baryons, and we certainly want the external baryons to emerge from the bootstrap.

One can still ask the question as to whether the 56-plet can bootstrap reciprocally with any other single multiplet. Equation (8) shows that the only such possibility is the pair 56 and 700. In this case, however, the ratio of the reduced coupling constants is

$$\Gamma_{700}/\Gamma_{56}=4/25,$$

which is rather small, and implies that if the 700-plet did exist at all, it would occur at a high energy, insofar as the magnitude of the coupling constant is any indication. Within the region of validity of this model, therefore, we may discard this possibility, just as we discarded the possibility of a 20-540 reciprocal bootstrap in the last section.

It is to be remarked that the 700-plet can in principle bootstrap reciprocally with any one of the other three multiplets, or with all of them together (in the latter case, all Γ's can be equal), and is in fact the only multiplet having this possibility. This has to do with the fact that it is the multiplet having the highest weight in the decomposition (5), and the quite general result that the highest weight multiplet gets an attractive contribution from the exchange of each of the multiplets in the decomposition.[14]

We conclude that the 56-plet self-bootstrap is the simplest possible bootstrap in the p-wave scattering of baryons by the meson 35-plet in the eightfold path of $SU(6)$.

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[14] This is seen here from the fact that all the elements of the last row of the C matrix are positive.

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Second-Order Perturbation in Intrinsically Broken $\tilde{U}(12)$

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In second-order perturbation theory in intrinsically broken $\tilde{U}(12)$, it is found that in many cases the $\tilde{U}(12)$ symmetry is recovered in the forward direction. The consequences of this are discussed, as is the asymptotic behavior of amplitudes and differential cross sections for the case of octet-baryon–pseudoscalar-meson scattering.

1. INTRODUCTION

THE noncompact symmetry group $\tilde{U}(12)$, which can be regarded as a relativistic generalization of $SU(6)$, has been proposed by several authors[1] as the underlying symmetry group of strong interactions. Exact $\tilde{U}(12)$ invariance has significance only in a highly idealized sense and to give any physical interpretation to the theory it is necessary to break the symmetry by the identification of the content of the $\tilde{U}(12)$ irreducible representations with physical particle states. This intrinsic symmetry breaking is most conveniently done with the aid of the Wigner-Bargmann (W.B.) equations, which eventually break the symmetry to $SU(3)\otimes\mathcal{L}_4$.

In the study of three- and four-point functions the approach of many authors[1,2] has been to write the general amplitude formally in terms of $\tilde{U}(12)$ invariant amplitudes and then to identify the initial and final states with physical states by means of the Wigner-Bargmann equations, maintaining the $\tilde{U}(12)$ invariant amplitudes. This approach has met with relative success, preserving most of the consequences of $SU(6)$, both good and bad, and leading to a number of new predictions. In particular, predictions about baryon-antibaryon annihilation into mesons have met with qualitative success, but predictions about baryon polarization seem to be at variance with experiment provided only "regular" couplings are included.[3]

Since it has been shown to be possible to derive the Wigner-Bargmann equations and propagators from a Lagrangian approach[4] we will consider second-order perturbation theory in baryon-meson scattering, baryon-baryon scattering and meson-meson scattering

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where propagated particles belong to **364** and **143** representations.

It must be emphasized that, in general, the amplitudes corresponding to the above perturbation diagrams have only $SU(3)\otimes\mathfrak{L}_4$ transformation properties and hence no expansion of amplitudes can be made in terms of $\tilde{U}(12)$ invariant amplitudes. In other words, the introduction of propagators forces particle identification from the start in the intermediate states.

Under certain circumstances some of the $\tilde{U}(12)$ properties can be retrieved, and it is these that we first discuss. Because the consequences of $\tilde{U}(12)$ so far discussed are mainly confined to octet-baryon—pseudoscalar-meson scattering the perturbation amplitudes discussed will be mainly confined to this case. Sections 2 and 3 will be devoted to perturbation amplitudes in the limit of forward scattering and Sec. 4 to general differential cross sections for the above case with special reference to asymptotic behavior.

2. FORWARD SCATTERING

We will consider forward scattering for the cases in which the collinear initial and final momenta belong to particles on the *same* mass hyperbola [in the $\tilde{U}(12)$ limit]. For a channel in which this is possible (e.g., this is not possible in the annihilation channel in which baryon-antibaryon annihilate into two mesons) let the initial and final momenta of the particles be p_1, p_2 and p_3, p_4, respectively, where

$$p_1+p_2=p_3+p_4$$

and $p_1^2=p_3^2$, $p_2^2=p_4^2$. Let the forward scattering in this channel correspond to $p_1=p_3$, $p_4=p_2$.

Omitting the contact terms (the momentum dependence of the contact terms in the **364** propagator do not affect the following discussion) the **364** and **143** propagators are, respectively,

$$\begin{aligned}\Delta_{ABC}{}^{A'B'C'}(q)&=\Delta_{\alpha a,\beta b,\gamma c}{}^{\alpha' a',\beta' b',\gamma' c'}(q)\\&=\tfrac{1}{6}\sum\frac{(\mathbf{q}+m)_\alpha{}^{\alpha'}\delta_a{}^{a'}(\mathbf{q}+m)_\beta{}^{\beta'}\delta_b{}^{b'}(\mathbf{q}+m)_\gamma{}^{\gamma'}\delta_c{}^{c'}}{m^2(q^2-m^2)}\,,\end{aligned}\tag{2.1}$$

$$\begin{aligned}\Delta_{EC}{}^{FD}(q)&=\Delta_{\epsilon e,\gamma c}{}^{\varphi f,\delta d}(q)=\frac{\delta_c{}^f\delta_d{}^e(\mathbf{q}+\mu)_\epsilon{}^\delta(\mathbf{q}-\mu)_\gamma{}^\varphi}{\mu^2(q^2-\mu^2)}\,,\end{aligned}\tag{2.2}$$

where m and μ are the masses of the **364** and **143** representations. The Greek indices take the values 1,2,3,4 and the Roman the values 1,2,3. Since there are now only two independent momenta p_1 and p_2, the propagated momentum q is either p_1+p_2, p_1-p_2 or zero.

(a) For any diagram for which $q=0$, the $\tilde{U}(12)$ transformation properties of the propagator are immediately recovered, since it is the presence of the momentum in the numerator of the propagator that destroys these

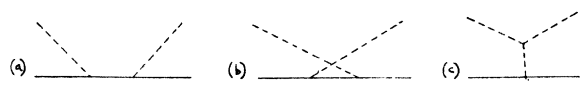

FIG. 1. Pole diagrams in baryon-meson scattering.

properties. The amplitudes corresponding to such diagrams are thus immediately expressible formally in terms of $\tilde{U}(12)$ amplitudes.

(b) Suppose for a diagram under question $q=p_1+p_2$. Then each bracket in the numerator of the propagator will be sandwiched between two multispinors. With the application of the W.B. equations $\mathbf{q}$ can be expressed as the product of a scalar and a $\tilde{U}(4)$ δ function. For example,

$$\begin{aligned}&\bar{\Psi}^{\alpha\beta\gamma,abc}(p_1)(\mathbf{q})_\alpha{}^{\alpha'}\Psi_{\alpha'\delta\epsilon,ade}(p_2)\\&\qquad=2m\bar{\Psi}^{\alpha\beta\gamma,abc}(p_1)\delta_\alpha{}^{\alpha'}\Psi_{\alpha'\delta\epsilon,ade}(p_2)\,,\\&\bar{\Psi}^{\alpha\beta\gamma,abc}(p_1)(\mathbf{q})_\alpha{}^{\alpha'}\Psi_{\alpha'\delta\epsilon,ade}(p_1)\\&\quad=(1/m)(p_1p_2+m^2)\bar{\Psi}^{\alpha\beta\gamma,abc}(p_1)\delta_\alpha{}^{\alpha'}\Psi_{\alpha'\delta\epsilon,ade}(p_1)\,,\end{aligned}\tag{2.3}$$

where the completely symmetric multispinor corresponding to the **364** representation has been used as an example. Similarly for $q=p_1-p_2$.

Thus in the limit of forward scattering for the case when this implies equality of four-momenta a reduction of the perturbation amplitude in terms of $\tilde{U}(12)$ invariant amplitudes can be made (the presence of blob form factors at the vertices does not affect this conclusion).

To be more precise, let us consider baryon-meson scattering. In second-order perturbation theory, three diagrams contribute (Fig. 1). The solid lines denote the baryon **364** representation and the dashed lines the meson **143** representation.

In the now familiar multispinor notation these diagrams give respective contributions to the second-order amplitude of

$$\begin{aligned}A&=g^2\bar{\Psi}^{ABD}(p_3)\Phi_D{}^C(-p_4)\Delta_{ABC}{}^{A'B'C'}(p_1+p_2)\\&\qquad\times\Psi_{A'B'D'}(p_1)\Phi_{C'}{}^{D'}(p_2)\,,\\A^c&=g^2\bar{\Psi}^{ABD}(p_3)\Phi_D{}^C(p_2)\Delta_{ABC}{}^{A'B'C'}(p_1-p_4)\\&\qquad\times\Psi_{A'B'D'}(p_1)\Phi_{C'}{}^{D'}(-p_4)\,,\\C&=fg\bar{\Psi}^{ABC}(p_3)\Psi_{ABD}(p_1)\Delta_{EC}{}^{FD}(p_4-p_2)\\&\quad[\Phi_F{}^G(-p_4)\Phi_G{}^E(p_2)+\Phi_G{}^E(-p_4)\Phi_F{}^G(p_2)]\,,\end{aligned}\tag{2.4}$$

where f,g are the three-meson and baryon-antibaryon-meson coupling constants, respectively.

Take $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, $u=(p_1-p_4)^2$. If the general form of the amplitude for meson-baryon scattering is

$$\begin{aligned}&\bar{\Psi}^{ABC}(p_3)\Phi_E{}^D(-p_4)\Psi_{ABC}(p_1)\Phi_D{}^E(p_2)A_1(s,t)\\&\quad+\bar{\Psi}^{ABC}(p_3)\Phi_C{}^E(-p_4)\Psi_{ABD}(p_1)\Phi_E{}^D(p_2)A_{143}(s,t)\\&\quad+\bar{\Psi}^{ABC}(p_3)\Phi_E{}^D(-p_4)\Psi_{ABD}(p_1)\Phi_C{}^E(p_2)A_{143'}(s,t)\\&\quad+\bar{\Psi}^{ADE}(p_3)\Phi_D{}^B(-p_4)\Psi_{ABC}(p_1)\Phi_C{}^E(p_2)A_{5940}(s,t)\,,\end{aligned}\tag{2.5}$$

then, for forward scattering in the baryon-meson channel ($t=0$), second-order perturbation theory corresponds to taking

$$A_1(s,0)=0\,,$$

$$A_{143}(s,0)=\frac{g^2}{24}\frac{(s+3m^2-\mu^2)^2(s+\mu^2-m^2+2\mu m)}{m^4\mu(s-m^2)}+\frac{fg}{\mu^2}\,,\tag{2.6}$$

$$A_{143'}(s,0)=\frac{g^2}{24}\frac{(u+3m^2-\mu^2)^2(u+\mu^2-m^2+2\mu m)}{m^4\mu(u-m^2)}+\frac{fg}{\mu^2}\,,$$

$$A_{5940}(s,0)=\frac{(2m+\mu)^2g^2}{3m^2}\left[\frac{s+3m^2-\mu^2}{s-m^2}+\frac{u+3m^2-\mu^2}{u-m^2}\right].$$

Because of the baryon-meson mass difference, the perturbation amplitude will *not* reduce to $\tilde{U}(12)$ form for forward scattering in the annihilation channel. For baryon-baryon or meson-meson scattering, it is apparent that the second-order perturbation amplitude recovers $\tilde{U}(12)$ for forward scattering in any channel.

This recovery of greater symmetry in the forward direction for these processes is a consequence of second-order perturbation theory and would not be maintained for higher order perturbation diagrams.

3. CONSEQUENCES

Since it is the octet-baryon–pseudoscalar meson four point function that has been considered in most detail we will be primarily concerned with the consequences of the preceding section for this case.

(1) Octet-Baryon Pseudoscalar Meson Scattering

The $\tilde{U}(12)$ relations for forward scattering are maintained.[2]

If $f(M)$ is the forward scattering amplitude for $pM\rightarrow pM$ elastic scattering, then

$$\tfrac{1}{2}[f(K^+)-f(K^-)]=[f(\pi^+)-f(\pi^-)]=[f(K^0)-f(\bar{K}^0)]\tag{3.1}$$

independent of form factors. The consequences of this are now rather different than when the full four-point function was considered since there is no optical theorem available. The most that can be said is that relation (3.1) holds when $f(M)$ is replaced by $+(d\sigma(t=0))^{1/2}$.

In addition, other $\tilde{U}(12)$ results are obtained for $t=0$,[1] e.g.,

$$d\sigma(\pi^-p\rightarrow K^+\Sigma^-;\,t=0)=d\sigma(K^-p\rightarrow\Sigma^-\pi^+;\,t=0)\,.\tag{3.2}$$

Because of the maintenance of $SU(3)$ invariance, all $SU(3)$ results will be obtained for arbitrary s and t.

(2) Polarization of Final Baryons

One of the predictions of $\tilde{U}(12)$ that seems to be at variance with experiment is that for unpolarized initial baryons, the final baryon will be unpolarized for several processes, including

$$K^-+p\rightarrow\Xi^-+K^+$$

and

$$\pi^++n\rightarrow\Sigma^++K^0$$

for which there exists experimental evidence to the contrary. With no form factors at the vertices in the second-order perturbation theory, no polarization can be expected for any processes purely as a result of the model. However, if form factors are introduced at the vertices and imaginary parts given to them by some phenomenological or dispersion argument, then there will be polarization for all processes, including those for which $\tilde{U}(12)$ gives a selection rule, for general s and t. If the polarization $\langle\boldsymbol{\sigma}\rangle$ in the c.m. system with scattering angle θ and energy E be expressed as

$$\langle\boldsymbol{\sigma}\rangle=\mathbf{n}\,\sin\theta P(\theta,E)\,,\tag{3.3}$$

where $\mathbf{n}$ is a unit vector perpendicular to the momentum plane, then for processes for which $\tilde{U}(12)$ forbids polarization of the final baryon, polarization will tend to zero faster than $\sin\theta$ for small-angle scattering, i.e.,

$$P(\theta,E)\rightarrow 0\quad\text{as}\quad\theta\rightarrow 0\,.\tag{3.4}$$

In those processes for which polarization is allowed by $\tilde{U}(12)$, second-order perturbation theory gives

$$P(\theta,E)\rightarrow\text{const}(\neq 0)\quad\text{as}\quad\theta\rightarrow 0\tag{3.5}$$

for constant E.

(3) Baryon-Baryon Scattering

One of the consequences of $\tilde{U}(12)$ is that octet-baryon antibaryon scattering at threshold is purely elastic, owing to the survival of only the **126412** amplitude in the four-point function. Because threshold corresponds to

$$s=4m^2,\quad t=u=0\,,$$

it follows that the same conclusion holds when second-order perturbation amplitudes are considered. To obtain nonelastic scattering from a perturbation amplitude at threshold it is necessary to go to higher orders than second.

This should be compared with baryon-antibaryon annihilation into two mesons at threshold, which is forbidden by $\tilde{U}(12)$, but which is given with reasonable quantitative accuracy by the second-order perturbation amplitude.[5]

4. GENERAL SCATTERING (OCTET-BARYON–PSEUDOSCALAR-MESON)

For octet-baryon–pseudoscalar-meson scattering the general second-order perturbation amplitude is of the

[5] R. Delbourgo, Y. C. Leung, M. A. Rashid, and J. Strathdee, Phys. Rev. Letters **14**, 609 (1965).

form

$$\bar{N}(p_3)F(p_1,p_2,p_3,p_4)N(p_1)\,, \tag{4.1}$$

where F is a sum of products of γ matrices. It is most convenient to express this amplitude in the form

$$\bar{N}(p_3)[c(s,t)+d(s,t)\gamma_\mu r_\mu]N(p_1)\,, \tag{4.2}$$

where

$$r_\mu=-\tfrac{1}{4}\gamma_5\epsilon_{\mu\nu\rho\sigma}(p_1+p_3)_\nu(p_2+p_4)_\rho(p_1-p_3)_\sigma\,. \tag{4.3}$$

The scalar amplitudes $c(s,t)$ and $d(s,t)$ are given in terms of F as

$$c(s,t)=\tfrac{1}{4}\,\mathrm{Tr}[(\mathbf{p}_3+m)F(\mathbf{p}_1+m)]/(2m^2-t/2)\,, \tag{4.4}$$

$$d(s,t)=\frac{\Delta}{2m}\frac{(u-s)}{(2m^2-t/2)}\tfrac{1}{4}\,\mathrm{Tr}[(\mathbf{p}_3+m)F(\mathbf{p}_1+m)] +\frac{\Delta}{m^2}\tfrac{1}{4}\,\mathrm{Tr}[(\mathbf{p}_3+m)F(\mathbf{p}_1+m)\mathbf{p}_2]\,, \tag{4.5}$$

where

$$\Delta^{-1}(s,t) =-(1/4m^2)t\{[s+(m^2-\mu^2)]^2+s(t-4m^2)\}\,. \tag{4.6}$$

For an amplitude expressed in the form (4.2) the differential cross section in the c.m. system is given by

$$d\sigma/d\Omega=(m/4\pi E)^2(1-t/4m^2) \times[|c(s,t)|^2+q^4E^2\sin^2\theta|d(s,t)|^2]\,, \tag{4.7}$$

where q is the magnitude of the three-momentum.

Because of the fact that in general F is a sum of terms that are the product of seven γ matrices the differential cross sections are extremely long and unwieldy. With unit form factors the typical behavior for asymtotically large s is

$$c(s,t)\sim s^3\,, \tag{4.8}$$

$$d(s,t)\sim s^2\,, \tag{4.9}$$

and hence

$$d\sigma/d\Omega\sim s^5 \tag{4.10}$$

which implies that form factors vanishing asymptotically as fast as $s^{-5/4}$ are needed to give physically sensible answers. This strong asymptotic behavior arises because the propagator for both the spin-$\frac{1}{2}$ and -$\frac{3}{2}$ baryons in this theory behaves as q for large momenta.

It may be worthwhile comparing this behavior with that obtained by considering just the $\tilde{U}(12)$ part of the amplitude arising from these three perturbation graphs (with unit form factors). This corresponds to taking

$$\begin{aligned} A_1(s,t)&=0\,,\\ A_{143}(s,t)&=\tfrac{1}{3}g^2m\frac{1}{s-m^2}-fg\frac{1}{t-\mu^2}\,,\\ A_{143'}(s,t)&=\tfrac{1}{3}g^2m\frac{1}{u-m^2}-fg\frac{1}{t-\mu^2}\,,\\ A_{5940}(s,t)&=\tfrac{2}{3}g^2m\left(\frac{1}{s-m^2}+\frac{1}{u-m^2}\right)\,, \end{aligned} \tag{4.11}$$

which gives characteristic behavior of

$$c(s,t)\sim s\,, \tag{4.12}$$

$$d(s,t)\sim 1\,, \tag{4.13}$$

and hence

$$d\sigma/d\Omega\sim s\,. \tag{4.14}$$

That is, the $\tilde{U}(12)$ part of the amplitude behaves as that due to the propagation of spinless particles and the full amplitude as that due to the propagation of the highest-spin particles in the representation to which the intermediate states belong. To consider just the $\tilde{U}(12)$ part of the amplitude arising from the second-order perturbation graphs is likely to be a dangerous procedure in that by neglecting higher spin behavior in the propagators, indefinite metrics are probably introduced.

5. CONCLUSION

We have shown that for second-order perturbation theory the perturbation amplitudes recover their $\tilde{U}(12)$ properties for forward scattering where the collinear momenta belong to particles on the same mass hyperbola, and that in general the perturbation amplitude has the asymptotic behavior associated with the propagated particles of highest spin in the propagated multiplet whereas the $\tilde{U}(12)$ part of the amplitude in general has the asymptotic behavior associated with propagation of spinless particles.

The case that superficially suggests the greatest recover of symmetry is weak interactions in which diagrammatically the incoming meson is converted into a tadpole spurion. Since the spurion carries zero momentum, $\tilde{U}(12)$ amplitudes are formally recovered but to obtain any physical results it is necessary to introduce Okubo mass splitting within the $SU(3)$ multiplets and it is as yet difficult to see how to do this within a $\tilde{U}(12)$ framework.

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