Possible C, T Noninvariance in the Electromagnetic Interaction*

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The experimental foundation of the various discrete symmetry properties for the strong and the electromagnetic interactions is reviewed. It is found that there is strong evidence that both interactions are invariant under P and CPT, and good evidence that the strong interaction is invariant under C and T. However, at present, there is a complete lack of evidence that the electromagnetic interaction of the strongly interacting particles is invariant under C and T. Possible experiments that can test such C, T invariance or noninvariance are discussed. It is pointed out that if the electromagnetic interaction of the strongly interacting particles has strong violations of C, T invariance, then, through second-order processes, C-violating and CP-violating effects can be observed in the usual strong and weak processes, respectively. In particular, the decay $K_2^0 \to \pi^+ + \pi^-$ may occur with an amplitude $\sim(\alpha/\pi)$ times that of $K_1^0 \to \pi^+ + \pi^-$.

I. INTRODUCTION

SINCE the discovery[1] that

$$K_2^0 \to \pi^+ + \pi^-, \qquad (1)$$

in apparent violation of CP invariance, it has become clear that the experimental foundation of all discrete symmetries should be re-examined for all the interactions. Such a study will be made in this paper for both the strong and electromagnetic interactions. We shall see that there is strong evidence supporting the premise that both interactions conserve P and CPT to a great accuracy; furthermore, the strong interaction is invariant under T to a fair degree of accuracy ($\sim$2%), and through CPT and P invariance the same accuracy applies also to C conservation in the strong interaction. The situation is, however, very different for the electromagnetic interactions. As will become clear, at present there exists *no* evidence that the electromagnetic interactions of the *strongly interacting particles* are invariant under C and T. Indeed, all existing experimental results are compatible with the possibility of a very large violation of C and T in the electromagnetic interaction of these strongly interacting particles. Throughout this paper, the three operators C, P, and T denote, respectively, the usual particle-antiparticle conjugation, space inversion, and time reversal.

For definiteness, let us write the nonleptonic part of the electromagnetic current operator $\mathcal{J}_\mu$ as

$$\mathcal{J}_\mu = J_\mu + K_\mu, \qquad (2)$$

where

$$CJ_\mu C^{-1} = -J_\mu \qquad (3)$$

and

$$CK_\mu C^{-1} = +K_\mu. \qquad (4)$$

Both J_μ and K_μ are assumed to have the same transformation properties under CPT, and both are vector currents under P (in the absence of the weak interaction). The condition of C, or T invariance is

$$K_\mu = 0. \qquad (5)$$

We note that if $K_\mu \neq 0$ and if some of its matrix elements are comparable in magnitude to those of J_μ (i.e., large C, T violation), then, through virtual electromagnetic processes, all strong reactions may violate C and T to order of α where $\alpha = (137)^{-1}$.

It has been suggested recently,[2,3] in connection with reaction (1), that the violation of CP invariance is not due to the usual weak interaction, but, rather, is due to the possible existence of a new C, T noninvariant interaction called H_F, whose coupling constant F is much stronger than the Fermi constant G_V of the usual weak interaction H_{wk}. It is estimated that

$$F \sim 10^3 G_V; \qquad (6)$$

or, the dimensionless constant (Fm_p^2) is given by

$$Fm_p^2 \sim 10^{-2}, \qquad (7)$$

where m_p is the mass of the proton. The usual weak interaction violates C invariance and P invariance, but it is assumed to be invariant under CP and T. The new interaction H_F is assumed to violate C invariance and T invariance, but it conserves strangeness, and is invariant under CT and P. The decay $K_1^0 \to 2\pi$ can occur through

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[2] T. D. Lee and L. Wolfenstein, Phys. Rev. 138, B1490 (1965).

[3] L. B. Okun (unpublished report). See also J. Prentki and M. Veltman, Phys. Letters 15, 88 (1965), in which they consider the possibility that the C, T-noninvariant interaction is simply the usual SU_3-violating, but SU_2-conserving part of the strong interaction. This possibility seems to encounter several difficulties, especially in view of the present accuracy ($\sim$2% in relative amplitude) of T invariance in many nuclear reactions (see Ref. 12). In all such reactions, large violations of SU_3 symmetry may occur because of the difference between the Compton wavelengths of π and K. It is also difficult to see how such a strong violation of C, T invariance manages to produce only a very small CP-noninvariant amplitude in the decay $K_2^0 \to \pi^+ + \pi^-$.

H_{wk} alone, but reaction (1) can occur only through the second-order CP-noninvariant term $H_F H_{\mathrm{wk}}$; thus, its amplitude is much smaller than that of $K_1^0 \to 2\pi$.

If such an H_F exists, then, at least to order (Fm_p^2), an "effective" C, T-noninvariant current K_μ will appear. However, total lack of evidence for C, T invariance for the nonleptonic part of the electromagnetic interaction suggests another more interesting possibility. The electromagnetic interaction may itself show a large violation of C and T, and the "new" interaction H_F may well be simply a manifestation of the second-order electromagnetic effects. This explains why the magnitude of (Fm_p^2) is $\sim\alpha$.

While these are merely some hypothetical possibilities, they do reflect our present state of ignorance and should, therefore, provide an incentive for a critical study of the various experimental implications of possible C, T violations in the electromagnetic interactions of strongly interacting particles. To set a sensitive limit on the magnitude of possible C noninvariances, experiments should be done to measure the rates of

$$\eta^0 \to \pi^0 + e^+ + e^-\,,$$

or

$$\phi^0 \to \omega^0(\text{or } \rho^0) + \gamma\,,$$

and to study the possible $\pi^\pm$ asymmetry in

$$\omega^0(\text{or } \eta^0) \to \pi^+ + \pi^- + \gamma\,,$$

etc. For a direct limit on possible T noninvariance in the electromagnetic interactions, a reaction such as

$$\Sigma^0 \to \Lambda^0 + e^+ + e^-$$

is important. A systematic phenomenological discussion of these and similar reactions will be given in the subsequent sections.

The electromagnetic interaction is usually thought to be invariant under C, P, and T separately. There exists a so-called "minimal" principle, which states that if, in the absence of the electromagnetic field A_μ, the Lagrangian $\mathcal{L}$ is known, then the replacement of $(\partial/\partial x_\mu)$ by $(\partial/\partial x_\mu - ieA_\mu)$ changes $\mathcal{L}$ to another Lagrangian $\mathcal{L}_A$, which contains all the dependence on the electromagnetic field. A consequence of this "minimal" principle is that if $\mathcal{L}$ is invariant under T, so must $\mathcal{L}_A$ be, since $\partial/\partial x_\mu$ and $(\partial/\partial x_\mu - ieA_\mu)$ transform in the same way under T.

It must be pointed out that the validity of such a "minimal" principle for the strongly interacting particles is, as yet, unclear, and it will not be assumed in this paper.[3a] Throughout our discussions, we assume that the electromagnetic interactions of the leptons are invariant[4] under C, P, and T separately, but the electromagnetic interactions of the nonleptons can have strong violations of C, T invariance.

II. PRESENT EXPERIMENTAL LIMITS ON C, P, T INVARIANCES FOR STRONG INTERACTIONS AND ELECTROMAGNETIC INTERACTIONS

Since the discovery[5,6] of the nonconservation of parity in weak interactions,[7] there have been many experiments to search for possible P-violating effects in nuclear reactions. These experiments[8] establish that the magnitudes of the P-nonconserving amplitudes are smaller than that of the corresponding P-conserving amplitude by, at least, a factor $\sim 10^{-5}$, which is about the order of magnitude of the dimensionless weak coupling constant $G_V m_p^2$. Thus, we should regard both strong and electromagnetic interactions to be P conserving (neglecting the weak-coupling constant).

Invariance under CPT implies[9] mass and lifetime equalities between any particle and its antiparticle. In view of the possibly large violations of C and CP invariances, we may use such equalities as evidence for CPT invariance. Among such equalities, the most accurate one is that between K^0 and $\bar{K}^0$:

$$\langle K^0 | H_{\mathrm{st}} + H_\gamma + H_{\mathrm{wk}} | K^0 \rangle = \langle \bar{K}^0 | H_{\mathrm{st}} + H_\gamma + H_{\mathrm{wk}} | \bar{K}^0 \rangle, \quad (8)$$

where H_{st} and H_γ are, respectively, the Hamiltonians for the strong and the electromagnetic interactions. From the experimental mass difference[10] Δm between K_1^0 and K_2^0, we conclude that Eq. (8) holds to $\sim |\Delta m/m_K| \sim 10^{-14}$. In the following, we take H_{st} and H_γ to be invariant under P and CPT; therefore, to the same great accuracy, both interactions are also invariant under CT.

The present experiments[11] on the polarization and

[3a] *Note added in proof.* It turns out that the principle of minimal electromagnetic interaction is compatible with the violation of C invariance where C is the particle-antiparticle conjugation operator determined by the strong interaction. For a complete discussion, see T. D. Lee, Phys. Rev. (to be published).

[4] The very precise experimental results on $(g-2)$ and Lamb shift show that the electromagnetic interactions of both e and μ are accurately described by the usual form $ie\psi^\dagger\gamma_4\gamma_\mu\psi A_\mu$ which is invariant under C, P, and T separately.

[5] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, Phys. Rev. **105**, 1413 (1957).

[6] R. L. Garwin, L. M. Lederman, and M. Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, *ibid.* **105**, 1681 (1957).

[7] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[8] We list but a few of such relatively recent experiments: F. Boehm and E. Kankeleit, University of California Report No. Calt-63-13 (unpublished); Yu G. Abov, P. A. Krupchitsky, and Yu. A. Oratovsky, *Proceedings of the International Congress on Nuclear Physics, Paris, 1964* (to be published); L. Grodzins and F. Genovese, Phys. Rev. **121**, 228 (1961); R. E. Segel *et al.*, Phys. Rev. **123**, 1382 (1961); D. E. Alburger *et al.*, Phil. Mag. **6**, 171 (1961); R. Haas, L. B. Leipuner, and R. K. Adair, Phys. Rev. **116**, 1221 (1959); F. Boehm and U. Hauser, Nucl. Phys. **14**, 615 (1959); D. A. Bromley *et al.*, Phys. Rev. **114**, 758 (1959).

[9] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).

[10] B. Aubert *et al.*, Phys. Letters **10**, 215 (1964); U. Camerini *et al.*, Phys. Rev. **128**, 362 (1962); J. H. Christenson *et al.*, Conference on Fundamental Aspects of Weak Interactions (Brookhaven National Laboratory, New York, 1963) (unpublished); J. H. Christenson, Princeton University, Technical Report 34, 1964 (unpublished); V. L. Fitch, Nuovo Cimento **22**, 1160 (1961); R. H. Good *et al.*, Phys. Rev. **124**, 1223 (1961).

[11] A. Abashian and E. M. Hafner, Phys. Rev. Letters **1**, 255 (1958).

angular asymmetry in the p-p scattering give an upper limit on the magnitude of the T-noninvariant amplitude of not more than a few percent of the T-invariant amplitude. A corresponding upper limit of $\sim 2\%$ has also been obtained from the existing experiments on reciprocity relations[12] in low-energy nuclear reactions. Thus, the strong interaction has been found to be invariant under the time-reversal operation to within a few percent. By using CT invariance, we can conclude that the strong interaction is also invariant under C to within a few percent.

The question of C, T invariance in the electromagnetic interaction for the strongly interacting particles is, however, a completely open one. We note that the electromagnetic interaction of a single physical nucleon is characterized by the matrix element $\langle N'|\mathcal{J}_\mu(x)|N\rangle$, which, by invariance under the continuous Lorentz transformations and space reflection, must be of the form

$$\langle N'|\mathcal{J}_\mu(x)|N\rangle = ieU_{N'}^\dagger\gamma_4[\gamma_\mu F_1+i(N_\mu'+N_\mu)F_2+(N_\mu'-N_\mu)F_3]U_N \times\exp[-i(N_\lambda'-N_\lambda)x_\lambda], \quad (9)$$

where N_μ and N_μ' are, respectively, the 4-momenta of the initial and the final single physical nucleon states $|N\rangle$ and $|N'\rangle$, U_N and U_N' are the solutions of the free Dirac equations with N_μ and N_μ' as their respective 4-momenta. The γ_1, γ_2, γ_3, γ_4 are the usual Dirac matrices and F_1, F_2, F_3 are functions of the square of the 4-momentum transfer

$$q^2=(N_\mu'-N_\mu)^2 \quad (10)$$

only. [We use for a 4-vector $N_\mu=(\mathbf{N},iN_0)$, where N_0 is real.] From Hermiticity, we find F_1, F_2, and F_3 to be all real. If T invariance holds, then F_1 and F_2 are real, but F_3 is pure imaginary; thus,

$$F_3\neq 0 \quad (11)$$

implies T noninvariance. However, because of the conservation of current, the T-noninvariant term F_3 actually vanishes on the nucleon mass shells.[13] The electromagnetic form factors of the nucleon can be measured by studying the scattering of

$$l^\pm+N\rightarrow l^\pm+N, \quad (12)$$

where $l=e$ or μ, in which the nucleons *are* on their mass shells. Thus, neglecting higher order electromagnetic effects, no information concerning the possible C, T noninvariance in the operator $\mathcal{J}_\mu$ can be obtained from reaction (12).

The electromagnetic property of a nucleus is determined by the corresponding properties of its constituents. It is a good approximation (i) to regard the nucleus as a collection of physical nucleons interacting through their strong nuclear forces, and (ii) to neglect other terms which can occur in the nucleon-photon vertex when the nucleons are off the mass shell. Within these approximations, and neglecting higher order radiative processes, there are still no T-noninvariant terms which contribute to any nuclear γ transition. The accuracy of these two approximations may be crudely estimated to be $\sim O(q^2/m_\pi^2)$ for (i), at least for T-violating effects, and $\sim O(V/m_N)$ for (ii), where V is the nuclear potential energy and m_N, m_π are, respectively, the masses of the nucleon and the pion. Thus, to the accuracy of $\sim$a few percent, no information concerning C, T invariance of H_γ can be obtained by studying the nuclear photoprocesses; rather, such processes can only be used to establish more firmly the C, T invariance of the strong interaction.

If H_γ strongly violates C, T invariance, then, through higher order effects, the reciprocity relation for any strong reaction may be violated with a fractional difference of the order of 10^{-2}–10^{-3}. All nuclear matrix elements in either γ or β transitions may also contain a small admixture of T-noninvariant amplitude which is $\sim(10^{-2}$–$10^{-3})$ times the T-invariant amplitude. Such a T-noninvariant amplitude can be observed by measuring the relative phase[14] between G_V and G_A in β decay, or by studying, e.g., the $E2$, $M1$ interference term through the detection of a $[\mathbf{k}_\beta\cdot(\mathbf{k}_\gamma\times\mathbf{k}_\gamma')](\mathbf{k}_\gamma\cdot\mathbf{k}_\gamma')$ term in an appropriate β-γ-γ transition.[15]

Reciprocity relations such as $\gamma+A\leftrightarrows B+C$ can be used to test possible T noninvariance in H_γ, where A, B, and C are any strongly interacting particle states. However, because of Hermiticity, a detailed spin-momentum analysis is necessary to test the reciprocity relation. As possible examples for such reciprocity tests, we may list $\gamma+n\leftrightarrows p+\pi^-$ or $\gamma+p\leftrightarrows n+\pi^+$.

Other direct tests of the C, T invariance properties of H_γ can be obtained by studying the relevant decays of mesons and hyperons. These will be analyzed in detail in the subsequent sections.

III. DECAYS OF THE PSEUDOSCALAR MESONS

Since H_{st} does conserve C, its eigenstates such as π^0 and η^0 are also eigenstates of C. In view of the possibility that H_γ may not be invariant under C, we cannot use the γ decay modes to determine the C values of these mesons. For definiteness, we assume that *virtual*

[12] L. Rosen and J. E. Brolley, Jr., Phys. Rev. Letters **2**, 98 (1959); D. Bodansky *et al.*, Phys. Rev. Letters **2**, 101 (1959).

[13] We would like to thank Professor R. Serber and Professor G. C. Wick for pointing this out, and for enlightening discussions.

[14] M. T. Burgy *et al.*, Phys. Rev. Letters **1**, 324 (1958). It should be emphasized that, independent of T invariance, (G_A/G_V) must be real if the nonleptonic current $(V_\mu+A_\mu)$ in the weak interaction satisfies the charge symmetry condition; i.e., $[V_\mu+A_\mu]^*=-[\exp(i\pi I_y)][V_\mu+A_\mu][\exp(-i\pi I_y)]$, where I_y is the y component of the isospin operator, * denotes the Hermitian conjugation if $\mu\neq 4$ and (-1) times the Hermitian conjugation if $\mu=4$.

[15] For a detailed discussion, see E. M. Henley and B. A. Jacobsohn, Phys. Rev. **113**, 225 (1959); B. A. Jacobsohn and E. M. Henley, *ibid.* **113**, 234 (1959).

transitions such as

$$N+\bar{N}\leftrightarrows\pi^0 \qquad (13)$$

can occur through H_{st}; thus,

$$C_{\pi^0}=+1. \qquad (14)$$

A further confirmation of Eq. (14) is given by Eq. (46) below. (The validity of many of our subsequent conclusions on tests of C, T invariance depends, however, only on the relative C values of different mesons states.)

It is convenient to decompose the $C=\mp1$ currents J_μ and K_μ into various components which have different transformation properties under the isotopic spin rotations:

$$J_\mu=J_\mu{}^s+J_\mu{}^v \qquad (15)$$

and

$$K_\mu=K_\mu{}^s+K_\mu{}^v+\cdots, \qquad (16)$$

where the superscripts s and v indicate that these currents transform as isoscalar ($I=0$) or isovector ($I=1$), respectively.[15a] If

$$K_\mu\neq0, \qquad (17)$$

then H_γ is not invariant under either C or T. A conclusive test of C, T noninvariance in H_γ would be the discovery of any process whose existence implies Eq. (17).

1. π^0 Decay

To first order in $\mathcal{J}_\mu$, the decay $\pi^0\rightarrow e^++e^-$ is forbidden because of current conservation. To second order in $\mathcal{J}_\mu$, the decays

$$\pi^0\rightarrow2\gamma, \qquad (18)$$

$$\pi^0\rightarrow\gamma+e^++e^-, \qquad (19)$$

and

$$\pi^0\rightarrow2e^++2e^-, \qquad (20)$$

contain only matrix elements of $J_\mu\cdot J_\nu$ and $K_\mu\cdot K_\nu$. Therefore, these reactions do not give direct tests of possible C, T noninvariance.

The decay

$$\pi^0\rightarrow3\gamma \qquad (21)$$

can occur only if C invariance is violated. Assuming that $K_\mu\neq0$, its rate can be estimated. We find

$$\frac{\text{Rate}(\pi^0\rightarrow3\gamma)}{\text{Rate}(\pi^0\rightarrow2\gamma)}\sim e^2\frac{(\text{phase space})_{3\gamma}}{(\text{phase space})_{2\gamma}}(kR)^6\sim3\times10^{-6}, \qquad (22)$$

where $(e^2/4\pi)=\alpha$, k is the average momentum of the γ ray, the ratio of phase space is $\sim(4\pi^2)^{-1}(m_\pi R)^2$, and R is taken to be $\sim m_\pi{}^{-1}$. The present experimental limit[16] is only

$$\left[\frac{\text{Rate}(\pi^0\rightarrow3\gamma)}{\text{Rate}(\pi^0\rightarrow2\gamma)}\right]_{\text{expt}}<3.8\times10^{-4}. \qquad (23)$$

Thus, pion decays seem to be particularly insensitive to any possible violation of C, T in H_γ.

2. η^0 Decays

From the decays

$$\rho^0\rightarrow\pi^++\pi^-, \qquad (24)$$

and

$$\rho^0\nrightarrow\eta^0+\pi^0, \qquad (25)$$

we may conclude

$$C_\rho=-1, \qquad (26)$$

and

$$C_\rho=-C_\eta C_{\pi^0}. \qquad (27)$$

Thus, it follows that

$$C_\eta=C_{\pi^0}. \qquad (28)$$

The same conclusion would follow if we assume virtual transitions such as $N+\bar{N}\leftrightarrows\eta^0$ to be allowed.

The η^0 can only decay through H_γ. The decay

$$\eta^0\rightarrow\pi^0+e^++e^- \qquad (29)$$

through a single-photon intermediate state violates C invariance, and it can occur only if the isovector part $K_\mu{}^v\neq0$. The general matrix element of $\mathcal{J}_\mu$ for this transition is given by

$$\begin{aligned}\langle\pi^0|\mathcal{J}_\mu(x)|\eta^0\rangle&=\langle\pi^0|K_\mu{}^v(x)|\eta^0\rangle\\&=[f_1(\eta_\mu+\pi_\mu)+f_2(\eta_\mu-\pi_\mu)]\\&\quad\times[4m_\eta\omega_\pi]^{-1/2}\exp[i(\eta_\lambda-\pi_\lambda)x_\lambda],\end{aligned} \qquad (30)$$

where η_μ and π_μ are, respectively, the 4-momenta of the initial η^0 and the final π^0 states, m_η is the mass of η^0, ω_π is the energy of the pion, and f_1 and f_2 are form factors depending only on the square of the 4-momentum transfer,

$$q^2=(\eta_\lambda-\pi_\lambda)^2. \qquad (31)$$

Conservation of current requires that

$$f_1(m_\eta{}^2-m_\pi{}^2)=q^2f_2, \qquad (32)$$

where m_π is the mass of π^0. To obtain an estimate of the decay rate, we assume[17]

$$f_1=-\tfrac{1}{6}e\langle r^2\rangle q^2, \qquad (33)$$

[15a] *Note added in proof.* From a phenomenological point of view, one should also decompose $K_\mu=(K_\mu)_1+(K_\mu)_8+\cdots$, where $(K_\mu)_1$ and $(K_\mu)_8$ transform, respectively, like a unitary singlet and a member of the unitary octet under the SU_3 group of transformations. If $K_\mu=(K_\mu)_1+(K_\mu)_8$ only, then, in the limit of perfect SU_3 symmetry, $\eta^0\nrightarrow\pi^0+e^++e^-$ and there is no T-noninvariant effect in $\Sigma^0\rightarrow\Lambda^0+e^++e^-$. If $K_\mu=(K_\mu)_1$ only [and, consequently, K_μ is an isoscalar], then there are many additional forbidden processes such as $\phi^0\nrightarrow\omega^0+\gamma$, $\phi^0\nrightarrow\rho^0+\gamma$, $\omega^0\nrightarrow\rho^0+\gamma$ and the absence of π^+, π^- asymmetries in $\omega^0\rightarrow\pi^++\pi^-+\pi^0$ (or γ).

[16] D. Cline and R. M. Dowd, Phys. Rev. Letters 14, 530 (1965).

[17] Such a form factor, after eliminating the photon propagator, gives an "effective" C, T-noninvariant point interaction:

$$\langle r^2\rangle\tfrac{1}{6}e^2[\phi_\eta(\partial/\partial x_\mu)\phi_\pi-\phi_\pi(\partial/\partial x_\mu)\phi_\eta](i\psi_e{}^\dagger\gamma_4\gamma_\mu\psi_e),$$

where ϕ_π, ϕ_η, and ψ_e are the operators for π^0, η^0, and e^-. In this paper, we assume that there does not exist any additional direct strangeness conserving interaction between e^+, e^- and the strongly interacting particles. For a discussion on the limits of such point interactions, see G. Feinberg and M. Goldhaber, Proc. Natl. Acad. Sci. 45, 1301 (1959).

where $\langle r^2\rangle$ is the average of $(\text{radius})^2$ of the "mixed" charge distribution between η^0 and π^0. The decay rate is given by [neglecting the mass of $e^\pm$]

$$\text{Rate}(\eta^0 \to \pi^0+e^++e^-)=(1728\pi)^{-1}\alpha^2[\langle r^2\rangle m_\eta^2]^2 m_\eta \times[(1-\epsilon^2)(1-8\epsilon+\epsilon^2)-12\epsilon^2\ln\epsilon], \quad (34)$$

where

$$\epsilon=(m_\pi/m_\eta)^2. \quad (35)$$

The corresponding spectrum of $e^\pm$ is, apart from a normalization constant, given by

$$[4k_+k_- -2m_\eta(k_++k_-)+(m_\eta^2-m_\pi^2)]dk_+dk_-, \quad (36)$$

where $k_\pm$ is the energy of $e^\pm$. The rate of $\eta^0\to 2\gamma$ may be estimated by using SU_3 symmetry:

$$\text{Rate}(\eta^0\to 2\gamma)\sim\tfrac{1}{3}(m_\eta/m_\pi)^3\times\text{Rate}(\pi^0\to 2\gamma). \quad (37)$$

Combining Eqs. (34) and (37), we find the ratio

$$[\text{Rate}(\eta^0\to\pi^0+e^++e^-)/\text{Rate}(\eta^0\to 2\gamma)] \sim 0.04[\langle r^2\rangle m_\eta^2]^2, \quad (38)$$

which is ~ 1 if $\langle r^2\rangle$ is set arbitrarily to be the same as the mean-square radius of the proton.

If reaction (29) is observed, then it clearly indicates the C noninvariance in η^0 decay, and it shows that such violation also does not conserve the isospin I.

Another possibility is to study the spectrum of π^+ and π^- in the decays

$$\eta^0\to\pi^++\pi^-+\pi^0 \quad (39)$$

and

$$\eta^0\to\pi^++\pi^-+\gamma. \quad (40)$$

Let $dN(E_+,E_-)$ be the number of events in which the energies of π^+ and π^- are between $E^\pm$ and $E^\pm+dE^\pm$. In either of these two reactions, the observation of

$$dN(E_+=E_1,E_-=E_2)\neq dN(E_+=E_2,E_-=E_1) \quad (41)$$

for any energies E_1 and E_2 is an unequivocal proof of C noninvariance.

A possible $\pi^\pm$ asymmetry in reaction (39) has been discussed elsewhere[2,18] under the assumption that a C, T noninvariant H_F exists whose coupling constant is given by Eq. (7). If H_γ strongly violates C and T, then H_F represents simply the second-order electromagnetic effect. We note that if K_μ contains only an isoscalar part $K_\mu{}^s$, then H_F satisfies the $|\Delta\mathbf{I}|<2$ rule; otherwise, H_F may contain a $|\Delta I|=2$ and $C=-1$ term.

Any $\pi^\pm$ asymmetry in reaction (40) must be due to the existence of the $I=0$ (or 2) part of K_μ; in addition, there must be strong pion interactions. To analyze such asymmetries, we may make a partial-wave analysis of the (2π) system for reaction (40) and the decay

$$\eta^0\to 2\pi^0+\gamma \quad (42)$$

which violates C conservation. As an illustration, only the lowest possible angular-momentum states will be kept. The (2π) can be either in an $I=1$ p state produced by $J_\mu{}^v$, or in an $I=0$ d state produced by $K_\mu{}^s$. The amplitudes for these two states can be represented, respectively, by $\sqrt{2}a(\mathbf{p}\cdot\mathbf{H})$ and $i\sqrt{3}b(\mathbf{p}\cdot\mathbf{H})(\mathbf{p}\cdot\mathbf{k})$, where $\mathbf{p}$ is the relative momentum of the two pions, $\mathbf{k}$ is the momentum of the photon, and $\mathbf{H}$ is its magnetic field. The energy distribution of reaction (40) is, then, given by the invariant phase space times

$$k^2[p^2-(\mathbf{p}\cdot\hat{k})^2] \times[|a|^2+|b|^2(\mathbf{p}\cdot\mathbf{k})^2+i(a^*b-b^*a)(\mathbf{p}\cdot\mathbf{k})], \quad (43)$$

where $\mathbf{p}$ ranges over the entire momentum space and $\hat{k}$ is a unit vector along $\mathbf{k}$; the corresponding density for reaction (42) is given by

$$|b|^2[p^2-(\mathbf{p}\cdot\hat{k})^2][\mathbf{p}\cdot\mathbf{k}]^2k^2, \quad (44)$$

where $\mathbf{p}$ ranges only over half of the momentum space. From CPT invariance, we find that the relative phase between a and b is given by $(\delta_p-\delta_d)$ or $(\pi+\delta_p-\delta_d)$, where δ_p is the strong-interaction phase shift of the 2-pion system in the $I=1$ p state and δ_d is the same in the $I=0$ d state. Thus, the asymmetry, if it exists, can also be used to obtain information concerning the pion interactions.

3. Weak Decays

We consider first the $K_1{}^0$-$K_2{}^0$ system. To make our analysis definite, we will assume here that H_{wk} is invariant under both T and CP, and H_γ has a large C, T noninvariant part (i.e., $K_\mu\neq 0$). To zeroth order in $\mathcal{g}_\mu$, there is no CP-violating effect. Thus,

$$CP=+1, \quad \text{for } K_1{}^0$$

and

$$CP=-1, \quad \text{for } K_2{}^0. \quad (45)$$

If we use Eq. (45) and note that the three pions in the decay $K_2{}^0\to 3\pi$ are produced predominantly in s states, it follows, then, that

$$(CP)_{\pi^0}=-1. \quad (46)$$

Thus, $C_{\pi^0}=+1$ which confirms Eq. (14).

To first order in $\mathcal{g}_\mu$, since H_{wk} does not conserve C, the decay

$$K_i{}^0\to\pi^0+e^++e^- \quad (47)$$

can occur via either J_μ or K_μ, where $i=1$ or 2. For either decay, there is only one form factor, due to current conservation; therefore, it does not yield any direct information concerning possible T or CP noninvariance. On the other hand, the decay

$$K_i{}^0\to\pi^++\pi^-+\gamma \quad (48)$$

can be used to test possible CP noninvariance. Let $\boldsymbol{\varepsilon}\cdot\mathbf{M}_J{}^i(\mathbf{p},\mathbf{k})$ and $\boldsymbol{\varepsilon}\cdot\mathbf{M}_K{}^i(\mathbf{p},\mathbf{k})$ be, respectively, the transition amplitudes due to $(J_\mu A_\mu)H_{\text{wk}}$ and $(K_\mu A_\mu)H_{\text{wk}}$, where A_μ is the electromagnetic field operator, $\mathbf{p}$ is the relative momentum between π^+ and π^-, $\mathbf{k}$ is the

[18] T. D. Lee, Phys. Rev. **139**, B1415 (1965).

momentum of γ, and $\boldsymbol{\varepsilon}$ is its polarization vector. By applying CP, we find

$$\mathbf{M}_J{}^i(\mathbf{p},+\mathbf{k})=\pm\mathbf{M}_J{}^i(\mathbf{p},-\mathbf{k}) \tag{49}$$

and

$$\mathbf{M}_K{}^i(\mathbf{p},+\mathbf{k})=\mp\mathbf{M}_K{}^i(\mathbf{p},-\mathbf{k})\,, \tag{50}$$

where the upper signs are for $i=1$, and the lower signs for $i=2$. The probability distribution, after summing over the two polarization directions of γ, is proportional to

$$\sum_{\alpha,\beta}\left[(\mathbf{M}_\alpha{}^i)^*\cdot\mathbf{M}_\beta{}^i-(\mathbf{M}_\alpha{}^i\cdot\hat{k})^*(\mathbf{M}_\beta{}^i\cdot\hat{k})\right], \tag{51}$$

where the sum extends over α (and β)$=J$ and K, and $\hat{k}$ is a unit vector along $\mathbf{k}$. By using Eqs. (49) and (50), we find that in expression (51) the interference term between $\mathbf{M}_J{}^i$ and $\mathbf{M}_K{}^i$ is an odd function in $\mathbf{k}$, and, therefore, it is also an odd function in $\mathbf{p}$; the other terms are even in $\mathbf{p}$. Thus, the assumption that $K_\mu\neq0$ can result in a $\pi^\pm$ asymmetry, similar to that given by Eq. (41):

$$dN(E_+=E_1,E_-=E_2)\neq dN(E_+=E_2,E_-=E_1)\,.$$

Conversely, the observation of such an asymmetry gives also a direct proof of CP noninvariance in the $K_i{}^0$ decays.

To second order in $\mathcal{J}_\mu$, decays such as $K_2{}^0\to2\pi$ and $K_1{}^0\to3\pi^0$ can occur. The smallness of the observed rate of reaction (1) is attributed to the smallness of the fine-structure constant. To the same order, explicit CP- and T-noninvariant effects can also be observed in K_{l3} and K_{l4} decays.

For the weak decays of charged mesons, to second order in $\mathcal{J}_\mu$, there exist explicit T-noninvariant observables, such as $\boldsymbol{\sigma}_\mu\cdot(\mathbf{p}_\mu\times\mathbf{p}_\pi)$ in the $K_{l3}{}^\pm$ decay, etc. The T-noninvariant amplitudes are, however, smaller than the corresponding T-invariant amplitudes by a factor $\sim O(\alpha)$. In the radiative decay $\pi^\pm$ (or $K^\pm$) $\to\mu^\pm$ $+$neutrino$+\gamma$, we may consider only the inner-bremsstrahlung process. Since both the lepton current and the weak interaction are assumed to be invariant under T, no T-noninvariant term such as $\boldsymbol{\sigma}_\mu\cdot(\mathbf{p}_\mu\times\mathbf{p}_\gamma)$ can be observed.

From CPT invariance, we have (neglecting higher order terms in $H_{\rm wk}$, but valid to all orders in H_γ)

$$\textstyle\sum_n\left[\text{Rate}(K^+\to n\pi)+\text{Rate}(K^+\to n\pi+\gamma)\right]$$
$$=\textstyle\sum_n\left[\text{Rate}(K^-\to n\pi)+\text{Rate}(K^-\to n\pi+\gamma)\right], \tag{52}$$

$$\textstyle\sum_n\left[\text{Rate}(K^+\to n\pi+l^++\nu_l)+\text{Rate}(K^+\to n\pi+l^++\nu_l+\gamma)\right]$$
$$=\textstyle\sum_n\left[\text{Rate}(K^-\to n\pi+l^-+\bar\nu_l)+\text{Rate}(K^-\to n\pi+l^-+\bar\nu_l+\gamma)\right], \tag{53}$$

$$\text{Rate}(K^+\to l^++\nu_l)+\text{Rate}(K^+\to l^++\nu_l+\gamma)$$
$$=\text{Rate}(K^-\to l^-+\bar\nu_l)+\text{Rate}(K^-\to l^-+\bar\nu_l+\gamma)\,, \tag{54}$$

where $l=e$ or μ, and γ stands for any numbers of photons and (l^+,l^-) pairs. For decays that are allowed by $H_{\rm wk}$, from CP invariance, each individual partial decay rate of K^+ is equal to that of K^-, provided virtual effects of H_γ are neglected. Thus, e.g., neglecting $O(\alpha)$, we obtain

$$\text{Rate}(K^+\to\pi^++2\pi^0)=\text{Rate}(K^-\to\pi^-+2\pi^0)\,, \tag{55}$$

$$\text{Rate}(K^+\to2\pi^++\pi^0)=\text{Rate}(K^-\to2\pi^-+\pi^0)\,, \tag{56}$$

etc. The decay $K^\pm\to\pi^\pm+\pi^0$ is forbidden by $H_{\rm wk}$, if $H_{\rm wk}$ rigorously satisfies the $|\Delta\mathbf{I}|=\frac{1}{2}$ selection rule. In this case, the $K_{2\pi}{}^\pm$ decay can only occur through $H_{\rm wk}H_\gamma H_\gamma$ which violates CP invariance. However, by using Eqs. (52)–(56) and the experimental limits on radiative decays, we find that the equality

$$\text{Rate}(K^+\to\pi^++\pi^0)=\text{Rate}(K^-\to\pi^-+\pi^0) \tag{57}$$

should hold to an accuracy $\sim O(10^{-2})$. Thus, any lack of difference between the $K_{2\pi}{}^+$ and $K_{2\pi}{}^-$ branching ratios does not reflect the T invariance or T noninvariance of H_γ.

IV. DECAYS OF THE VECTOR MESONS

The vector mesons all decay through $H_{\rm st}$. If H_γ is not invariant under C and T, then, through second-order processes, C, T-noninvariant effects, such as $\pi^\pm$ asymmetry in $\omega^0\to\pi^++\pi^-+\pi^0$, $\phi^0\to2K_1{}^0$, $\rho^0\to\eta^0+\pi^0$, etc., can occur. The C, T-noninvariant amplitudes are smaller than the corresponding C, T-invariant amplitudes by a factor $\sim O(\alpha)$. In the following, we will discuss in detail two types of C, T-noninvariant processes that can occur to the first order in $\mathcal{J}_\mu$.

1. $A^0(1-)\to B^0(1-)+\gamma$

In this class, there are

$$\phi^0\to\omega^0+\gamma\,, \tag{58}$$

$$\phi^0\to\rho^0+\gamma\,, \tag{59}$$

and

$$\omega^0\to\rho^0+\gamma\,. \tag{60}$$

We observe that by using Eq. (45) and the decay $\phi^0\to K_1{}^0+K_2{}^0$, it follows that

$$C_\phi=-1\,. \tag{61}$$

From the fact that the π^+,π^- in the decay $\omega^0\to\pi^+$ $+\pi^-+\pi^0$ are produced in an antisymmetric state, or $\omega^0\nrightarrow3\pi^0$, we may conclude

$$C_\omega=-C_{\pi^0}=-1\,. \tag{62}$$

The C_ρ is, according to Eq. (26), also -1. Decays such as $\omega^0\to\pi^0+\gamma$ or $\omega^0\to\pi^0+e^++e^-$ conserve C. However, reactions (58)–(60) all violate C invariance. Thus, only the current K_μ can contribute to these decays. Furthermore, reaction (58) can occur only if $K_\mu{}^s\neq0$ and reactions (59) and (60) can occur only if $K_\mu{}^v\neq0$.

By using invariance under space reflection and the continuous Lorentz transformations, the general matrix element of $\mathcal{J}_\nu$ between any two physical (1−) states $|A\rangle$ and $|B\rangle$ is given by

$$\langle B|\mathcal{J}_\lambda|A\rangle = ie\varphi_\mu'[(p'+p)_\lambda(F_1\delta_{\mu\nu}+F_2q_\mu q_\nu) + F_3\delta_{\lambda\nu}q_\mu + F_4\delta_{\lambda\mu}q_\nu + q_\lambda(F_5\delta_{\mu\nu}+F_6q_\mu q_\nu)]\varphi_\nu, \quad (63)$$

where p_λ, p_λ' are, respectively, the 4-momenta of the states $|A\rangle$ and $|B\rangle$, and $q_\lambda = p_\lambda' - p_\lambda$. The polarizations of the initial and final states are given by φ_μ' and φ_ν, respectively, which satisfy the normalization conditions

$$\varphi_\nu^2 = (2\omega_A)^{-1} \quad \text{and} \quad \varphi_\nu'^2 = (2\omega_B)^{-1}, \quad (64)$$

where ω_A and ω_B are the energies of A and B. From current conservation, we have

$$(m_A^2 - m_B^2)F_1 + q^2F_5 = 0 \quad (65)$$

and

$$(m_A^2 - m_B^2)F_2 + q^2F_6 + F_3 + F_4 = 0, \quad (66)$$

where m_A and m_B are, respectively, the masses of A and B. For the radiative decay

$$A \to B + \gamma, \quad (67)$$

the longitudinal part $q_\lambda(F_5\delta_{\mu\nu}+F_6q_\mu q_\nu)$ cannot contribute. Assuming that all form factors are regular at $q^2=0$ and dropping the longitudinal part, we find, at $q^2=0$, $\langle B|\mathcal{J}_\lambda|A\rangle$ becomes

$$ie\varphi_\mu'[a(p'+p)_\lambda q_\mu q_\nu + b(\delta_{\lambda\nu}q_\mu - \delta_{\lambda\mu}q_\nu) - \tfrac{1}{2}a(m_A^2-m_B^2)(\delta_{\lambda\nu}q_\mu + \delta_{\lambda\mu}q_\nu)]\varphi_\nu, \quad (68)$$

where a and b are related to these form factors by

$$a = F_2(0) \quad (69)$$

and

$$b = \tfrac{1}{2}[F_3(0) - F_4(0)]. \quad (70)$$

[If $m_B = m_A = m$, then as $q_\lambda \to 0$, expression (68) corresponds to a system of two spin-1 neutral particles with a "mixed" magnetic moment $= (eb/2m)\times$spin and a "mixed" quadrupole moment $= (e/m^2)(2am^2 - b)$. If A and B are the same particle, then $\langle A|\mathcal{J}_\lambda|A\rangle = 0$ by CPT invariance and Hermiticity.] The rate for reaction (67) is given by

$$\text{Rate}(A\to B+\gamma) = (24)^{-1}\alpha m_A^{-5}m_B^{-2}(m_A^2+m_B^2) \times (m_A^2-m_B^2)^3g^2, \quad (71)$$

where

$$g^2 = \tfrac{1}{4}(m_A^2-m_B^2)^2|a|^2 + |b|^2 - \tfrac{1}{2}(m_A^2+m_B^2)^{-1}(m_A^2-m_B^2)^2(a^*b+ab^*). \quad (72)$$

If H_γ violates C, T invariance strongly and if $K_\mu{}^s$ exists, then the parameter g^2 for $\varphi^0 \to \omega^0+\gamma$ may be ≈ 1, and the corresponding branching ratio becomes $\approx 1.9\%$. Similarly, if $K_\mu{}^v$ exists and if the corresponding g^2 is taken to be ≈ 1, then the branching ratio for reaction (59) is $\approx 2.4\%$. The reaction (60) has an extremely small branching ratio, so that it is unlikely to be of any practical use.

If H_γ does strongly violate C, T invariance, observations of reaction (58) and (59), though difficult, may become feasible. The detections of these decay modes are unambiguous proofs of C noninvariance.

2. $A^0(1-) \to \pi^+ + \pi^- + \gamma$

There are three such decays:

$$\phi^0 \to \pi^+ + \pi^- + \gamma, \quad (73)$$

$$\omega^0 \to \pi^+ + \pi^- + \gamma, \quad (74)$$

and

$$\rho^0 \to \pi^+ + \pi^- + \gamma. \quad (75)$$

If H_γ is not invariant under C, then there would be an asymmetry in the π^+ and π^- distribution in these decays [see Eq. (41)],

$$dN(E_+ = E_1, E_- = E_2) \neq dN(E_+ = E_2, E_- = E_1).$$

Such an asymmetry can occur in reactions (73) and (74) if $K_\mu{}^v$ exists, and in reaction (75) if $K_\mu{}^s$ exists.

The analysis of these asymmetries is similar to that in the case of η^0 decay. We make a partial-wave analysis of the (2π) system for these reactions together with

$$A^0 \to 2\pi^0 + \gamma, \quad (76)$$

where A^0 stands for either one of the three mesons: ϕ^0, ω^0, and ρ^0. Reaction (76) does not violate C invariance.

As an illustration of the form of asymmetry that might be observed in these decays, we keep only the lowest possible angular-momentum states. The (2π) system can be either in a $I=0$ s state via J_μ, or in a $I=1$ p state via K_μ. The amplitudes for these two states can be represented by $\sqrt{3}a\mathbf{s}\cdot\mathbf{E}$ and $i\sqrt{2}b\mathbf{s}\cdot(\mathbf{p}\times\mathbf{H})$, respectively, where $\mathbf{s}$ is the polarization vector of A^0, $\mathbf{p}$ is the relative momentum of the two pions, and $\mathbf{E}$ and $\mathbf{H}$ are, respectively, the electric and magnetic field of the γ. The complex constants a and b are two phenomenological parameters; they are comparable in magnitude, if the C, T violation is a large one. The pion spectrum, after summing and averaging over the polarizations of A^0 and γ, is given by the invariant phase space times

$$k^2\{|a|^2 + \tfrac{1}{2}|b|^2[p^2 + (\mathbf{p}\cdot\hat{k})^2] - i(a^*b - ab^*)(\mathbf{p}\cdot\hat{k})\} \quad (77)$$

for reaction $A^0 \to \pi^+ + \pi^- + \gamma$, and the same phase space times

$$k^2|a|^2 \quad (78)$$

for reaction (76), where $\hat{k}$ is a unit vector along the momentum direction of γ. In the evaluation of the phase space $\mathbf{p}$ ranges over the entire momentum space in (77), but only over half of the momentum space in (78). From CPT invariance, the relative phase between a and b is given by $(\delta_s - \delta_p)$ or $(\delta_s - \delta_p + \pi)$, where δ_s and δ_p are, respectively, the appropriate average phase shifts of the (2π) system in the $I=0$ s state and the $I=1$ p state.

Before leaving the meson systems, we remark that the identical analysis can be applied to the annihilation of the nucleon and antinucleon system at rest; e.g.,

$$p+\bar{p}\to\pi^{+}+\pi^{-}+\gamma. \tag{79}$$

The initial ${}^1S_0$ state has the identical symmetry properties as a mixture of η^0 and π^0, and the ${}^3S_1$ state has the same properties as a mixture of ω^0 (or ϕ^0) and ρ^0.

V. $\Sigma^0\to\Lambda^0+e^++e^-$

For a baryon system, tests of possible T noninvariance in H_γ become more accessible than that of C noninvariance. A suitable case is[19]

$$\Sigma^0\to\Lambda^0+e^++e^-. \tag{80}$$

Let $\boldsymbol{\sigma}_\Sigma$, $\boldsymbol{\sigma}_\Lambda$, $\mathbf{p}$, $\mathbf{k}_+$, and $\mathbf{k}_-$ denote, respectively, the Pauli spin matrix of Σ, spin matrix of Λ, momentum of Λ^0, momentum of e^+, and momentum of e^-. Since the lepton currents conserve C, P, and T, the distribution, as well as any T-noninvariant term, must be symmetric with respect to the exchange between $\mathbf{k}_+$ and $\mathbf{k}_-$ in the one-photon approximation. We define the normal vector of the decay plane to be parallel to

$$\mathbf{N}\equiv\hat{p}\times(\hat{k}_++\hat{k}_-), \tag{81}$$

where $\hat{p}$, $\hat{k}_+$, and $\hat{k}_-$ are unit vectors along $\mathbf{p}$, $\mathbf{k}_+$, and $\mathbf{k}_-$. Neglecting the possibility of any spin measurements for $e^\pm$, the only T-noninvariant, but P-invariant, observables are

$$\boldsymbol{\sigma}_\Sigma\cdot\mathbf{N}\quad\text{and}\quad\boldsymbol{\sigma}_\Lambda\cdot\mathbf{N}, \tag{82}$$

which are even functions with respect to the exchange between e^+ and e^-. If either one of these averages $\langle\boldsymbol{\sigma}_\Sigma\cdot\mathbf{N}\rangle$ and $\langle\boldsymbol{\sigma}_\Lambda\cdot\mathbf{N}\rangle$ is not zero, then it clearly proves T noninvariance, and consequently, also C noninvariance in this decay.

Sometimes, in order not to lose any information contained in the limited statistics available, it may be helpful to use the detailed form of the spin-momentum distribution for an experimental analysis, rather than a simple over-all average. The theoretical results of these distribution functions for reaction (80) are given below.

The matrix element of $\mathcal{J}_\mu$ is given by

$$\begin{aligned}\langle\Lambda|\mathcal{J}_\mu|\Sigma\rangle&=\langle\Lambda|J_\mu^{\gamma}+K_\mu^{\gamma}|\Sigma\rangle\\&=ieU_\Lambda^\dagger\gamma_4[\gamma_\mu F+i(\Lambda_\mu+\Sigma_\mu)F'\\&\qquad+i(\Lambda_\mu-\Sigma_\mu)F'']U_\Sigma,\end{aligned} \tag{83}$$

where Λ_μ, Σ_μ are, respectively, the 4-momenta of the states $|\Lambda\rangle$ and $|\Sigma\rangle$, U_Σ and U_Λ are spinor solutions of the free-particle Dirac equations with the same 4-momenta as the physical Σ^0 and Λ^0. If T invariance holds, then F, F', and F'' are all real functions of q^2, where

$$q^2=(\Lambda_\mu-\Sigma_\mu)^2. \tag{84}$$

In the following, we will, however, assume these functions to be complex so as to violate T invariance.

From current conservation, these functions satisfy the following relation

$$F=(m_\Sigma+m_\Lambda)F'+(m_\Sigma-m_\Lambda)^{-1}q^2F'', \tag{85}$$

where m_Λ and m_Σ are the masses of Λ^0 and Σ^0, respectively. Thus, there are, in general, only two independent complex form factors. Let F_0 and F_0' be the values of F and F' at $q^2=0$. We have

$$F_0=(m_\Sigma+m_\Lambda)F_0' \tag{86}$$

which may be regarded as the "mixed" gyromagnetic ratio between Σ^0 and Λ^0. The value of F_0 is related to the decay $\Sigma^0\to\Lambda^0+\gamma$:

$$\begin{aligned}\text{Rate}(\Sigma^0\to\Lambda^0+\gamma)=\tfrac{1}{2}\alpha|F_0|^2m_\Sigma^{-3}(m_\Sigma-m_\Lambda)^3\\\times(m_\Sigma+m_\Lambda).\end{aligned} \tag{87}$$

The initial Σ^0 may be produced in any reaction, or any combination of reactions. The spin state of Σ^0 in its rest system is completely characterized by a (2×2) density matrix:

$$D_\Sigma=\tfrac{1}{2}(1+\boldsymbol{\sigma}_\Sigma\cdot\mathbf{s}_\Sigma), \tag{88}$$

where $\mathbf{s}_\Sigma$ is a real vector whose direction and magnitude determine the average spin direction and polarization of the initial Σ^0.

By using Eqs. (83) and (88), it is straightforward to evaluate the resulting (2×2) density matrix D_Λ for the Λ^0. It is convenient to introduce G which is a linear function of F and F':

$$G=(m_\Lambda+m_\Sigma)F-2m_\Sigma(m_\Lambda+E)F', \tag{89}$$

where

$$E=(p^2+m_\Lambda^2)^{1/2} \tag{90}$$

and

$$p=|\mathbf{p}|. \tag{91}$$

In terms of the form factors G and F, the density matrix for the Λ^0 is given by (neglecting the masses of $e^\pm$, but without any nonrelativistic approximation)

$$\begin{aligned}D_\Lambda=GG^*(1+\hat{k}_+\cdot\hat{k}_-)+2FF^*p^2[1-(\hat{p}\cdot\hat{k}_+)(\hat{p}\cdot\hat{k}_-)]\\+i(GF^*-G^*F)p\mathbf{N}\cdot(\mathbf{s}_\Sigma+\boldsymbol{\sigma}_\Lambda)+D_\Lambda',\end{aligned} \tag{92}$$

where $\mathbf{N}$ is defined in Eq. (81). We note that the coefficients of $\mathbf{N}\cdot\mathbf{s}_\Sigma$ and $\mathbf{N}\cdot\boldsymbol{\sigma}_\Lambda$ are the same, and can be nonzero only if G and F are not relatively real, which violates the condition of T invariance.

The term D_Λ' contains all the spin-spin correlations; it is, therefore, zero if we average over either the initial spin direction $\mathbf{s}_\Sigma$ or the final spin direction of Λ^0. The detailed form of D_Λ' is somewhat complicated, and is given by

$$D_\Lambda'=\boldsymbol{\sigma}_\Lambda\cdot\mathbf{S}_\Lambda', \tag{93}$$

[19] The rate for this decay has been analyzed by G. Feinberg, Phys. Rev. **109**, 1019 (1958); G. Feldman and T. Fulton, Nucl. Phys. **8**, 106 (1958).

where

$$\mathbf{S}_\Lambda'=\mathbf{s}_\Sigma\{GG^*(1+\hat{k}_+\cdot\hat{k}_-)+2FF^*[p^2(\hat{k}_+\cdot\hat{k}_-)-(\mathbf{p}\cdot\hat{k}_+)(\mathbf{p}\cdot\hat{k}_-)]\} +\mathbf{p}\{-(GF^*+FG^*)[\mathbf{s}_\Sigma\cdot(\hat{k}_++\hat{k}_-)]+2FF^*[-(\mathbf{s}_\Sigma\cdot\mathbf{p})(1+\hat{k}_+\cdot\hat{k}_-)+(\mathbf{p}\cdot\hat{k}_+)(\mathbf{s}_\Sigma\cdot\hat{k}_-)+(\mathbf{p}\cdot\hat{k}_-)(\mathbf{s}_\Sigma\cdot\hat{k}_+)]\} +\hat{k}_+\{(GF^*+FG^*)(\mathbf{p}\cdot\mathbf{s}_\Sigma)+2FF^*[-p^2(\mathbf{s}_\Sigma\cdot\hat{k}_-)+(\mathbf{p}\cdot\hat{k}_-)(\mathbf{p}\cdot\mathbf{s}_\Sigma)]\} +\hat{k}_-\{(GF^*+FG^*)(\mathbf{p}\cdot\mathbf{s}_\Sigma)+2FF^*[-p^2(\mathbf{s}_\Sigma\cdot\hat{k}_+)+(\mathbf{p}\cdot\hat{k}_+)(\mathbf{p}\cdot\mathbf{s}_\Sigma)]\}. \quad (94)$$

Combining Eqs. (92) and (94), we may write

$$D_\Lambda=T+\boldsymbol{\sigma}_\Lambda\cdot\mathbf{S}_\Lambda, \quad (95)$$

where

$$\mathbf{S}_\Lambda=i(GF^*-FG^*)p\mathbf{N}+\mathbf{S}_\Lambda' \quad (96)$$

and

$$T=\tfrac{1}{2}\,\mathrm{tr}D_\Lambda, \quad (97)$$

which is related to the decay rate by

$$\mathrm{Rate}(\Sigma_0\to\Lambda^0+e^++e^-) =\alpha^2\int[4\pi^3E(m_\Lambda+E)]^{-1}(q^2)^{-2}T \times d^3k_+d^3k_-\delta(k_++k_-+E-m_\Sigma). \quad (98)$$

In the above, all momenta are measured in the rest system of the Σ^0. Let us first choose the z axis to be parallel to $\mathbf{p}$, and then make a Lorentz transformation in the (z,t) subspace to a particular rest system of Λ^0. In this system, the spin direction of Λ^0 is parallel to $\mathbf{S}_\Lambda$ and its magnitude is $|T^{-1}\mathbf{S}_\Lambda|$.

In this decay, the range of $|q^2|$ is $\leqq(m_\Sigma-m_\Lambda)^2$; thus, we may expand G and F as power series in q^2. By using

$$q^2=2Em_\Sigma-(m_\Sigma^2+m_\Lambda^2) \quad (99)$$

and Eqs. (86) and (89), we find $G(q^2=0)=0$. For small q^2, G and F are given by

$$F=F_0+O[q^2] \quad (100)$$

and

$$G=(dG/dq^2)_0q^2+O[(q^2)^2]. \quad (101)$$

Substituting these expressions into Eq. (92), we find that if $\mathbf{s}_\Sigma=0$, the approximate amount of Λ^0 polarization along $\mathbf{N}$ is given by

$$2\sin\phi\left|F_0^{-1}\left(\frac{dG}{dq^2}\right)_0\right|\times\frac{k_+k_-(k_+-k_-)\sin\theta}{k_+^2+k_-^2-k_+k_-(1-\cos\theta)}, \quad (102)$$

where $k_\pm=|\mathbf{k}_\pm|$, θ is the angle between $\mathbf{k}_+$ and $\mathbf{k}_-$,

$$\cos\theta=(2k_+k_-)^{-1} \times[(m_\Sigma^2-m_\Lambda^2)-2m_\Sigma(k_++k_-)+2k_+k_-], \quad (103)$$

ϕ is the relative phase between the two parameters $(dG/dq^2)_0$ and F_0, and ϕ is 0 or π if T invariance holds. The dimension of $|(dG/dq^2)_0/F_0|$ is the same as m_Σ^{-1}. Assuming that H_γ strongly violates T invariance, and using only simple dimensional considerations, we may expect an over-all average of $\langle\mathbf{N}\cdot\boldsymbol{\sigma}_\Lambda\rangle$ to be about

$$\frac{m_\Sigma(m_\Sigma-m_\Lambda)\langle r^2\rangle}{\ln[(m_\Sigma-m_\Lambda)^2/q_{\min}^2]}, \quad (104)$$

where $q_{\min}^2$ is the minimum value of q^2 among the events and we have taken $(dG/dq^2)_0\sim\frac{1}{6}iF_0\langle r^2\rangle(m_\Sigma+m_\Lambda)$. Clearly, it is more favorable to take events with relatively large q^2, but this must be balanced against the small number of such events.

VI. COMPARISON BETWEEN BRANCHING RATIOS OF PARTICLES AND ANTIPARTICLES

We consider first the weak decays, and assume that H_{wk} conserves CP, but H_γ can have strong violations of C and T. Thus, the partial weak-decay rates of any particle are the same as that of its antiparticle, provided H_γ is neglected. For example, the equality

$$\mathrm{Rate}(\Sigma^+\to n+\pi^+)=\mathrm{Rate}(\bar{\Sigma}^-\to\bar{n}+\pi^-) \quad (105)$$

holds to $O(\alpha)$. Violation of the equality, Eq. (105), depends not only on H_γ being noninvariant under C and T, but also on the strong interactions in the final states. A more sensitive test is to compare the partial radiative decay rates. If H_γ violates C, T invariance strongly, then the radiative decay rate of a particle may be quite different from that of its antiparticle, provided the radiation is not just the inner-bremsstrahlung of the lepton currents. The observation of, say,

$$\mathrm{Rate}(\Sigma^+\to n+\pi^++\gamma)\neq\mathrm{Rate}(\bar{\Sigma}^-\to\bar{n}+\pi^-+\gamma) \quad (106)$$

is a conclusive proof of CP violation.

These considerations can be applied to any weak decays, and the particular case of K-meson decays has already been studied in Sec. III.3.

Next, we make a few brief remarks concerning strong reactions. The strong interaction H_{st} conserves C, P, T separately. Since H_γ may be strongly noninvariant under C, T, the differential cross section of

$$A+B\to\gamma+C+D+\cdots \quad (107)$$

can be very different from that of its C-conjugate process

$$\bar{A}+\bar{B}\to\gamma+\bar{C}+\bar{D}+\cdots, \quad (108)$$

where $A, B, C, D\cdots$ are any strongly interacting particle states and $\bar{A}$, $\bar{B}$, $\cdots$ are the corresponding antiparticle states. The most convenient case is probably the annihilation of p by $\bar{p}$; i.e.,

$$A=p \quad \text{and} \quad B=\bar{p}. \tag{109}$$

Another possible test is to compare the branching ratios, or detailed distributions, of the radiative decays of any resonance with that of its C-conjugate state. If H_γ violates C, T invariance, then the distribution and branching ratio of, say,

$$N^{*0} \rightarrow p+\pi^-+\gamma \tag{110}$$

may be different from that of

$$\bar{N}^{*0} \rightarrow \bar{p}+\pi^++\gamma. \tag{111}$$

From CPT invariance, such a difference can occur only if the strong final-state interaction is not neglected.

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Antiproton Annihilation in Hydrogen at Rest. I. Reaction $\bar{p}+p \rightarrow K+\bar{K}+\pi$ *

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In a study of 735 000 antiproton annihilations at rest in the hydrogen bubble chamber, 182 examples of the reaction $K_1K_1\pi^0$ and 851 examples of the reaction $K_1K^\pm\pi^\mp$ were recorded. The distributions in the internal variables of these reactions are presented. A substantial fraction of the latter reaction proceeds through an intermediate K^* state; $\bar{p}+p \rightarrow K+K^*$. The theory of the interference effects in this reaction is presented and compared with the experimental result. It is concluded that the KK^* annihilation proceeds dominantly from the ^3S, $I=1$ state of the $\bar{N}N$ system. The fraction of $\bar{p}p$ annihilations into KK^* is given as $f_{KK^*}=(2.1\pm0.3)\times10^{-3}$.

I. INTRODUCTION

IN the domain of strong-interaction physics, antinucleon-nucleon annihilation has several features which may lead one to hope that the study of these reactions may provide useful information. In particular, the nucleon number is zero, the capture at rest is known to proceed dominantly (~99%) from an S state; hence the parity is negative and the charge-conjugation eigenvalue is linked to the angular momentum [$C(^3S)=-1$, $C(^1S)=+1$]. Nevertheless, there is not a large amount of published experimental information on the details of the various capture channels, nor has there been a great deal of theoretical interest.

This paper is the first of several contemplated articles in which we intend to present the results of a study pursued by the Columbia-Rutgers groups on $\bar{p}$ annihilations in a liquid-hydrogen chamber, using an exposure at the Brookhaven National Laboratory AGS machine.

We present the results on the $K\bar{K}\pi$ annihilations. The following reactions are possible:

$$\bar{p}+p \rightarrow K^0+\bar{K}^0+\pi^0, \tag{1}$$

$$\bar{p}+p \rightarrow K^0+K^-+\pi^+, \tag{2}$$

$$\bar{p}+p \rightarrow K^++\bar{K}^0+\pi^-. \tag{3}$$

Reaction (1) can be further separated on the basis of the decay of the kaon:

$$K_1^0+K_1^0+\pi^0, \tag{1a}$$

$$K_2^0+K_2^0+\pi^0, \tag{1a'}$$

$$K_1^0+K_2^0+\pi^0. \tag{1b}$$

Reaction (1a′) is not observed because of the long K_2 lifetime, but is presumably identical with reaction (1a). Reaction (1b) is, in general, inaccessible to hydrogen-bubble-chamber study because there are two missing

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