PHYSICAL REVIEW VOLUME 139, NUMBER 6B 20 SEPTEMBER 1965

Experimental Study of K_{e4}^+ Decay*

Robert W. Birge, Robert P. Ely, Jr., George Gidal, George E. Kalmus,
Anne Kernan, and Wilson M. Powell
Lawrence Radiation Laboratory, University of California, Berkeley, California

and

Ugo Camerini, David Cline, William F. Fry, James G. Gaidos, David Murphree,
and C. Thornton Murphy†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(Received 13 May 1965)

The rare K_{e4}^+ decay modes $\pi^+\pi^-e^+\nu$, $\pi^+\pi^+e^-\bar{\nu}$, and $\pi^+\pi^+\mu^-\bar{\nu}$ were searched for in a sample of approximately 3×10^6 K^+ decays. A total of 69 events of the mode $\pi^+\pi^-e^+\nu$ were observed. The other two modes are forbidden by the $\Delta S=\Delta Q$ rule and no examples of them were found. This experiment gives an upper limit for the amplitude ratio $X=|[A(\Delta S=-\Delta Q)]/[A(\Delta S=\Delta Q)]|$, for the axial-vector current of 0.25, with the assumption of no final-state π-π interaction. The effects of final-state interactions in the π-π system modify the interpretation of this result and may be large. The rate for the $\pi^+\pi^-e^+\nu$ mode was determined to be $(2.9\pm0.6)\times10^3$ sec^{-1}. The angular correlations between the planes formed by the lepton pair and the pion pair, and the angular distribution of π^+ with respect to the dipion line of flight in the dipion rest system for the 69 events, were studied. The dipion mass spectrum is presented for the 69 $\pi^+\pi^-e^+\nu$ events. The value of the ratio of the two form factors in K_{e4} decay (p wave to s wave) was found to be $\eta=0.8\pm0.30$ from an analysis of these correlations. An estimate of the low-energy π-π phase shift was determined to be "$\delta_0-\delta_1$" $=35°\pm30°$ from the angular correlations. The difference "$\delta_0-\delta_1$" between the s-wave (δ_0) and p-wave (δ_1) phase shift is averaged over the π-π invariant mass spectrum. Since this spectrum is peaked at low-dipion invariant mass, the average pertains mainly to an invariant mass in the region of 179.2 to 350 MeV. The shape of the dipion invariant mass spectrum does not agree with the shape expected if a σ resonance exists and has a mass of approximately 400 MeV and a width of approximately 100 MeV.

I. INTRODUCTION

The rare decay modes of the K^+ meson

$$K\to\pi^+\pi^-e^+\nu,\ K_{e4}(e^+)\,, \tag{1}$$

$$\to\pi^+\pi^+e^-\bar{\nu},\ K_{e4}(e^-)\,, \tag{2}$$

permit a test of the $\Delta Q=\Delta S$ selection rule[1] for axial-vector currents in weak interactions.[2,3] Reaction (1) is permitted by this rule, but (2) is forbidden.

Several authors[2] have discussed the general form of the matrix element for these decays and have shown, assuming a $V-A$ theory, that reaction (2) proceeds almost entirely through the axial-vector current, whereas (1) is a mixture of vector and axial-vector. Arguments can be made that show that the vector portion of the latter is two orders of magnitude smaller than the axial-vector part.[4]

The rate of the $K_{e4}(e^+)$ decay mode has been estimated by many authors,[2–7] and the effects of final-state interactions on the decay rate have been considered by Ciochetti,[5] Brown and Faier,[6] and Kacser, Singer, and Truong.[7]

The isotopic-spin state of the two pions in reaction (2) is pure $T=2$. In reaction (1), if the dipion is emitted in an even-angular-momentum state, $T=0$ and $T=2$ states are possible, but an odd-angular-momentum state will have $T=1$ associated with it. The relative rates of reactions (1) and (2) will, therefore, depend upon the amplitudes of the isotopic-spin states as well as the strengths of the $\Delta S=-\Delta Q$ and $\Delta S=+\Delta Q$ currents.

The presence of two pions and two leptons in the final state offers an excellent opportunity for pion-pion interactions to be studied without the complication caused by additional strongly interacting particles.[8–10]

If there is a final-state interaction between the π^+ and π^- in reaction (1), then interference between the s and p waves brings about a correlation between the dilepton

---

* Work performed under the auspices of the U. S. Atomic Energy Commission.

† Present address: the University of Michigan, Ann Arbor, Michigan.

[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[2] V. S. Mathur, Nuovo Cimento **14**, 1322 (1959); L. B. Okun and E. P. Shabalin, Zh. Eksperim. i Teor. Fiz. **37**, 1775 (1959) [English transl.: Soviet Phys.—JETP **10**, 1252 (1960)]; A. Sirlin, Phys. Rev. **129**, 1377 (1963).

[3] K. Chadan and S. Oneda, Phys. Rev. Letters **3**, 292 (1959); E. P. Shabalin, Zh. Eksperim. i Teor. Fiz. **39**, 345 (1960) [English transl.: Soviet Phys.—JETP **12**, 245 (1961)]; R. E. Behrends and A. Sirlin, Phys. Rev. Letters **8**, 221 (1962); B. Sakita, M. Kato, and E. McClement, University of Wisconsin (unpublished report); B. A. Arbazov, Nguyen Van Hieu, and R. N. Faustov, Zh. Eksperim. i Teor. Fiz. **44**, 329 (1963) [English transl.: Soviet Phys.—JETP **17**, 225 (1963)].

[4] J. Iliopolous, Orsay Report No. TH/84 and Erratum (unpublished).

[5] G. Ciocchetti, Nuovo Cimento **25**, 385 (1962).

[6] L. M. Brown and H. Faier, Phys. Rev. Letters **12**, 514 (1964).

[7] C. Kacser, P. Singer, and T. N. Truong, Phys. Rev. **137**, B1605 (1965); **139**, AB5(E) (1965).

[8] E. P. Shabalin, Zh. Eksperim. i Teor. Fiz. **44**, 765 (1963) [English transl.: Soviet Phys.—JETP **17**, 517 (1963)].

[9] N. Cabibbo and A. Maksymowicz, Phys. Rev. **137**, B438 (1965).

[10] R. H. Dalitz, Proceedings of the International School of Physics, "Enrico Fermi," Varenna Lectures, 1964 (to be published).

and the dipion decay planes. This correlation is equivalent to an up-down asymmetry of the e^+ with respect to the plane formed by the two pions. An asymmetry of this type can also be induced by a violation of time-reversal invariance. In this paper we have assumed time-reversal invariance. Interference between the s and p waves also brings about a forward-backward asymmetry of the π^+ in the dipion rest frame, with respect to the dipion line of flight. This asymmetry is reflected in a difference in the π^+ and π^- energy spectra. Cabibbo and Maksymowicz have shown that the ratio of these two asymmetries is proportional to the tangent of the phase between the s and p waves.[9] The effects of a pion-pion interaction should be exhibited in the dipion mass spectrum irrespective of the presence of any p wave.

Preliminary results of this experiment have been reported previously.[11,12]

II. PROCEDURE

A. Beam and Chamber

Three million K decays were observed from stopping K^+ mesons, the K^+ mesons being stopped in the Berkeley heavy-liquid bubble chamber.[13] The beam used was produced at 26° from an internal target and had two stages of separation. The beam momentum was 800 MeV/c with a momentum bite of $\pm 2\%$. The pion contamination before the degrader was less than 1%. The beam was degraded by a copper sawtooth degrader at the bubble-chamber entrance window, so that the K^+ stopping points were well spread out in the chamber. The stopping volume of the K^+ was approximately 30 cm long by 20 cm wide by 8 cm deep. This spreading of the K^+ decay points had the advantage of separating the origins for ease of scanning. A total of 240 000 pictures containing about 3.0×10^6 stopping K^+, an average of about 13 K^+/picture, was taken.

The chamber was filled with Freon (C_3F_8), which has a density of 1.22 g/cm^{-3} and a radiation length of 28 cm under operating conditions. The choice of this liquid was influenced by a number of considerations, the most important of which, its high stopping power, meant that the majority of the pions from K_{e4} decay and all the pions from τ decays stop in the chamber. An excessive stopping power, on the other hand, seriously reduced the resolution in the experiment, because of the difficulty of observing low-momentum particles. The second most important consideration was the identification of the electron from K_{e4} decay; this requirement dictates a short radiation length, but clearly, a compromise had to be made because of the negative effect of a short radiation length on the measurability of an electron track. Finally, there were also considerations that involved other experiments with the same film and that made a low-Z material desirable (small depolarization and μ^- capture).[14] On the basis of these considerations, Freon C_3F_8 was chosen.

The film was divided equally between the two institutions (LRL and UW) for scanning and preliminary selection of events. Final selection of the events was carried out jointly. All the film at both institutions was rescanned, in order for the scanning efficiencies to be evaluated. The scanning efficiencies were found to be approximately the same and the K_{e4} rates obtained from each institution were found to agree.

B. Scanning

The film was scanned for K_{e4} candidates and for τ decays. The τ decays provided a measurement of the number of stopped K^+ in the experiment, which number was used in calculations of the absolute branching ratio for K_{e4} decays.

An event had to satisfy the following criteria on the scan table to be taken as a K_{e4} *candidate*:

(a) There were three tracks from the origin.

(b) The ionization of the K^+ track near the decay point was such as to be consistent with a stopping particle.

(c) If all three particles from the decay stopped in the chamber, one had to be identified as an electron (by curling up) and two as pions (by increase in ionization near the end of the track and formation of a $\pi^+ \to \mu^+ \to e^+$ chain if the track was positive).

(d) Only one track could leave the chamber without providing evidence as to whether it was a fast π or an electron. An electron that had gone through a maximum radius vector (see Fig. 1), or a π that had scattered with a visible recoil before leaving the chamber, was considered as being identified. Positive tracks that disappeared in flight (without recoil) were counted as unidentified as they could have been made either by a positron that annihilated on an electron or by a π^+ charge exchange on a neutron.

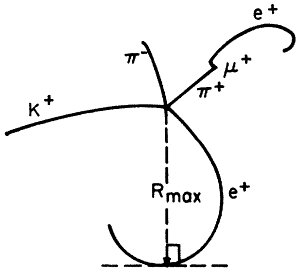

FIG. 1. K_{e4}^+ decay. The vector **R** from the K decay vertex goes through a maximum value.

---

[11] R. W. Birge, R. P. Ely, G. Gidal, G. E. Kalmus, A. Kernan, W. M. Powell, U. Camerini, W. F. Fry, J. Gaidos, R. H. March, and S. Natali, Phys. Rev. Letters **11**, 35 (1963).

[12] R. W. Birge, R. P. Ely, G. Gidal, G. E. Kalmus, A. Kernan, W. M. Powell, U. Camerini, W. F. Fry, J. Gaidos, D. Murphree, and C. T. Murphy, Lawrence Radiation Laboratory Report UCRL-11549, 1964 (unpublished) and reported at the *12th Annual International Conference for High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[13] W. M. Powell, L. Oswald, G. Griffin, and F. Swartz, Rev. Sci. Instr. **34**, 1426 (1963).

[14] G. Gidal, W. M. Powell, R. March, and S. Natali, Phys. Rev. Letters **13**, 95 (1964).

(e) The track that left the chamber was not allowed to have the opposite sign to a track already identified as being from a positron (or electron). This criterion eliminated Dalitz pairs from τ', $K_{\pi 2}$, and $K_{\mu 3}$ decays.

These scanning criteria produced about 800 K_{e4} candidates, all of which were examined by physicists.

C. Selection Criteria for $K_{e4}(e^+)$

Among the candidates selected on the scan table, the major sources of background events were: (1) collinear τ's and (2) τ decays in flight. Several lesser sources of background were also considered.

1. Collinear τ Decay

These events were characterized by the feature that the π^- and a π^+ went off in nearly opposite directions with approximately equal momenta. This configuration leads to a small residual momentum for the other π^+. If the range of this π^+ was sufficiently small and the μ decay from the π decay was hidden or went in such a direction as to make the e^+ from the $\pi^+ \to \mu^+ \to e^+$ chain appear to originate from the K^+ decay point, the event was then topologically the same as the $K_{e4}(e^+)$. Even though most of these events were easy to recognize, all of them were measured to make sure that they did fit the "collinear τ" hypothesis; this was done by calculation of the missing mass and missing momentum from the measured π^+ and π^- momenta and angles. For a τ decay, the missing mass should be 140 MeV (π^+ mass) and the missing momentum $P_N < 35$ MeV (corresponding to a range of approximately 0.1 cm). For steep π^+ tracks it would, in fact, be possible for one to not observe a range of up to 0.5 cm or approximately 50-MeV/c momentum. In practice, we found it necessary to eliminate all candidates for which $130 < M_N < 150$ MeV (where M_N is the missing mass) and $P_N < 50$ MeV/c (see Fig. 2).[15]

Of the approximately 800 K_{e4} candidates, about 700 were rejected as being collinear τ's by the above criteria.

Furthermore, all the K_{e4} candidates, which were not rejected after measurement as collinear τ's, were examined to see if they might have been τ decays in which some unusual occurrence led to an incorrect value of M_N or P_N. Examples of such occurrences are (a) $\pi^\pm \to \mu^\pm$ decays in flight, (b) π interactions in flight, (c) the prong from a π^- absorption star being almost collinear with the track and being included in the range of the pion, (d) π scattering near the origin so that the measured angles were wrong, and (e) collinear $\pi^+ \to \mu^+ \to e^+$ chains in which these tracks were so steep that the change in ionization at the μ decay point was not observable. Enlargements of all the origins were made in order to make this examination easier. In all cases it was required that a rejected event be consistent both kinematically and visually with this explanation. Altogether, only 20 additional events were rejected by application of these criteria.

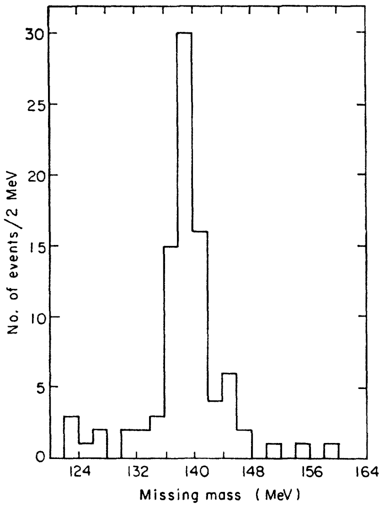

FIG. 2. Missing mass plot for a random sample of 91K_{e4} *candidates* in the range 120 to 160 MeV. The majority of these are collinear τ decays. The missing mass was calculated by measurement of the momenta of the π^+ and π^- from range and with the assumption that the decay was of the type $K^+ \to \pi^+ + \pi^- + \pi^+$. Some of the events in the tail of the distribution are $K_{e4}{}^+$; the others are mismeasures and appear in the peak when correctly measured.

The 800 candidates were divided by the physicists into two categories at the scan table, collinear τ's and likely candidates. Of the approximately 700 events designated as collinear τ's, none was accepted as K_{e4} decay after measurement.

That these criteria were stringent enough to eliminate virtually all collinear τ's can be seen from Fig. 3, which is a plot of P_N versus M_N for all remaining candidates and shows no events in the region $P_N < 60$ MeV/c (well beyond the collinear τ region). This depopulation is indicative of the clean separation of K_{e4} decays from collinear τ decays.

2. Tau Decays in Flight

All events for which it was obvious, from ionization, that the K^+ decayed in flight were rejected, as were all events for which all the decay products stopped and could be identified as pions. When only two tracks could be identified as being made by pions, the momenta of the K and of the missing mass were calculated from the two identified prongs on the assumption that they were made by pions from a τ decay in flight. The event was rejected if the momentum of the K was <200 MeV/c and the direction of the missing momentum was within two standard deviations of the actual direction of the unidentified track. By this criterion about 10 events were rejected as being τ decays in flight.

3. Other Sources of Background Events

(a) Candidates for which the dip of the negative track was $>60°$ were rejected because it was frequently difficult for us to differentiate a π from a steep low-energy electron associated with an asymmetric Dalitz pair. By this criterion we rejected four events that

---

[15] This is because of the large ratio of numbers of collinear τ to K_{e4} decays, even though the width of the M_N distribution was found to be 3 MeV.

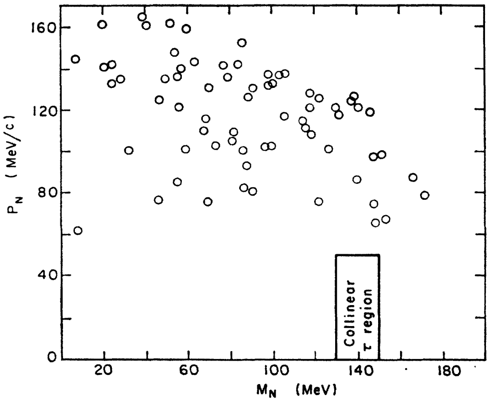

FIG. 3. Scatter diagram of P_N versus M_N for 69 K_{e4}^{+}.

otherwise were ambiguous, and also five events that were otherwise acceptable as K_{e4} decays. As described in Sec. IID3c, a correction was made for these rejected events.

(b) Candidates for which the projected length of the negative track was less than 0.2 cm and did not have an associated recoil proton were rejected as τ', $K_{\mu 3}$, or $K_{\pi 2}$ decays with an asymmetric Dalitz pair, because of the high probability that the track was really of a low-energy electron rather than a π^-.

Criteria 3(a) and 3(b) have some overlap.

4. Kinematical Constraint

The 72 remaining K_{e4} candidates were constrained kinematically and those with a χ^2 greater than 3 (1 degree of freedom) were rejected. As three events failed this test, the final sample of K_{e4}'s numbered 69.

D. Losses

In order for us to calculate the decay rate, we must correct both for the loss of events due to the scanning efficiency and for the loss of real events rejected by the scanning and selection criteria:

1. Scanning Efficiency for K_{e4} Decays

The scanning efficiency for the two scans combined was calculated to be $(85\pm5)\%$. (The events found on the second scan but missed in the first scan did not appear to differ in configuration from events found on the first scan.)

2. Scanning Efficiency for τ Decays

Approximately 5% of every scanner's film, spread evenly through the scan, was rescanned for τ decays. The over-all scanning efficiency for τ's was calculated from this to be $(94\pm1)\%$.

3. Real K_{e4} Events Rejected by the Selection Criteria

(*a*) *Collinear τ [Sec. IIC1].* In order for us to calculate how many real K_{e4}^{+}'s were rejected by the conditions $130<M_N<150$ MeV and $P_N<50$ MeV/c, we generated a sample of 1000 K_{e4} decays by a Monte Carlo program, using a weak matrix element with pure s-wave dipion state (Shabalin[3]) and also an equal mixture of s and p wave. The fraction of these events rejected by the collinear τ conditions was, respectively, 0.5 and 1%. The smallness of these values resulted mainly from the fact that small values of P_N are improbable in K_{e4} decay.

(*b*) *Tau decay in flight* (*Sec. IIC2*). The same Monte-Carlo-generated events were then subjected to the in-flight rejection criteria. We found that 0.5% of the events would have been rejected if the two pions had been in a pure s state and 2% would have been rejected for equal mixtures of s and p wave.

(*c*) *Dip of the π^-* (*Sec. IIC3a*). For π^- events with a $>60°$ dip, the geometric correction is 13% when the decay of the K^+ is at rest.

(*d*) *Short π^-* (*Sec. IIC3b*). Using the Monte-Carlo-generated events, we evaluated a correction for K_{e4} in which the π^- track was <4 mm long. This correction included all events with a projected π^- length of <2 mm and a dip $<60°$. Those with dip $>60°$ were corrected for under (*c*). The correction varied from 9% for pure s wave to 11% for equal mixtures of s and p wave. From this we subtracted those four events (6%) in which the π^- was less than 4 mm but a π^- absorption star was visible; this subtraction left a correction of 5%.

(*e*) *π^- leaves chamber [Sec. IIB, item (e)].* The probability that a π^- meson of a given energy and a dip angle $<60°$ left the chamber was calculated with a Monte-Carlo program. This calculation gave the detection efficiency for the π^-'s stopping as a function of E_π. A 5% correction is obtained if the experimentally found π^- energy spectrum is corrected for the detection efficiency.

(*f*) *Both e^+ and π^+ leave chamber [Sec. IIB, item (d)].* By means of a Monte-Carlo electron-detection efficiency program similar to that described in Ref. 16, the total probability that the e^+ from the K_{e4} decay would leave the chamber before curling up was calculated to be 20%. About 10% of the π^+'s leave the chamber before stopping. With the assumption of no correlation between the e^+ and π^+, the total correction is 2%. A correlation between the π^+ and e^+ cannot bring this to $>5\%$.

After application of the scanning and selection criteria, 69 events were left. Table I summarizes the losses due to these criteria. As 69 events represent 0.62 of the events in the film, therefore the total number of K_{e4} in the film is 110 ± 14. The uncertainty of some of the corrections brings this error up to ±20.

Several biases have not been mentioned but may exist in the data, for example, that against identifying

---

[16] R. P. Ely, G. Gidal, G. E. Kalmus, L. O. Oswald, W. M. Powell, W. J. Singleton, F. W. Bullock, C. Henderson, D. J. Miller, and F. R. Stannard, Phys. Rev. **131**, 868 (1963).

TABLE I. Summary of losses due to scanning and selection criteria.

Section number	Fractional acceptance
1	0.85
3*a*	0.99
3*b*	0.98
3*c*	0.87
3*d*	0.95
3*e*	0.95
3*f*	0.95
Product	0.62

steep electrons. None of these biases is included in any of the selection criteria and the data available are insufficient to show any significant effect. The effect of the biases is to increase the number of K_{e4}'s in the film and therefore to increase the branching ratio. However, since none of them could be shown to be statistically significant, we feel that the stated error should cover them.

E. Selection Criteria for $K_{e4}(e^-)$ Candidates

The scanning criteria for these events were the same as for the $K_{e4}(e^+)$ events. Candidates were tested in the same way as $K_{e4}(e^+)$.

The sources of background, however, for these events is somewhat different than those for $K_{e4}(e^+)$. Since $\pi^- \rightarrow \mu^- \rightarrow e^-$ decays are rather improbable, collinear τ background was very much reduced. Neither of the pions was allowed to leave the chamber [by Sec. IIB, item (*e*)]; this condition decreased the detection efficiency by about 10%. Events in which one of the π^+'s was very short or steep and decayed without it or the μ^+ being visible were rejected as being τ's with a Dalitz pair. This rejection further reduced the detection efficiency by an estimated amount of up to 10%. On the basis of these considerations, we estimate that the detection efficiency for $K_{e4}(e^-)$ (insofar as the detection efficiency of events that have not been found can be determined) is about 80% of that for $K_{e4}(e^+)$.

No $K_{e4}(e^-)$ events passed all the selection criteria.

III. RESULTS

A. K_{e4} Branching Ratios

On the first scan the number of τ decays found at rest was 155 000, with a scanning efficiency of $(94\pm1)\%$. The τ/K^+ branching ratio is taken to be 0.0546 ± 0.0009.[17]

1. $K_{e4}(e^+)$

$$(\Gamma[K_{e4}(e^+)])/(\Gamma(K^+))$$
$$=(110\times0.0546\times0.94)/155\,000=(3.6\pm0.8)\times10^{-5}.$$

2. $K_{e4}(e^-)$

Only an upper limit for this can be established since no events were seen among 3×10^6 stopping K^+ particles. After all the cuts were made to the data as discussed in Secs. IID and IIE, the total probability of our finding a $K_{e4}(e^-)$ was about 50%. Therefore,

$$(\Gamma[K_{e4}(e^-)])/(\Gamma(K^+))<2\times10^{-6}$$

at the 95% confidence level.

B. $\Delta Q=\Delta S$ Rule

The decay $K_{e4}(e^+)$ obeys the $\Delta Q=\Delta S$ rule but $K_{e4}(e^-)$ violates it.

The isotopic-spin state of the dipion in Eq. (2) is pure $T=2$, whereas in (1) the dipion can be $T=0$ or 2 for even-angular-momentum states and $T=1$ for odd states. If we make the assumption that the $\Delta T=\frac{1}{2}$ rule is not strongly violated for (1), then the only isotopic-spin states are $T=0$ and $T=1$ [$T=2$ for (2) does, of course, mean that $\Delta T=\frac{3}{2}$]. Since this is a relatively low-energy interaction, we will assume that only *s* and *p* waves are important. Reaction (2) is pure axial-vector whereas (1) is a mixture of axial-vector and vector. There are arguments that show that the vector part is small, of the order of a few percent.[4] In order for us to obtain a meaningful value for the violation parameter $X=[A(\Delta Q=-\Delta S)]/[A(\Delta Q=+\Delta S)]$ (where A is the amplitude of the current) for axial-vector currents, not only must the vector part of (1) be small, but also the enhancement factor due to the final-state interaction between the two pions must either be small or calculable. Kacser, Singer, and Truong calculate values of the enhancement factor as a function of the *s*-wave $\pi-\pi$ scattering length.[7]

For no *s*-wave $\pi-\pi$ interaction in the final state, and a negligible amount of vector in (1), we obtain from the branching ratios

$$X<0.25 \text{ (95\% confidence level)}.$$

Until a reliable calculation of the enhancement factor is made, it is difficult for us to interpret the above result. Estimates for this enhancement factor are $\lesssim 4$ (Refs. 5, 7), which give $X<0.5$.

C. $K_{\mu4}(\mu^-)$ Branching Ratio

The decay mode $K^+\rightarrow\pi^++\pi^++\mu^-+\nu_\mu$ was also searched for in this experiment. This decay mode can exist, in principle, even if the corresponding $K_{e4}(e^-)$ decay mode does not because decay mode (1) involves an additional form factor [e in Eq. (3b)].

$K_{\mu4}(\mu^-)$ was searched for by our scanning for τ-like decays of stopped K^+ mesons where the negative track decayed into an electron track. Approximately 72% of all stopped μ^- mesons will decay in C_3F_8. A total of 461 candidates for $K_{\mu4}(\mu^-)$ were found in the scan. All of these events were consistent with the interpretation of

---

[17] A. Callahan, R. March, and R. Stark, Phys. Rev. 136, B1463 (1964).

τ decay with the π^- meson's decaying in flight into a μ^-, and the stopped μ^- decaying in the chamber. In this experiment we expect 1300 τ decays with $\pi^- \rightarrow \mu^-$ decays in flight. Some $K_{\mu 4}(\mu^-)$ decays will fit the hypothesis of τ decay and therefore cannot be separated from the τ background. The fraction of $K_{\mu 4}(\mu^-)$ that fit τ decay was determined by generation of fake $K_{\mu 4}(\mu^-)$ decays according to a Lorentz-invariant phase space. Folding in the detection efficiency and the probability of $K_{\mu 4}(\mu^-)$ faking a τ decay, an effective sample of 9.3×10^5 K^+ decays was examined in the search for $K_{\mu 4}(\mu^-)$ decay. Thus, the upper limit on this decay mode is

$$[\Gamma(\pi^+\pi^+\mu^-\nu)]/[\Gamma(K^+ \rightarrow \text{all modes})] < 3 \times 10^{-6}$$
$$(95\% \text{ confidence level}).$$

D. Analysis of Correlations

Since pions are the only strongly interacting particles in the final state of the decay $K^+ \rightarrow \pi^+\pi^-e^+\nu$, the form factors in the matrix element are expected to be dominated by the $\pi-\pi$ interaction.

Following the development of Cabibbo and Maksymowicz,[9] the matrix element for $K^+ \rightarrow \pi^+\pi^-e^+\nu$ in first-order perturbation theory is (aside from the usual kinematic factor)

$$M(K_{e4}) = |G/\sqrt{2}|[\bar{\nu}\gamma^\lambda(1+\gamma_5)e] \times \langle \pi^+\pi^- | J_\lambda{}^V + J_\lambda{}^A | K^+ \rangle, \quad (3)$$

where J^V and J^A are the vector and axial-vector currents of the strongly interacting particles. From invariance considerations

$$\langle \pi^+\pi^- | J_\lambda{}^V | K^+ \rangle = (ih/M^3{}_K)\epsilon_{\lambda\mu\nu\sigma} P_K{}^\mu (p_+ + p_-)^\nu (p_+ - p_-)^\sigma \quad \text{(3a)}$$

and

$$\langle \pi^+\pi^- | J_\lambda{}^A | K^+ \rangle = (f/M_K)(p_+ + p_-)_\lambda + (g/M_K)(p_+ - p_-)_\lambda + (e/M_K)(p_K)_\lambda, \quad \text{(3b)}$$

where p_K, p_+, p_-, p_e, and p_ν are the four-momenta of the K, π^+, π^-, e, and neutrino, respectively, and the form factors f, g, h, and e are in general functions of $R^2 = (p_+ + p_-)^2$, $(p_K \cdot p_+)$, and $(p_K \cdot p_-)$.

Because our data are few, only the largest terms in the above expression are included in the expression for the rate. Therefore the term $(e/M_K)p_K$ is dropped because it is proportional to (M_e/M_K) and the form factors are assumed to be functions of R^2 (or $M_{\pi\pi}{}^2$) only, since the singularities in the variables $(p_K \cdot p_+)$ and $(p_K \cdot p_-)$ are far from the physical region.[7]

In addition, we have assumed that only the $T=\frac{1}{2}$ currents are important and that only s and p waves will contribute to the final-state interaction. With these assumptions, the form factors by the Watson-Fermi theorem[18] can be written

$$f = \tilde{f} e^{i\delta_0(R^2)}, \quad g = \tilde{g} e^{i\delta_1(R^2)}, \quad h = \tilde{h} e^{i\delta_1(R^2)}, \quad (4)$$

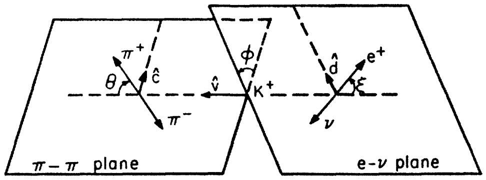

FIG. 4. Definition of angles used.

where $\tilde{f}$ and $\tilde{g}$ and $\tilde{h}$ are real, and δ_0 and δ_1 are the $I=0$, $l=0$ and $I=1$, $l=1$ $\pi-\pi$ scattering phase shifts. Cabibbo and Maksymowicz have carried out the integration and found the rate to be

$$\begin{aligned} d\Gamma(x^2) = [G^2\pi^2 M^5{}_K/8(2\pi)^8] dx^2 d\cos\theta\, d\phi \{ & \tilde{f}^2 A(x^2) \\ & + \tilde{g}^2[B(x^2) + C(x^2)\cos^2\theta + D(x^2)\sin^2\theta\sin^2\phi] \\ & + \tilde{h}^2 E(x^2)\sin^2\theta(1+2\cos^2\phi) \\ & + \tilde{f}\tilde{g}\cos(\delta_0-\delta_1)S(x^2)\cos\theta \\ & + \tilde{f}\tilde{g}\sin(\delta_0-\delta_1)T(x^2)\sin\theta\sin\phi \\ & + \tilde{f}\tilde{h}\cos(\delta_0-\delta_1)U(x^2)\sin\theta\cos\phi \\ & + \tilde{g}\tilde{h}V(x^2)\cos\theta\sin\theta\cos\phi \}, \end{aligned} \quad (5)$$

where $x^2 = R^2/M^2{}_K$, and θ and ϕ are the angles shown in Fig. 4. The functions A, B, C, D, E, S, T, U, and V are well-defined functions of x^2 given in Ref. 9.[19]

1. $\pi-\pi$ *Invariant-Mass Distribution*

The $\pi-\pi$ invariant-mass distribution, obtained by integrating Eq. (5) over θ and ϕ, is

$$d\Gamma(x^2) = \frac{4\pi G^2\pi^2 M^5{}_K}{8(2\pi)^8} f^2\{A(x^2) + \eta^2[\tfrac{5}{3}B(x^2) + \tfrac{1}{3}C(x^2)]\}, \quad (6)$$

where $\eta = \tilde{g}/\tilde{f}$ and terms proportional to h^2 have been dropped because the vector term has been estimated to be smaller than the axial vector by two orders of magnitude.[4] The terms involving $U(x^2)$ and $V(x^2)$ are included in Eq. (6) because their angular dependence provides a test of this estimate. The terms proportional to $S(x^2)$ and $T(x^2)$ provide, through their unique angular dependence, measures of the real and the imaginary parts of the s-p wave interference.

Figures 5, 6, and 7 show the distributions of the invariant mass of the two pions, (R), of $\cos\theta$ and ϕ, respectively. Figure 5 contains 69 events, but in Figs. 6 and 7 eight events were eliminated because the track lengths were too short to provide good measurements of the angle. In order to investigate possible biases in these distributions introduced by the cuts discussed in Sec. IIC, we have applied the same cuts to 1000 events generated by a Monte Carlo program. The results showed no detectable effects in either the $M_{\pi\pi}$ or ϕ distributions, and a small increase in the asymmetry in $\cos\theta$ due to the elimination of short π^-. The

---

[18] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

[19] There is an additional term in $\cos\theta$, as pointed out by Dalitz (Ref. 10), which we have neglected. This term is down by about a factor of 5 from kinematical considerations, and Kacser *et al.* (Ref. 7) estimate it to be less than 5%.

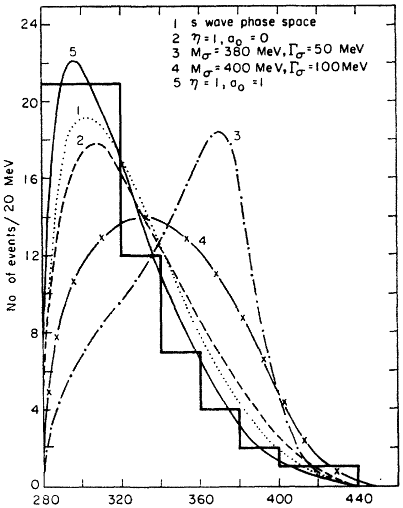

FIG. 5. $\pi-\pi$ invariant-mass plot. The histogram shows the experimental distribution of 69 events. Curve 1 shows s-wave phase space; curve 2 shows $\eta=1$, $a_0=0$; curve 3 shows $M_\sigma=380$, $\Gamma_\sigma=50$; curve 4 shows $M_\sigma=400$, $\Gamma_\sigma=100$; curve 5 shows $\eta=1$, $a_0=1$. Curves 1, 2, and 5 have $>10\%$ probability of fitting the data; curves 3 and 4 have $<0.1\%$ chance of fitting.

latter has been neglected because the change is less than the error in the asymmetry.

In the $\pi-\pi$ invariant-mass plot (Fig. 5), the fitted curves numbered 1 and 2 are the s-wave phase space ($f=1$, $g=0$) and a mixture of s and p wave corresponding to $\tilde{g}/\tilde{f}=1$. Our data are clearly insufficient to distinguish between them. There is no evidence for a $\pi-\pi$ resonance between 380 and 400 MeV such as that reported by Samios *et al.*[20] and proposed by Brown and Singer.[21] Curve 3 shows the mass spectrum from the decay $K\rightarrow\sigma+e^-+\nu$ and $\sigma\rightarrow\pi^++\pi^-$, with $M_\sigma=380$ MeV and $\Gamma_\sigma=50$ MeV as calculated by Brown and Faier. Later papers have suggested that the parameters which best fit the η branching ratios may be $M_\sigma=400$ and $\Gamma_\sigma=100$ MeV.[22] Curve 4 shows the spectrum predicted for these parameters, and although this fits the data better than for curve 3, the χ^2 (which equals 29 for 4 degrees of freedom) gives a probability of less than 0.1%.

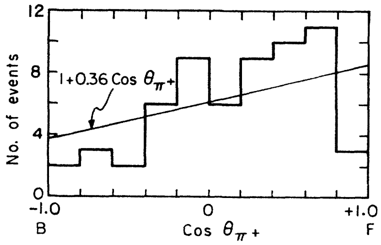

FIG. 6. Plot of $\cos\theta_{\pi^+}$ for 61 events, where θ_{π^+} is the angle shown in Fig. 4.

We have also fitted the $\pi-\pi$ mass spectrum, assuming the s-wave $\pi-\pi$ interaction to be described by the Chew-Mandelstam effective-range formula. The best fit is for an s-wave scattering length of $a_0=1.0$ (Fig. 5, curve 5) although it is clear that, with our statistics, the $\pi-\pi$ mass distribution is not a sensitive test of the scattering length and is, in fact, quite compatible with $a_0=0$.

2. *Angular Correlation*

The presence of both s- and p-wave $\pi-\pi$ interactions leads to asymmetries in the angles θ and ϕ, the relative magnitude depending on the difference of the phase shifts. These are the $S(x^2)$ and $T(x^2)$ terms in Eq. (5). Figures 6 and 7 show the distributions of $\cos\theta$ and ϕ for 61 events for which the curves are the best fit to the forms $1+a\cos\theta$ and $1+b\sin\phi$ ($a=0.36\pm0.12$ and $b=0.19\pm0.14$). Given an infinite supply of data we could determine a and b for each value of x^2. Then from Eq. (5)

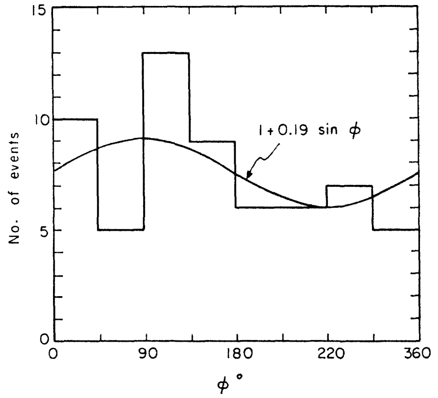

FIG. 7. Plot of ϕ for 61 events where ϕ is defined in Fig. 4.

$$a(x^2)=\frac{3\int\cos\theta\,d\Gamma(x^2)}{\int d\Gamma(x^2)}=\frac{\eta S(x^2)\cos(\delta_0-\delta_1)}{A+\eta^2[(5/3)B+\frac{1}{3}C]},\qquad(7)$$

$$b(x^2)=\frac{\int\sin\phi\,d\Gamma(x^2)}{2\pi\int d\Gamma(x^2)}=\frac{\pi}{4}\eta\frac{T(x^2)\sin(\delta_0-\delta_1)}{A+\eta^2[(5/3)B+\frac{1}{3}C]},\qquad(8)$$

and therefore

$$\delta_0-\delta_1=\tan^{-1}\left(\frac{4}{\pi}\frac{b}{a}\frac{S}{T}\right)$$

[η can be obtained from either (7) or (8)]. In fact our data are so few that we have taken all values of x^2 in

---

[20] N. P. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shepherd, Phys. Rev. Letters 9, 139 (1962).

[21] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962); L. M. Brown and P. Singer, Phys. Rev. 133, B812 (1964).

[22] H. Faier and L. M. Brown, Bull. Am. Phys. Soc. 10, 467 (1965).

Figs. 6 and 7 and substituted values of A, B, C, S, and T averaged over all x^2, i.e., $\bar{A}=\int A(x^2)dx^2$. The result is

$$``\delta_0-\delta_1"=35°\pm30°\,,$$

$$``\eta"=0.8\pm0.3\,,$$

where quotation marks have been used because "$\delta_0-\delta_1$" is a weighted average that is not easily interpreted unless the energy dependence is small. In view of the large error it does not seem necessary to investigate further the detailed nature of the average. We have divided the data into two equal samples by x^2; there is no significant change in "$\delta_0-\delta_1$" calculated for each half separately.

In addition we have fitted the data using the Chew-Mandelstam effective-range formula, which gives the energy dependence of the $T=0$, $l=0$ phase shift.

$$\cot\delta_0=\frac{1}{\beta a_0}+\frac{2}{\pi}\ln\left[\frac{(R^2)^{1/2}}{2M_\pi}(1+\beta)\right],$$

where a_0 is the $\pi-\pi$ scattering length for the $I=0$, $l=0$ state in units of the pion Compton wavelength and $\beta=(1-4M^2{}_\pi/R^2)^{1/2}$. Assuming δ_1 to be constant in this energy range, Cabibbo and Maksymowicz have derived an expression for the rate

$$d\Gamma(x^2)=dx^2d\cos\theta\, d\phi\frac{10^{-5}G^2\pi^2M^5{}_K\tilde{f}_0{}^2}{8(2\pi)^8}$$
$$\times[A_1(x^2)+\eta_0A_2(x^2)\cos\theta+\eta_0A_3(x^2)\sin\theta\sin\phi$$
$$+\eta_0{}^2A_4(x^2)\cos^2\theta+\eta_0{}^2A_5(x^2)(1+2\sin^2\theta\sin^2\phi)],\quad(9)$$

where $\tilde{f}_0=\tilde{f}(R^2=4M^2{}_\pi)$, $\eta_0=\tilde{g}_0/\tilde{f}_0$, and $A_i(x^2)$ are functions of x^2 and a_0 only. Since this form has only two unknowns that are truly constant, we have made a maximum likelihood plot using Eq. (7) as the likelihood function, and we find $a_0=(1\pm1)\lambda_\pi$ and $\eta_0=0.8\pm0.30$. Curve 5 in Fig. 5 shows the mass spectrum calculated with these parameters and it is a very good fit. Within errors, this value of a_0 is also consistent with that determined by Booth *et al.*[23]

The presence of a correlation in ϕ of the form $(1+c\cos\phi)$ would indicate interference between the vector and axial-vector currents [see the last two terms in Eq. (5)]. We find $c=0.06\pm0.14$, which is certainly compatible with a very small vector contribution, as predicted.

Figure 8 shows the dilepton-invariant-mass plot and Fig. 9 shows the angle between the two pions in the laboratory system.

IV. DISCUSSION OF RESULTS

The rate for $K^+\rightarrow\pi^++\pi^-+e^++\nu$ has been measured and found to be $(2.9\pm0.6)\times10^3$ sec^{-1}. All of the theoretical calculations that do not include final-state interactions contain an adjustable parameter and can be brought into agreement with our rate with reasonable values of these parameters. Of those calculations which do include the final-state interactions, that of Brown and Faier[6] (which assumes the decay to proceed through the production of a σ meson) agrees very well with our rate, but their predicted mass spectrum for the $\pi-\pi$ system fits very poorly.

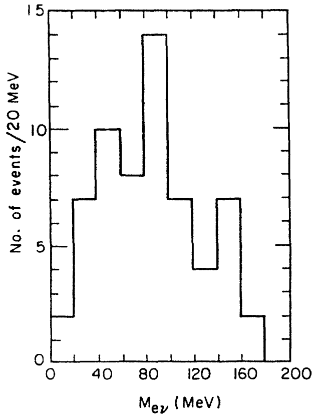

FIG. 8. Plot of the $e^+\bar{\nu}$-invariant mass for 61 events.

Our data are insufficient to exclude any reasonable $\pi-\pi$ interaction in the low-energy region. Our direct measurement of the difference between the s-wave and p-wave phase shifts yields $\delta_0-\delta_1=35°\pm30°$ with $\eta=0.8\pm0.30$; which are very difficult to interpret because they are averaged over such a large range of $\pi-\pi$ energies. Certainly another experiment with ten times the data could yield valuable information by measurement of $\delta_0-\delta_1$ as a function of the $\pi-\pi$ energy.

The effect of violation of time-reversal invariance on the relationship between the angular correlations and the $\pi-\pi$ interactions depends on the true nature of this violation.

In the total sample of stopping K^+'s, no examples of the decay $K^+\rightarrow\pi^++\pi^++e^-+\bar{\nu}$ were found, and when relative detection efficiencies are considered, the upper limit for this branching ratio is $<2\times10^{-6}$ or 1/20 the e^+ decay. The $K_{e4}(e^-)$ decay proceeds almost entirely via the axial-vector current and is strictly forbidden by the $\Delta S/\Delta Q=+1$ rule. However, it is difficult to

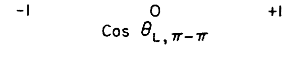

FIG. 9. Plot of $\cos\theta_{L\pi-\pi}$ for 61 events, where $\theta_{L\pi-\pi}$ is the angle between the π^+ and the π^- in the laboratory system.

---

[23] N. E. Booth and A. Abashian, Phys. Rev. **132**, 2314 (1963); and references therein.

compare this result with the results of the three-body K_{e3} decays,[24] which proceed through the vector interaction, because of the final-state interaction in K_{e4} decays. If we assume Eq. (1) to be dominated by the axial-vector current, as predicted by most theorists, and if there were no final-state interactions, then the violation parameter

$$X=[A(\Delta S/\Delta Q=-1)]/[A(\Delta S/\Delta Q=+1)]$$

corresponding to the parameter $a_0=0$ would be $X<0.25$. On the other hand, if there is no $T=2$ final-state interaction but the $T=0$ s-wave enhancement factor were as large as 4, then $X<0.5$.

ACKNOWLEDGMENTS

We would like to acknowledge the help of the Bevatron staff, Larry O. Oswald, and the 30-in. bubble-chamber crew at LRL, as well as the scanning-and-measuring staffs at both Laboratories.

We wish to thank the graduate students from Wisconsin and Berkeley who helped with this experiment, especially Miss Martha Dickenson and Andrew Callahan. We are also indebted to Dr. Nicola Cabibbo, Dr. Elliot Leader, Dr. Alexander T. Maksymowicz, Dr. Robert Sachs, and Dr. Bunji Sakita for many helpful discussions.

---

[24] R. P. Ely, W. M. Powell, H. White, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, O. Fabbri, F. Farini, C. Filippi, H. Huzita, G. Miari, U. Camerini, W. F. Fry, and S. Natali, Phys. Rev. Letters **8**, 132 (1962); G. Alexander, S. P. Almeida, and F. S. Crawford, Jr., *ibid.* **9**, 69 (1962); B. Aubert, L. Behr, J. P. Lowys, P. Mittner, and C. Pascaud, Phys. Letters **10**, 215 (1964); M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, C. Filippi, H. Huzita, F. Mattioli, and G. Miari, *Proceedings of the Sienna International Conference on Elementary Particles* (Societa Italionadi Fisica, Bologna, Italy, 1963); L. Kirsch, R. J. Plano, J. Steinberger, and P. Franzini, Phys. Rev. Letters **13**, 35 (1964).

PHYSICAL REVIEW VOLUME 139, NUMBER 6B 20 SEPTEMBER 1965

Spin Tests for Bosons

M. Ademollo, R. Gatto, and G. Preparata
Istituto di Fisica dell'Università, Firenze, Italy
and
Sezione di Firenze dell'I.N.F.N., Firenze, Italy

(Received 7 May 1965)

Tests of spin for an unstable boson which decays into three spinless particles, or into a spin-1 and a spin-0 particle, or into two spin-$\frac{1}{2}$ particles, are presented. The proposed spin tests should be useful for the spin-parity determination of the new boson resonances. Spin tests linear in the experimental test functions are discussed in particular, in view of their general applicability independently of the production kinematics. Explicit expressions of the test functions are reported for the lower spin assignments.

I. INTRODUCTION

THE recent discoveries of many resonant states which decay strongly or electromagnetically into baryons and bosons have stimulated the search for convenient spin-parity tests, which may allow for a determination of the spin and parity of the unstable particle, possibly avoiding dynamical hypotheses on the mechanisms of production and decay. Particularly useful have been the tests based on simultaneous analysis of angular and polarization distributions.[1]

In this paper we consider some possible methods for determining the spin of an unstable boson. We discuss its modes of decay, into three spinless particles, into a spin-1 and spin-0 particle, and into two spin-$\frac{1}{2}$ particles. In each case we look for relations among the coefficients of the final distributions which do not depend on the elements of the density matrix of the decaying boson. We obtain a general method for spin determination which appears to be more powerful than methods based on the reconstruction of the density matrix. The relations to be tested are in fact linear in the experimental averages and independent of the production process, making it possible to average on all the events, independent of the production kinematics. Such a possibility is especially useful when the number of events is relatively small.

In Sec. II we discuss the decay of a boson into three spinless particles. The spin tests we derive for this case could be of use for the spin-parity assignments to recently found three-body resonances such as 3π, $\eta\pi\pi$, $K\pi\pi$, etc. The final distributions are first expressed in terms of a suitable set of parameters which are subject to a number of constraints. Symmetry principles or possible identity between two of the final particles produce further relations. The different spin tests are discussed in Sec. 2.3 and are written down explicitly for spin one. Their explicit forms for spin two and three are reported in Appendix A. In Sec. III we discuss the mode of decay into a spin-1 and a spin-0 boson in view of applications to spin-parity tests of the $\pi\omega$ and $\pi\rho$

---

[1] R. Gatto and H. P. Stapp, Phys. Rev. **121**, 1553 (1961); N. Byers and S. Fenster, Phys. Rev. Letters **11**, 52 (1963); M. Ademollo and R. Gatto, Phys. Rev. **133**, B531 (1964); N. Byers and C. N. Yang, *ibid.* **135**, B796 (1964); S. M. Berman and R. J. Oakes, *ibid.* **135**, B1034 (1964).