Baryon Bootstrap Possibilities in $SU(6)$

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We discuss the various bootstrap possibilities arising in meson-baryon scattering in $SU(6)$ using the static model. The mesons are taken to belong to the 35-plet of $SU(6)$. It is shown that if one restricts oneself for the baryons to the "eightfold way"-type representations, i.e., to the 20-plet, 56-plet, 70-plet, etc., then the simplest bootstrap possibility for baryons is the self-bootstrap of the 56-plet in the scattering of 56-plet baryons and mesons. No simpler possibility arises when one considers the scattering of a lower dimensional 20-plet of baryons with mesons.

1. INTRODUCTION

IT has been known for some time[1] that in the p-wave scattering of a pseudoscalar meson octet by a spin-$\frac{1}{2}^+$ baryon octet, the baryon $\frac{1}{2}^+$ octet, and a $\frac{3}{2}^+$ decimet are capable of sustaining each other in a reciprocal bootstrap. One can then raise the following question within the framework of the eightfold way: Given the pseudoscalar meson octet, do the bootstrap equations for meson-baryon scattering imply a definite multiplet structure for the baryons? This question was discussed recently[2] within a simple model which is just the $SU(3)$ version of Chew's original reciprocal bootstrap[3] for N and N^*, and it was shown that the simplest bootstrap that could exist was a reciprocal one between the $\frac{1}{2}^+$ octet and the $\frac{3}{2}^+$ decimet. Further, if the external baryons form an octet, then this 8–10 reciprocal bootstrap is the only possible bootstrap involving not more than two multiplets. Now one knows[4] that these two multiplets together form the 56-plet of $SU(6)$, and so the interesting possibility arises that the same bootstrap model might single out the 56-plet as the *simplest* baryon multiplet sustaining itself in a simple bootstrap with a meson 35-plet.[5,6] The purpose of this note is to show that this is indeed the case.

2. THE MODEL

The physical meson 35-plet has a 0^- octet, a 1^- octet, and a 1^- singlet as its components. In the static model, we have to consider its scattering by the baryon 56-plet in the p wave so as to conserve parity. We can take care of this orbital angular-momentum state in the static approximation by attaching the angular momentum to the mesons and treating them effectively[7,8] as a 1^+ octet, a 0^+ octet, and a 1^+ singlet, respectively, so that the effective particles again form a 35-plet.

Let us now consider the scattering of this meson 35-plet by an arbitrary baryon multiplet (of dimension R_0, say) belonging to "the eightfold way" in $SU(6)$. The values that R_0 can have are 20, 56, 70, $\cdots$. We assume, as usual, that the dominant contribution to the forces comes from the exchange of various baryon multiplets in the crossed channel. Since we are looking for a bootstrap mechanism, we shall demand that the exchanged multiplets and the external baryon R_0-plet appear as bound states in the direct channel. Now if we make the decomposition[9]

$$35\otimes R_0=R_0\oplus R_1\oplus R_2\oplus\cdots\oplus R_{n-1}, \qquad (1)$$

it is clear that in the scattering of R_0 by 35 the only representations in which bound states can possibly occur are the representations R_0, R_1, $\cdots$, R_{n-1} which appear on the right-hand side of (1). We are therefore led to consider $35-R_0$ scattering with the exchange of R_0, R_1, $\cdots$, R_{n-1} and to look at the forces arising therefrom in the states R_0, R_1, $\cdots$, R_{n-1} in the direct channel. The discussion of forces and of the possibilities of bootstraps now centers around the crossing matrix C connecting the baryon-exchange channel to the direct channel. In fact, if we use the static N/D model with a linear approximation for the D function, a necessary condition[10] for the existence of a bootstrap involving

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[5] That 56 can bootstrap itself has been shown by R. H. Capps and by J. G. Belinfante and R. E. Cutkosky (Ref. 6).
[6] R. H. Capps, Phys. Rev. Letters 14, 31 (1965); J. G. Belinfante and R. E. Cutkosky, *ibid.* 14, 33 (1965).
[7] The physical 0^- meson, in the p wave, can give only a 1^+ state; the physical 1^- meson, in the p wave, can give 0^+, 1^+, or 2^+ states. Thus we are assuming that the 1^+ and 2^+ state interactions are negligible for the 1^- octet, and that the 0^+, 2^+ interactions are negligible for the 1^- singlet. This choice is not arbitrary, but is forced on us if we demand that the effective mesons also form an $SU(6)$ multiplet.
[8] R. H. Capps and J. G. Belinfante and R. E. Cutkosky (Ref. 6) suggest an alternative possibility of taking the external mesons as a 0^- octet, a 0^- singlet, and a 1^- octet. One would then have only 33 spin states for the physical mesons, and thus they would not form a multiplet of $SU(6)$. It appears to us, however, that the physical mesons must belong to a multiplet of $SU(6)$ since they are coupled to the baryon-antibaryon system.
[9] Note that for $R_0=20$ and 56, R_0 occurs once on the right-hand side of (1). For $R_0=20$ and 56, which are the only values we need consider in our discussion, no representation occurs more than once on the right-hand side of (1). Thus there are no multichannel problems needing special consideration as in the SU_3 case considered in Ref. 2, and it is possible to discuss bootstraps simply with the help of Eq. (2).
[10] See Ref. 2 for details.

all these multiplets is[11]

$$\Gamma = C\Gamma , \tag{2}$$

where Γ is an n-component column vector whose elements are the reduced coupling constants between the multiplets $R_i(i=0, 1, \cdots, n-1)$, the external baryon multiplet R_0, and the meson multiplet 35. If there is no particle existing in any particular state R_i, we put the corresponding $\Gamma_i=0$.

Our procedure now is to consider Eq. (2) for the various representations 20, 56, 70, $\cdots$, and to look for the simplest bootstrap—i.e., the one with the lowest value of R_0, and for a given R_0, one demanding the smallest number of baryon multiplets to support one another [i.e., the smallest number of nonzero Γ_i's in Eq. (2)]. We would also demand that the baryon multiplets have as small a dimensionality as possible.

3. SCATTERING OF THE MESON 35-PLET BY A BARYON 20-PLET

The lowest representation belonging to the eightfold path to which baryons could possibly belong in $SU(6)$ is the 20-plet. We know that

$$35\otimes 20 = 20\oplus 70\oplus \overline{70}\oplus 540 . \tag{3}$$

The corresponding crossing matrix is

$$C=\begin{array}{cccc|l} (20) & (70) & (\overline{70}) & (540) & \\ 3/7 & -1 & -1 & 18/7 & (20) \\ -2/7 & 1/2 & -1/2 & 9/7 & (70) \\ -2/7 & -1/2 & 1/2 & 9/7 & (\overline{70}) \\ 2/21 & 1/6 & 1/6 & 4/7 & (540) \end{array} \tag{4}$$

The condition for the 20-plet to bootstrap itself would be that $C_{20,20}$ be approximately equal to unity. We see that $C_{20,20}$ is only 3/7, and therefore conclude that the 20-plet is not capable of bootstrapping itself.

We then ask if there can be a reciprocal bootstrap involving the 20-plet and some other multiplet. For this we use Eq. (2), which yields[12]

$$\Gamma_{20}+ (7/4)(\Gamma_{70}+\Gamma_{\overline{70}}) = \tfrac{9}{2}\Gamma_{540} . \tag{5}$$

Equation (5) suggests the possibility of the 20-plet and the 540-plet reciprocally bootstrapping. But then [putting $\Gamma_{70}=\Gamma_{\overline{70}}=0$, in Eq. (5)]

$$\Gamma_{540}/\Gamma_{20}=2/9 ,$$

which is rather small. So, following the arguments of Ref. 2, we conclude that the 540-plet, if it exists at all, would appear at too high a value of energy for our model to be applicable, and that there is therefore no possibility of a reciprocal bootstrap between a 20-plet and a 540-plet at low energies. Equation (5) also shows that neither the 70-plet nor the $\overline{70}$-plet can bootstrap reciprocally with the 20-plet (Γ_{70}/Γ_{20} or $\Gamma_{\overline{70}}/\Gamma_{20}$ would be negative). The only possibilities of bootstraps in which the relevant coupling constants can be comparable are ones in which at least two more multiplets appear besides the 20-plet; e.g., 20-70-540, or 20-$\overline{70}$-540, or 20-70-$\overline{70}$-540; in the last case, we could have all the Γ's equal.

Having seen that there is no simple bootstrap possible with the 20-plet, we now consider the next higher representation possible for the baryons, namely the 56.

4. SCATTERING OF THE MESON 35-PLET BY A BARYON 56-PLET

We know that

$$56\otimes 35 = 70\oplus 1134\oplus 56\oplus 700 , \tag{6}$$

where the representations on the right-hand side have been written in order of increasing weights. The crossing matrix is[13]

$$C=\begin{array}{cccc|l} (70) & (1134) & (56) & (700) & \\ 1/4 & -27/20 & -2/5 & 5/2 & (70) \\ -1/12 & 17/20 & -2/45 & 5/18 & (1134) \\ -1/2 & -9/10 & 11/15 & 5/3 & (56) \\ 1/4 & 9/20 & 2/15 & 1/6 & (700) . \end{array} \tag{7}$$

As one notices, the only diagonal elements which are close to unity are the 56-56 and the 1134-1134 elements, indicating that one has the possibility of an approximate self-consistent bootstrap involving either a 56-plet alone or an 1134-plet alone. One is therefore led to consider the possibility of these two multiplets existing together in a reciprocal bootstrap.

We again discuss this possibility using Eq. (2), which, with C given by (5), yields[12]

$$\Gamma_{56}+ (15/8)\Gamma_{70}+ (27/8)\Gamma_{1134}= (25/4)\Gamma_{700} . \tag{8}$$

If one puts $\Gamma_{70}=\Gamma_{700}=0$ in Eq. (8), it reduces to

$$\Gamma_{56}/\Gamma_{1134}= -27/8 .$$

Thus Γ_{56} and Γ_{1134} cannot both be positive and hence the possibility of the self-consistent coexistence of the 56-plet and the 1134-plet is ruled out. Inspection of the crossing matrix shows that this is essentially because of the fact that the exchange of the 1134-plet produces a strong repulsion in the 56 state.

Since the 56-plet and the 1134-plet cannot coexist, we have to choose one of them, and we naturally choose the 56-plet, not only because it is the simpler multiplet, but also because it represents the external

[11] Note that while in the $SU(2)$ and $SU(3)$ problems, the crossing matrix is a direct product of the J crossing matrix and the symmetry crossing matrix, here we have only the symmetry crossing matrix to consider, since the spin is part of the symmetry, and the orbital angular momentum is attached to the meson spin.

[12] There is only one relation between the four Γ's. This is because of the fact that C has the eigenvalue 1 occurring thrice.

[13] The third column of the crossing matrix has been given by R. H. Capps and by J. G. Belinfante and R. E. Cutkosky (Ref. 6).

baryons, and we certainly want the external baryons to emerge from the bootstrap.

One can still ask the question as to whether the 56-plet can bootstrap reciprocally with any other single multiplet. Equation (8) shows that the only such possibility is the pair 56 and 700. In this case, however, the ratio of the reduced coupling constants is

$$\Gamma_{700}/\Gamma_{56}=4/25,$$

which is rather small, and implies that if the 700-plet did exist at all, it would occur at a high energy, insofar as the magnitude of the coupling constant is any indication. Within the region of validity of this model, therefore, we may discard this possibility, just as we discarded the possibility of a 20-540 reciprocal bootstrap in the last section.

It is to be remarked that the 700-plet can in principle bootstrap reciprocally with any one of the other three multiplets, or with all of them together (in the latter case, all Γ's can be equal), and is in fact the only multiplet having this possibility. This has to do with the fact that it is the multiplet having the highest weight in the decomposition (5), and the quite general result that the highest weight multiplet gets an attractive contribution from the exchange of each of the multiplets in the decomposition.[14]

We conclude that the 56-plet self-bootstrap is the simplest possible bootstrap in the p-wave scattering of baryons by the meson 35-plet in the eightfold path of $SU(6)$.

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[14] This is seen here from the fact that all the elements of the last row of the C matrix are positive.

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Second-Order Perturbation in Intrinsically Broken $\tilde{U}(12)$

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In second-order perturbation theory in intrinsically broken $\tilde{U}(12)$, it is found that in many cases the $\tilde{U}(12)$ symmetry is recovered in the forward direction. The consequences of this are discussed, as is the asymptotic behavior of amplitudes and differential cross sections for the case of octet-baryon–pseudoscalar-meson scattering.

1. INTRODUCTION

THE noncompact symmetry group $\tilde{U}(12)$, which can be regarded as a relativistic generalization of $SU(6)$, has been proposed by several authors[1] as the underlying symmetry group of strong interactions. Exact $\tilde{U}(12)$ invariance has significance only in a highly idealized sense and to give any physical interpretation to the theory it is necessary to break the symmetry by the identification of the content of the $\tilde{U}(12)$ irreducible representations with physical particle states. This intrinsic symmetry breaking is most conveniently done with the aid of the Wigner-Bargmann (W.B.) equations, which eventually break the symmetry to $SU(3)\otimes\mathcal{L}_4$.

In the study of three- and four-point functions the approach of many authors[1,2] has been to write the general amplitude formally in terms of $\tilde{U}(12)$ invariant amplitudes and then to identify the initial and final states with physical states by means of the Wigner-Bargmann equations, maintaining the $\tilde{U}(12)$ invariant amplitudes. This approach has met with relative success, preserving most of the consequences of $SU(6)$, both good and bad, and leading to a number of new predictions. In particular, predictions about baryon-antibaryon annihilation into mesons have met with qualitative success, but predictions about baryon polarization seem to be at variance with experiment provided only "regular" couplings are included.[3]

Since it has been shown to be possible to derive the Wigner-Bargmann equations and propagators from a Lagrangian approach[4] we will consider second-order perturbation theory in baryon-meson scattering, baryon-baryon scattering and meson-meson scattering

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