Possibility of Observing a Nonlinear Electromagnetic Effect*

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A fourth-order electromagnetic effect which may be observable with present experimental techniques is discussed. The effect is the interaction of a photon with an electrostatic field to produce two photons. A low-energy approximation to the cross section is given and other coherent processes are discussed.

IT has been recognized almost from the beginning of theoretical quantum electrodynamics that processes involving virtual electron-positron pairs can give rise to nonlinear electromagnetic effects. It is perhaps unfortunate that there is no decisive experimental evidence for such effects since they could provide an interesting confirmation of quantum electrodynamics. These effects are difficult to observe either because they are quite small, being fourth-order processes, or because they occur coherently with other processes so that the experimental results are obscure. The purpose of this note is to describe an experiment to observe such an effect which may be on the threshold of feasibility.

Several nonlinear effects which may be observable have been discussed. The fundamental one is the scattering of light by light[1,2] which is almost impossible to observe because of the low density of high-energy photon beams.[3] Other nonlinear effects which are probably not experimentally observable are two-quantum pair creation,[4] birefringence of the vacuum,[5] and dichroism of the vacuum.[6] The effect which is most promising for observation is Delbrück scattering, the elastic scattering of a photon by the electrostatic field of a nucleus.[7] The cross section for Delbrück scattering by a heavy nucleus may be quite substantial, but it is difficult to draw definite conclusions from the experimental results because this process is coherent with two others: Thompson scattering and Rayleigh scattering from the K electrons of the nucleus. At present there seems to be no conclusive experimental evidence of Delbrück scattering.[8] A further difficulty with Delbrück scattering is that the cross section is very difficult to compute, although considerable progress on this has been made recently.[8,9]

An effect which has not been extensively investigated is the process in which a photon interacts with an electrostatic field and two photons are radiated. If the electrostatic field is that of a nucleus, the process will be called the nuclear γ-2γ process. This effect has been discussed by Sannikov[10] at high energy and with one photon emitted in the forward direction, but no extensive calculations have been made. The cross section is proportional to $Z^2\alpha^3$ and so for large Z is of the same order of magnitude as double Compton scattering (Compton scattering with two photons in the final state).

It is conceivable that this effect could be observed with modern techniques by exposing a target of heavy nuclei to a γ-ray source and observing pairs of radiated γ rays in coincidence. The coincidence requirement reduces the difficulties with background perhaps rendering the experiment more favorable than Delbrück scattering for which the cross section is much larger. The nucleus absorbs negligible kinetic energy so that the emitted energies must sum to the incident energy; this fact permits the process to be distinguished from the electronic double Compton effect in which a substantial amount of energy is lost to the scattered electron in any feasible experimental configuration.

The theory of the nonlinear processes has been investigated by Euler[1] and has been discussed more extensively by Karplus and Neuman[11,2] using the more modern techniques of quantum field theory. One Feynman diagram for the process is shown in Fig. 1. There are five others similar to it differing only in the route of the electron line. Karplus and Neuman have shown that these diagrams give a finite result and have evaluated it in the case that the four photons are real. In the case that one of the photons is virtual, evaluation appears to be more difficult. Karplus and Neuman have given, however, an approximation to the matrix element which is valid at low photon energies. From this result a low-energy approximation to the cross section for the nuclear γ-2γ process may be obtained. If the polarizations of the outgoing photons are summed over and the polarization of the incident photon aver-

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aged, the result is

$$d\sigma=(2025\pi)^{-1}d\Omega_1 d\Omega_2 d\omega_1 Z^2\alpha^3 r_0^2\omega_1\omega_2\omega^{-1}(mc^2)^{-6}|\mathbf{q}|^{-4}$$
$$\times\{157|\mathbf{q}|^2[\omega^2(k_1\cdot k_2)^2+\omega_1^2(k\cdot k_2)^2+\omega_2^2(k\cdot k_1)^2 - (k\cdot k_1)(k\cdot k_2)(k_1\cdot k_2)]$$
$$-36|\mathbf{q}|^2[\omega_1\omega_2(k\cdot k_1)(k\cdot k_2)+\omega\omega_1(k\cdot k_2)(k_1\cdot k_2) +\omega\omega_2(k\cdot k_1)(k_1\cdot k_2)]$$
$$-278[(k_1\cdot k_2)^2(k\cdot k_2)^2+(k\cdot k_1)^2(k\cdot k_2)^2 +(k_1\cdot k_2)^2(k\cdot k_1)^2]\}.$$

In this expression ω, ω_1, and ω_2 are the energies of the incoming and two outgoing photons; k, k_1, and k_2 are their 4-momenta and $(a\cdot b)$ denotes the scalar product $\mathbf{a}\cdot\mathbf{b}-a_0b_0$ of two 4-momenta. The vector $|\mathbf{q}|$ is the momentum transferred to the nucleus; $\mathbf{q}=\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}$ and $|\mathbf{q}|^2=2(k_1\cdot k_2)-2(k\cdot k_1)-2(k\cdot k_2)$.

Experimentally, one will usually observe photons for all values of ω_1 so that it may be of interest to integrate the result over ω_1. Except for special configurations this is probably best done numerically. For the special case in which one of the photons is emitted in the forward direction (which is very difficult to observe) the integrated cross section is given by

$$d\sigma=139(81\,000\,\pi)^{-1}Z^2\alpha^3 r_0^2(\omega/mc^2)^6\sin^2\theta d\Omega_1 d\Omega_2,$$

where θ is the scattering angle of the second photon. If the photon directions are mutually orthogonal (a reasonable experimental configuration) the integrated cross section is given by

$$d\sigma=121(2025\pi)^{-1}(2\pi 3^{-3/2}-6/5)Z^2\alpha^3 r_0^2(\omega/mc^2)^6 d\Omega_1 d\Omega_2.$$

For $Z=82$ and $\omega=0.66$ MeV (the γ-ray energy from a Cs^{137} source) the last result is approximately 0.17 μb/(sr)^2.

In the problem of scattering of light by light Karplus and Neuman found that the low-energy approximation used here was valid almost up to the pair-production threshold and underestimated the cross section immediately below the threshold. It is plausible that a similar situation prevails for the γ-2γ process so that the cross section for Co^{60} γ rays incident on Pb can be estimated to be approximately 3 μb/(sr)^2.

It is important to discuss other processes which can proceed coherently with the nuclear γ-2γ process. The principal one of these is the process in which the incident photon interacts with the K electrons of the atom, causing them to radiate two photons, a process analogous to Rayleigh scattering. This process can also be viewed as double Compton scattering from a K electron with the scattered electron being captured by the nucleus. It is very difficult to compute the amplitude

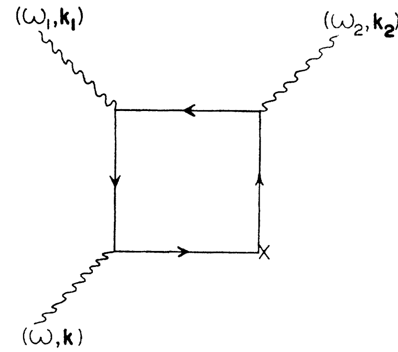

FIG. 1. A particular Feynman diagram for the $\gamma-2\gamma$ process. The cross at one vertex depicts the interaction with the electrostatic field.

for this process reasonably well. It can be estimated, however, to differ from the amplitude for the nuclear process by a factor $R=2(Z\alpha)^{-1}f_Z(|\mathbf{q}|)$ where $f_Z(|\mathbf{q}|)$ is a form factor which represents the probability that the K electron will remain bound to the nucleus.[12] The form factor is given by

$$f_Z(|\mathbf{q}|)=\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}|\psi(\mathbf{r})|^2 =(2y\sigma)^{-1}(1+y^2)^{-\sigma}\sin(2\sigma\tan^{-1}y),$$

where $\sigma=[1-(\alpha Z)^2]^{1/2}$ and $y=|\mathbf{q}|/(2mc\alpha Z)$. The last result is obtained if $\psi(\mathbf{r})$ is the Dirac wave function for the K electron. The form factor decreases fairly rapidly with increasing values of $|\mathbf{q}|$. In the configuration in which the three photon directions are orthogonal the minimum value of $|\mathbf{q}|$ is $(3/2)^{1/2}\omega/c$. For $\omega=0.66$ MeV, $Z=82$, $R=0.74$, so that the amplitude for the two effects are comparable in magnitude. The situation can be improved either by decreasing Z or increasing ω. If, for example, $\omega=1.26$ MeV and $Z=46$, $R=0.025$ so that in this case the nuclear process should dominate the amplitude.

Another coherent effect is nuclear double Compton scattering, or double Thompson scattering, but the amplitude for this is reduced relative to electronic double Compton scattering by a factor m/M, so that this effect is negligible.

We can conclude that the nuclear γ-2γ process may be observable in a carefully planned experiment and can provide definite evidence of a nonlinear electromagnetic effect.

The author's interest in this subject was stimulated by an interesting discussion with Professor William J. Knox.

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