^\dagger)_\mu\tau_+\sigma_\alpha(u_{N_i;\cdots M_i=\frac{3}{2}\cdots})_\mu=\sum_{\sigma=-\frac{1}{2},+\frac{1}{2}}(\chi_\sigma{}^\dagger\sigma_\alpha\chi_\sigma)|\langle1,\tfrac{3}{2}-\sigma;\tfrac{1}{2},\sigma|\tfrac{3}{2},\tfrac{3}{2}\rangle|^2=(\chi_{1/2}{}^\dagger\sigma_\alpha\chi_{1/2})=\delta_{\alpha,3}\quad\text{(A11)}$$

in agreement with Eq. (A4).

APPENDIX II

In this Appendix we shall derive a relation between $[G_A(N_i\to N_f)/G_A(n\to p)]^2$ as calculated on the basis of the impulse approximation based Eq. (22) generalized to any half-integral J and the magnetic moments of N_f, and N_i, $\mu(N_f)$ and $\mu(N_i)$ [Eqs. (A19) and (A18) below]; this relation is employed (apart from indicated exceptions) to obtain the values of $[G_A(N_i\to N_f)/G_A(n\to p)]^2_{\text{imp-approx theor}}$ in the sixth column of Table I and in the dashed

[13] W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1944).

curve of Fig. 1. We have, using Eq. (22) and Eq. (A6),

$$\left[\frac{G_A(N_i\to N_f)}{G_A(n\to p)}\right]^2_{\text{imp-approx theor}}=\frac{(1+\xi)^2\sum_{M_f=-J,\cdots,+J}|\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle|^2}{\sum_{M_f=-J,\cdots,+J}|u_{N_f;\cdots M_f\cdots}{}^\dagger\tau_+\boldsymbol{\sigma}u_{N_i;\cdots M_i\cdots}|^2}$$

$$=\frac{(1+\xi)^2\sum_{M_f=-J,\cdots,+J}|\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle|^2}{(J+1)/J} \tag{A12}$$

$$=(1+\xi)^2|\langle\boldsymbol{\sigma}\rangle_{fi}|^2J/(J+1);$$

$$|\langle\boldsymbol{\sigma}\rangle_{fi}|=\{\sum_{M_f=-J,\cdots,+J}|\langle\Psi_{N_f;\cdots M_f\cdots}|\sum_{a=1}^{A}\tau_+^{(a)}\boldsymbol{\sigma}^{(a)}|\Psi_{N_i;\cdots M_i\cdots}\rangle|^2\}^{1/2}$$

$$=\left(\frac{J+1}{J}\right)^{1/2}|\langle\Psi_{N_f;\cdots M_f=J\cdots}|\sum_{a=1}^{A}\tau_+^{(a)}\boldsymbol{\sigma}_3^{(a)}|\Psi_{N_i;\cdots M_i=J\cdots}\rangle|,$$

whence

$$\left[\frac{G_A(N_i\to N_f)}{G_A(n\to p)}\right]^2_{\text{imp-approx theor}}=(1+\xi)^2|\langle\Psi_{N_f;\cdots M_f=J\cdots}|\sum_{a=1}^{A}\tau_+^{(a)}\sigma_3^{(a)}|\Psi_{N_i;\cdots M_i=J\cdots}\rangle|^2; \tag{A13}$$

it is clear from Eq. (A12) that $|\langle\boldsymbol{\sigma}\rangle_{fi}|$ is the impulse-approximation Gamow-Teller matrix element.

We now note that

$$\Psi_{N_i;\cdots M_i=J\cdots}=(\sum_{b=1}^{A}\tau_-^{(b)})\Psi_{N_f;\cdots M_f=J\cdots},\quad \Psi_{N_f;\cdots M_f=J\cdots}=(\sum_{b=1}^{A}\tau_+^{(b)})\Psi_{N_i;\cdots M_i=J\cdots};$$
$$(\sum_{b=1}^{A}\tau_-^{(b)})\Psi_{N_i;\cdots M_i=J\cdots}=0,\quad (\sum_{b=1}^{A}\tau_+^{(b)})\Psi_{N_f;\cdots M_f=J\cdots}=0 \tag{A14}$$

since $\Psi_{N_i;\cdots M_i=J\cdots}$ and $\Psi_{N_f;\cdots M_f=J\cdots}$ are characterized by $T_i=\frac{1}{2}$, $T_i^{(3)}=-\frac{1}{2}$ and $T_f=\frac{1}{2}$, $T_f^{(3)}=+\frac{1}{2}$, respectively, and that

$$(\sum_{a=1}^{A}\tau_+^{(a)}\sigma_3^{(a)})(\sum_{b=1}^{A}\tau_-^{(b)})-(\sum_{b=1}^{A}\tau_-^{(b)})(\sum_{a=1}^{A}\tau_+^{(a)}\sigma_3^{(a)})=\sum_{a=1}^{A}\tau_3^{(a)}\sigma_3^{(a)}. \tag{A15}$$

Thus Eqs. (A12) and (A13) become

$$|\langle\boldsymbol{\sigma}\rangle_{fi}|=[(J+1)/J]^{1/2}|\langle\Psi_{N_f;\cdots M_f=J\cdots}|\tfrac{1}{2}\sum_{a=1}^{A}\tau_3^{(a)}\sigma_3^{(a)}|\Psi_{N_f;\cdots M_f=J\cdots}\rangle$$
$$-\langle\Psi_{N_i;\cdots M_i=J\cdots}|\tfrac{1}{2}\sum_{a=1}^{A}\tau_3^{(a)}\sigma_3^{(a)}|\Psi_{N_i;\cdots M_i=J\cdots}\rangle| \tag{A16}$$
$$=[(J+1)/J]^{1/2}|\langle f|S_3^{(p)}-S_3^{(n)}|f\rangle-\langle i|S_3^{(p)}-S_3^{(n)}|i\rangle|;$$

$$[G_A(N_i\to N_f)/G_A(n\to p)]^2_{\text{imp-approx-theor}}=(1+\xi)^2|\langle f|S_3^{(p)}-S_3^{(n)}|f\rangle-\langle i|S_3^{(p)}-S_3^{(n)}|i\rangle|^2$$

and it remains to relate $\langle f|S_3^{(p)}-S_3^{(n)}|f\rangle-\langle i|S_3^{(p)}-S_3^{(n)}|i\rangle$ to the magnetic moments $\mu(N_f)$ and $\mu(N_i)$.

These magnetic moments are given, on the basis of the customary impulse approximation, by

$$\mu(N_f)=(1+\xi_f)\langle\Psi_{N_f;\cdots M_f=J\cdots}|\sum_{a=1}^{A}\{[(1+\tau_3^{(a)})/2][\mu_p\sigma_3^{(a)}+(j_3^{(a)}-\tfrac{1}{2}\sigma_3^{(a)})]+[(1-\tau_3^{(a)})/2]\mu_n\sigma_3^{(a)}\}|\Psi_{N_f;\cdots M_f=J\cdots}\rangle$$
$$=(1+\xi_f)\{J/2+\tfrac{1}{2}\langle f|J_3^{(p)}-J_3^{(n)}|f\rangle+[\mu(p)+\mu(n)-\tfrac{1}{2}]$$
$$\times\langle f|S_3^{(p)}+S_3^{(n)}|f\rangle+[\mu(p)-\mu(n)-\tfrac{1}{2}]\langle f|S_3^{(p)}-S_3^{(n)}|f\rangle\},$$

$$\mu(N_i)=(1+\xi_i)\langle\Psi_{N_i;\cdots M_i=J\cdots}|\sum_{a=1}^{A}\{[(1+\tau_3^{(a)})/2][\mu_p\sigma_3^{(a)}+(j_3^{(a)}-\tfrac{1}{2}\sigma_3^{(a)})]+[(1-\tau_3^{(a)})/2]\mu_n\sigma_3^{(a)}\}|\Psi_{N_i;\cdots M_i=J\cdots}\rangle$$
$$=(1+\xi_i)\{J/2+\tfrac{1}{2}\langle i|J_3^{(p)}-J_3^{(n)}|i\rangle+[\mu(p)+\mu(n)-\tfrac{1}{2}] \tag{A17}$$
$$\times\langle i|S_3^{(p)}+S_3^{(n)}|i\rangle+[\mu(p)-\mu(n)-\tfrac{1}{2}]\langle i|S_3^{(p)}-S_3^{(n)}|i\rangle\};$$
$$(1+\xi_f)\approx(1+\xi_i)\equiv(1+\xi'),$$

where $(1+\xi_f)$, $(1+\xi_i)$ are pion-exchange corrections. Equation (A17) yields

$$\langle f|S_3^{(p)}-S_3^{(n)}|f\rangle-\langle i|S_3^{(p)}-S_3^{(n)}|i\rangle=\frac{1/(1+\xi')[\mu(N_f)-\mu(N_i)]}{[\mu(p)-\mu(n)-\frac{1}{2}]\pm(l+\frac{1}{2})};$$
$$\pm(l+\tfrac{1}{2})\equiv-\frac{1}{2}\frac{\langle f|J_3^{(p)}-J_3^{(n)}|f\rangle-\langle i|J_3^{(p)}-J_3^{(n)}|i\rangle}{\langle f|S_3^{(p)}-S_3^{(n)}|f\rangle-\langle i|S_3^{(p)}-S_3^{(n)}|i\rangle} \tag{A18}$$

so that, substituting into Eq. (A16),

$$|\langle\boldsymbol{\sigma}\rangle_{fi}|=\left(\frac{J+1}{J}\right)^{1/2}\frac{1}{(1+\xi')}\left|\frac{\mu(N_f)-\mu(N_i)}{[\mu(p)-\mu(n)-\frac{1}{2}]\pm(l+\frac{1}{2})}\right|;$$
$$\left[\frac{G_A(N_i\to N_f)}{G_A(n\to p)}\right]^2_{\text{imp-approx theor}}=\left(\frac{1+\xi}{1+\xi'}\right)^2\left|\frac{\mu(N_f)-\mu(N_i)}{[\mu(p)-\mu(n)-\frac{1}{2}]\pm(l+\frac{1}{2})}\right|^2. \tag{A19}$$

In a model in which $\Psi_{N_i;\cdots M_i=J\cdots}$ and $\Psi_{N_f;\cdots M_f=J\cdots}$ are such that N_i and N_f can be visualized as consisting of a "core plus or minus an odd nucleon," l and $j=l\pm\frac{1}{2}$ are the orbital angular momentum and total-angular-momentum quantum numbers of the odd nucleon (e.g., in ${}_1H_2{}^3$ and ${}_2He_1{}^3$: $l=0$, and $j=l+\frac{1}{2}=\frac{1}{2}$; in ${}_7N_6{}^{13}$ and ${}_6C_7{}^{13}$: $l=1$ and $j=l-\frac{1}{2}=\frac{1}{2}$; etc.); for the numerical values of $[G_A(N_i\to N_f)/G_A(n\to p)]^2{}_{\text{imp-approx theor}}$ in the sixth column of Table I and the dashed curve of Fig. 1 we have used such a model and also taken $[(1+\xi)/(1+\xi')]^2=1$. Comparison of Eqs. (A19) and (A18) with Eqs. (30) and (27), viz.:

$$\left[\frac{G_A(N_i\to N_f)}{G_A(n\to p)}\right]^2_{\text{G-T theor; anom-mag-mom theor}}=\left|\frac{[\mu(N_f)-Z(N_f)/A]-[\mu(N_i)-Z(N_i)/A]}{[\mu(p)-1]-[\mu(n)-0]}\right|\frac{1}{A^{1/3}} \tag{A20}$$

shows that, in spite of the not too great differences between corresponding numerical values, the functional dependence of $[G_A(N_i\to N_f)/G_A(n\to p)]^2$ on $\mu(N_f)$ and $\mu(N_i)$ in the customary impulse-approximation theory is very different from that in our Goldberger-Treiman-type theory.