unambiguous evidence of departure from Curie's law. The theoretical results show that deviations from Curie's law are proportional to the difference between the virial coefficient calculated for He^3 and the virial coefficient calculated for a hypothetical He^3 with zero spin; therefore, the relatively small observed deviations from Curie's law are experimental evidence of a relatively small difference between the two calculated virial coefficients. For both quantities the standard of comparison is the corresponding quantity calculated for a perfect gas. From the theoretical results it is concluded that much lower temperatures ($T<0.002$°K) are required to assure that the perfect-Fermi-gas theory gives a good approximation to the deviations from Curie's law. Moreover, because of the limited range of vapor density any deviations from Curie's law are expected to be less than 1% in He^3 vapor at any temperature and density whenever only binary collisions need to be considered.

The theoretical results presented in this paper are in agreement with the results of the previous measurements[4] made at 3°K in this laboratory. The discrepancy between Romer's results[3] and our results remains unexplained.

ACKNOWLEDGMENT

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Angular-Momentum Experiments with Liquid Helium*

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Angular-momentum measurements on helium I and helium II contained in a cylindrical vessel suspended from a virtually frictionless magnetic bearing are reported. Helium I behaves as a classical viscous fluid but the results for helium II are quite different. The latter shows two distinct types of behavior, either assuming an equilibrium state of solid-body-type rotation or else forming a metastable state wherein the liquid refuses to be excited into rotation. Both types of behavior are shown to be in accordance with predictions of the Onsager-Feynman hypothesis of quantized circulation in the superfluid. Data are presented which seem to imply that macroscopic turbulence already present in the superfluid is a necessary condition for the generation of quantized vortex lines in this type of geometry.

INTRODUCTION

LIQUID helium, according to current theory, consists of a superfluid phase of zero viscosity and entropy plus excited states (phonons and rotons) at all temperatures greater than 0°K and less than about 2.19°K. These quantum excitations possess both entropy and viscosity and are called the "normal" fluid component. Formally their density can be represented by a number ρ_n and, if the ordinary density of the liquid be ρ, the density of the unexcited (superfluid) states is $\rho_s=\rho-\rho_n$. The ratio ρ_n/ρ is a function of the temperature in the above range.

The ρ_s component, in addition to having zero viscosity and entropy, is thought to satisfy another criterion, namely, $\nabla\times\mathbf{v}_s=0$, where $\mathbf{v}_s$ is the flow velocity of the ρ_s fraction. This condition implies some interesting consequences. For a cylindrical geometry, the only solutions with cylindrical symmetry are

$$v_s=0 \quad \text{and} \quad v_s=A/r,$$

where A is a constant and r is the radial distance in the cylindrical vessel. In a simply connected geometry the only possibility for the second type of velocity field is a vortex with a hollow core in the region $r=0$. If pure superfluid were placed in a cylindrical vessel (bucket) and rotated about its vertical axis of symmetry, then either the superfluid would not be excited into rotation or it would form a hollow-core vortex. The free energy of the vortex mode, however, is rather high, so the first alternative would appear to be the preferred one.

The experiment when tried some years ago[1–3] failed to confirm this expectation. The superfluid either rotated as a solid body or, less commonly, produced a hollow-core vortex.

The reasons for this behavior became clearer when Onsager[4] and, independently, Feynman[5] showed that circulation in the superfluid should be quantized in units of $h/m\cong10^{-3}$ cm²/sec. ($h=$Planck's constant,

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† Part of this work was submitted by JDR to the Yale University Graduate School in partial fulfillment of the requirements for ments for the Ph.D. degree.

[1] D. V. Osborne, Proc. Phys. Soc. (London) **63**, 909 (1950).
[2] E. Andronikashvili and I. Kaverkin, Zh. Eksperim. i Teor Fiz. **28**, 126 (1955) [English transl.: Soviet Phys.—JETP **1**, 174 (1955)].
[3] R. Meservey, Phys. Rev. **133**, A1471 (1964).
[4] L. Onsager, Nuovo Cimento **6**, Suppl. 2, 249 (1949).
[5] R. P. Feynman, *Progress in Low Temperature Physics* (Interscience Publishers, Inc., New York, 1955), Vol. I. Chap. 2.

m = mass of the He^4 atom). A free-energy calculation based on this model[6] indicates that, in rotation experiments of the type described above, the macroscopic velocity field should be of the solid-body type. This result is produced by a spatially uniform distribution of quantized vortex lines, the number density being $2\omega/(h/m)$ per unit area of cross section, where ω is the angular velocity of the container. These lines which form part of the superfluid component rotate with the same angular velocity as the fluid as a whole. Since the core radius of each line is of the order of a few angstroms, the fine structure of the velocity field would not have been observed in the early experiments. Some recent experiments by Rayfield and Reif[7] have shown, in a particularly unambiguous way, that quanta of circulation h/m do in fact exist in superfluid helium and this work, therefore, lends very strong support to the correctness of the Onsager-Feynman hypothesis.

The mechanism via which the lines are produced in liquid helium is not, at the moment, entirely clear. In the Rayfield-Reif experiment, positive and negative ions were produced in the helium by means of alpha-particle bombardment. Vortex lines, of the smoke-ring type, were created somehow and attached themselves to these ions. In the case of rotating helium, where the lines are parallel to the axis of rotation, the creation mechanism is equally obscure. As the experiments reported here tend to show, it appears likely that macroscopic turbulence already present in the superfluid plays a key role in their creation.

EXPERIMENTAL METHOD

In principle, the best way to explore the macroscopic velocity field of a rotating fluid is to measure its angular momentum. But this presents some special difficulties when we are dealing with a light liquid, such as helium, at a very low temperature.

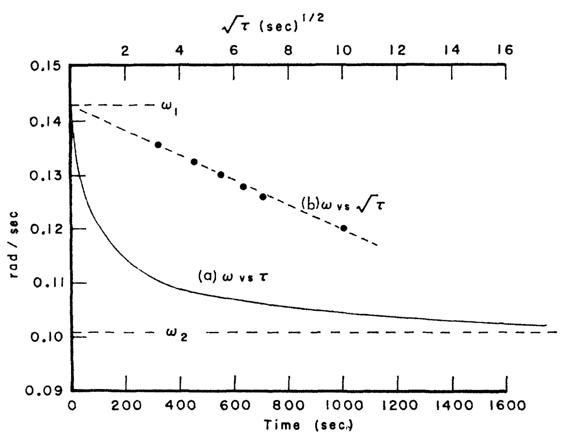

FIG. 1. Helium I at 2.54°K. The lower curve (a) shows the angular velocity of the bucket versus the time. In the upper curve (b), the angular velocity of the bucket is plotted against the square root of the time.

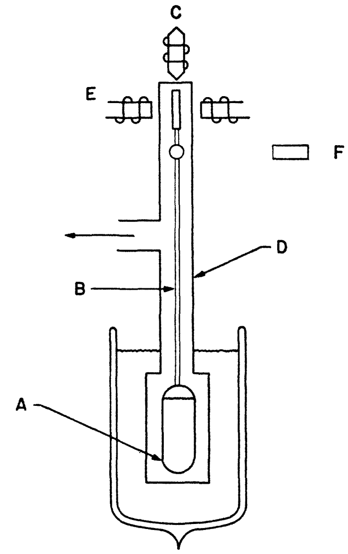

FIG. 2. Schematic drawing of the experimental apparatus.

The present experiment was designed to provide information on the angular momentum and acceleration of liquid helium in a rotating vessel. The method used is conceptually simple. A technique was devised by which a cylindrical bucket of liquid helium, with vertical axis, is supported in such a manner that it can rotate about this axis with negligible friction. Starting with the container and liquid at rest, the container is given an angular-momentum impulse ΔL. Since there is no frictional torque, angular momentum is conserved in the system, and we have: $\Delta L = I_B\omega + L$, where I_B is the moment of inertia of the bucket plus suspension, ω the angular velocity of the bucket, and L the angular momentum absorbed by the liquid. If the liquid is initially at rest, then $\Delta L = I_B\omega_1$, where ω_1 is the value of ω immediately after the impulse is applied. Then the quantity of interest L is given by

$$L = I_B(\omega_1 - \omega).$$

Thus after the initial impulse, the liquid and bucket interact and we observe a decrease in the angular velocity of the latter as a function of time until a steady, time-independent value for ω is finally achieved. Another quantity of interest, the torque necessary to accelerate the liquid, is immediately available from the time derivative of the above expression.

In the case of a classical viscous fluid it is well known that the final equilibrium state will be one in which the fluid is moving with solid-body rotation at the same angular speed as the container. This fact permits us to test the method by means of an experiment with He I. Such a calibrating run is shown in Fig. 1, curve (a), made at a temperature of 2.54°K, well above the λ point.

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[6] H. E. Hall, Phil. Mag. Suppl. **9**, 89 (1960).

[7] G. Rayfield and F. Reif, Phys. Rev. Letters **11**, 305 (1963); Phys. Rev. **136**, A1194 (1964).

When the liquid is rotating as a solid body, it is relevant to introduce the moment of inertia I_{SB} defined by $L_{SB}=I_{SB}\omega_2$ where ω_2 is the final steady-state value of the angular velocity ω. It follows, therefore, that

$$I_{SB}=I_B(\omega_1-\omega_2)/\omega_2.$$

Since I_{SB} can be computed from the dimensions of the bucket and the density of the liquid and I_B obtained from a separate calibration of the bucket and suspension, we confirm that, in the case of He I, the final state is indeed solid-body motion.

DESCRIPTION OF THE APPARATUS

The apparatus developed to perform this experiment is shown schematically in Fig. 2. The liquid helium is contained in a thin-walled cylindrical glass vessel (bucket) A, of typical dimensions, 14 cm long by 2.5 cm diameter. Sealed into the bucket is a stainless steel tube (0.032-in. o.d. with 0.002-in. wall), approximately 1 m long (B). This tube terminates in a special probe through which helium can be condensed into the bucket. The tube and bucket are connected by a flexible link to a soft iron armature which, in turn, forms part of a Beams-type magnetic suspension.[8] This consists of a support coil (C) with an iron core, plus a position-sensing coil wound on the lower end of the support-coil core.

The position-sensing element forms the grid coil of a 5 Mc/sec tuned-plate, tuned-grid oscillator. When the oscillator is tuned for marginal oscillation, the amplitude of oscillation is very sensitive to the position of the armature. If the armature approaches the sensing coil, the Q of the circuit is reduced, and the amplitude of oscillation decreases. This change in amplitude is detected, passed through one stage of dc amplification, and then applied to the control grids of six CD6 tubes which regulate the current through the support coil.

The bucket and suspension are enclosed in a vacuum-tight housing (D) which can be pumped to a high vacuum (2×10^{-6} mm Hg), or, alternatively, can be filled with He exchange gas at a low pressure.

A magnetic support of this type is free to rotate about the vertical axis with very little friction. With our apparatus, as finally developed, the parasitic drag on the rotating system is less than 5×10^{-6} dyne cm.

In order to accelerate the bucket system to the desired initial speed of rotation, a set of external coils, E, forms with the iron armature, an induction motor. The range of acceleration can be varied from 10^{-4} to 1 rad/sec.[2] Exchange gas is used to provide thermal contact between the external helium bath and the bucket. An eddy-current brake prevents the bucket from rotating during the period required for thermal equilibrium.

[8] J. W. Beams, J. D. Ross and J. F. Dillon, Rev. Sci. Instr. 22, 77 (1951).

The angular velocity of the bucket is measured by a system of mirrors and photocells (F). During rotation, each of 12 mirrors mounted on the suspension reflects the image of an illuminated slit, first into one photocell, then into another. The time lapse between these two events is proportional to the period of rotation. A Hewlett-Packard 405B counter measures the time interval between signals and presents it through a Hewlett-Packard 560A digital recorder. The recorder produces a printed record and a voltage analog of the measured time interval. The analog is displayed on a Brown strip-chart recorder. The strip-chart record is collated with the printed record during data reduction in order to provide a time base.

For this experiment 1% accuracy in determination of the angular speed is quite sufficient. This accuracy is easily achieved by the above method. With 12 mirrors the speed is sampled 12 times during each revolution. This frequency of sampling is desirable since the period of rotation may be more than 100 sec.

EXPERIMENTAL PROCEDURE

The usual practice is to precool overnight and fill the helium bath the next morning. After filling, the temperature of the bath is reduced to near the λ point, and about 10-mμ Hg pressure of He exchange gas is introduced. The bucket is filled with liquid helium to the desired height by condensation of pure helium gas. The filling rate is approximately 20 cc of liquid per hour. When the liquid level in the bucket has reached the desired height, condensation is stopped, the fill line connected to an oil manometer, and the temperature of the liquid in the bucket is determined. The temperature of the bucket gradually falls to near that of the bath, generally about 0.1°K higher. For most of the measurements the bath is kept at 1.1°K so that the bucket temperature will be about 1.2°K. Several hours are required for the bucket to cool down to this temperature. The level of the liquid in the bucket is measured with a cathetometer.

Once the bucket temperature has reached the desired value, the suspension is retracted from the filling connection, sealed off, and suspended from the magnetic bearing. During this operation a magnetic clamp prevents any unwanted rotation of the bucket. There now ensues a waiting period during which pendulum motions, which are unavoidably introduced during handling, are slowly damped out by the exchange gas. When these oscillations have died away sufficiently, the exchange gas is pumped out and the system is ready for a measurement.

When the exchange-gas pressure reaches 5×10^{-6} mm Hg the brake is released and the drive pulse is initiated. Its duration may be adjusted from 0.1 to 1.0 sec. Once the drive pulse ends, the system rotates freely and its angular speed is recorded twelve times during each revolution. The length of time during which data is recorded depends on the behavior of the liquid in the particular run and may vary from 10^3 to 10^4 sec.

When sufficient data has been taken, the magnetic brake is turned on, stopping the suspension, and about 25 μ of exchange gas is introduced to re-establish thermal contact with the bath. The suspension is usually clamped for at least 3000 sec to allow the liquid to come to rest before starting another run. The bucket may be connected to the manometer at this point for temperature measurement.

The quantities which must be determined by calibration are I_B and I_{SB}. The first is obtained from a torsional oscillation experiment in which the bucket and suspension are hung from a fiber of known torsion constant. The angular momentum of the liquid is then found from the data. In order to compare this with solid-body motion at ω_2, the radius of gyration of the bucket void must be found. This can be obtained either by calculation or from a torsion-pendulum experiment with the bucket filled with a highly viscous fluid such as glycerine.

RESULTS AND DISCUSSION

The experimental procedure used below the λ point is the same as that used above. The normal fluid behaves classically and will rotate as a solid body once equilibrium is reached. The angular momentum carried by the normal component is $(\rho_n/\rho)I_{SB}\omega_2$ where ω_2 is the equilibrium angular velocity of the bucket. Since we are mainly interested in the angular momentum absorbed by the superfluid, most runs were made at 1.2°K where ρ_n/ρ is about 3%.

Returning to Fig. 1 curve (a), it is apparent that the initial torque exerted by the fluid on the bucket in the case of He I is large, and this makes the precise determination of ω_1 somewhat difficult. Generally speaking, the same quantity in the case of He II is very much smaller. In the classical case, and in the initial stages of the bucket's deceleration where only the liquid close to the wall is in motion, we may treat the problem by approximating the cylindrical wall of the bucket by an infinite plane. Under these conditions, the solution of the Navier-Stokes equation gives

$$\omega \cong \omega_1[1-2k(\nu t/\pi)^{1/2}],$$

where ν is the kinematic viscosity and $k=4I_{SB}/R^2I_B$. By plotting ω versus t we do, in fact, obtain a straight line [Fig. 1(b)] which, by extrapolation, is useful in obtaining an accurate value for ω_1. In addition, by using the slope of the line plotted in Fig. 1 curve (b) plus the measured radius R of the bucket and the known liquid density, a value for the viscosity of He I can be found. This yields a value of 33 μpoise and is comparable with the value of 36.5 μpoise obtained by Dash and Taylor[9] at the same temperature.

[9] J. G. Dash and R. D. Taylor, Phys. Rev. **105**, 7 (1957).

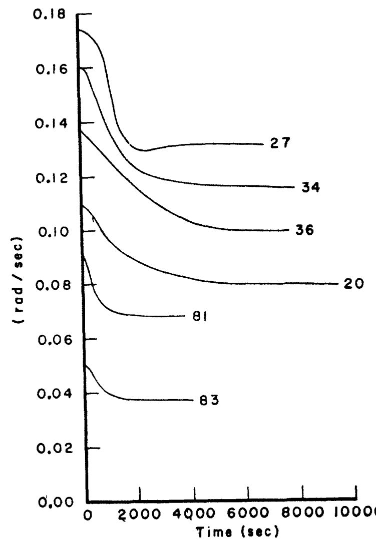

FIG. 3. Helium II at 1.2°K, where the normal-fluid component is approximately 3%. Six runs showing the angular velocity of the bucket as a function of time. Attached to each curve is our serial number designating the run. In runs 81 and 83 macroscopic turbulence was induced in the fluid (see text).

For the runs made below the λ point, the superfluid behavior falls into two categories. In the first of these, the superfluid interacts with the bucket and absorbs angular momentum; whereas in the second it fails to absorb angular momentum and remains at rest in the laboratory frame throughout part or all of the experiment. A number of runs of the first type are shown in Fig. 3. A striking feature of these is the wide variation in the interaction process from run to run. The total angular momentum absorbed by the fluid depends only on the initial and equilibrium values of the angular velocity. The ratio of the angular momentum absorbed, L to L_{SB}, for a number of different runs is recorded in Table I. The value of L/L_{SB} is equal to one, within experimental error, even for runs at the lowest speeds. This result is just what is expected on the basis of the Onsager-Feynman theory.

We have employed another method to verify that the superfluid does indeed possess solid-body angular momentum. Once the superfluid has absorbed angular

TABLE I. The ratio of the angular momentum L of He II to that for solid-body motion L_{SB} at various temperatures and initial bucket velocities (ω_1). The calibration factor $I_B/I_{SB}=2.62$.

Run	°K	ω_1 (rad/sec)	ω_2 (rad/sec)	L/L_{SB}
116	2.03	0.233	0.170	0.97
122	1.82	0.230	0.167	0.99
109	1.40	0.223	0.160	1.02
119	1.37	0.219	0.159	0.99
117	2.17	0.217	0.158	0.99
121	1.70	0.206	0.151	0.96
108	1.53	0.180	0.131	1.00
105	2.10	0.175	0.129	0.93
106	2.03	0.175	0.127	0.99
34	1.20	0.161	0.115	1.06
36	1.20	0.138	0.100	1.00
67	1.20	0.054	0.039	1.05
83	1.20	0.050	0.036	1.00

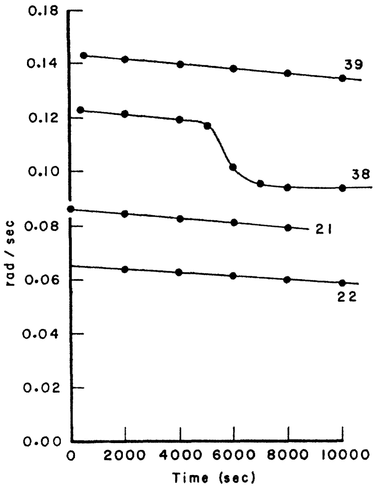

FIG. 4. Helium II at 1.2°K. The angular velocity of the bucket plotted against time showing a series of "non-interacting" runs.

momentum from the bucket and the equilibrium speed is reached, the temperature of the liquid is raised above the λ point by shining a light (radiant heat) on the bucket. Since this method exerts no torque on the system, conservation of angular momentum holds. Passage through the λ point is marked by the formation of bubbles in the liquid, and it is necessary to wait for these transient effects to die away. After this waiting period, the liquid He I will be rotating as a solid body. Any change in speed of the bucket would imply that the liquid below the λ point did not possess solid-body angular momentum. Only a few measurements of this type were made, since they were very expensive in time. No significant change in speed was observed in this type of run.

A series of runs of the second category are shown in Fig. 4. No apparent difference in the experimental procedure occurred for the runs shown in Figs. 3 and 4. In other words, the different behavior recorded in Fig. 4 cannot be attributed to any conscious act of the experimenter; this special behavior occurred at random and comparatively infrequently.

These runs show a nearly constant bucket speed over the entire period of observation. It will be noted, however, that there is, in all runs, a small but steady decline in the bucket angular velocity. We believe that this effect can be explained almost entirely by a small unavoidable heat leak into the bucket. This results in the formation of normal fluid which interacts with the container, slowing it down. The magnitude of the heat leak has been estimated and within experimental uncertainty can account for the entire change in speed.

Since this small change in bucket speed may be accounted for by the creation of normal fluid, then we may conclude that the superfluid, initially at rest, remains at rest for the duration of the run, a period of nearly three hours. We believe this implies that no vortex lines were generated in the superfluid during the course of the experiment. The assumption that there is no interaction between the superfluid component and solid objects forms the basis for the Andronikashivili experiment which measures the ratio ρ_n/ρ. In the majority of our experiments, however, such an interaction did, in fact, occur and it is necessary to inquire as to the possible mechanism responsible for this. It has been suggested[10] that forces, of the Magnus type, may exist between rotons and phonons on the one hand and the vortex lines on the other when the former are in relative motion to the latter. Since the normal fluid (rotons and phonons) always exerts forces on solid objects, it appears plausible that this may provide the required mechanism. According to this view, then, a lack of interaction at finite temperatures could arise only because there were no vortices present in the superfluid.

It is known however, that "persistent currents" can be induced in He II.[11,12] These might be caused by purely potential flow (curl $\mathbf{v}=0$), but in the experiments to date this appears unlikely. If these current rings were composed of vortices, then, according to the above views, it would be necessary for such vortices to be at rest with respect to the normal component which is always present at finite temperatures.

The persistent-current experiments involve geometries different from that employed here. The buckets contained various solid materials so that the helium was confined to narrow places. After a persistent current was formed, the bucket was stopped and the current converted to normal fluid by radiant heating and then the angular momentum acquired by the bucket measured. In view of the above ideas, it would clearly be necessary for the vortices and the normal fluid to be at

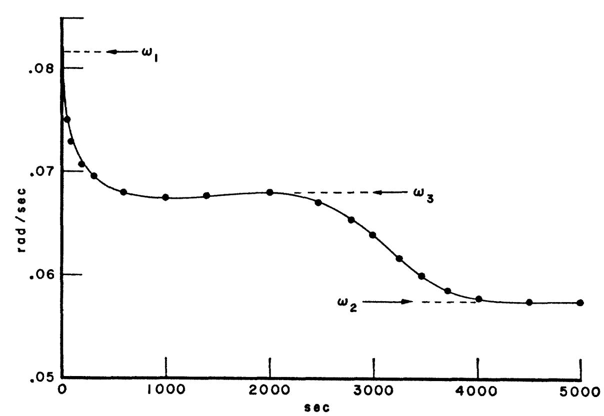

FIG. 5. Helium II at about 2°K, where approximately 50% normal fluid component is present, showing the angular velocity of the bucket as a function of time. ω_1, ω_2, and ω_3 are discussed in the text.

[10] W. F. Vinen, *Proceedings of the International School of Physics*, edited by G. Careri (Academic Press Inc., New York, 1963), Course 21, p. 336.

[11] J. D. Reppy, D. Depatie, and C. Lane, *Proceedings of the Eighth International Conference on Low Temperature Physics, 1962* (Butterworths Scientific Publications Ltd., London, 1964).

[12] J. D. Reppy and D. Depatie, Phys. Rev. Letters **12**, 187 (1964).

rest relative to each other. Due to the large surface area provided by the solid material in the bucket it seems likely that the vortices are "pinned" to protuberances in this material. Their destruction would, of course, result in an observed angular momentum communicated to the bucket.

Figure 5 presents the data for a run taken at a temperature of 2.0°K where ρ_n/ρ is about 55% and reveals a rather interesting feature. We attribute the initial sharp decay to the normal fluid component exclusively, the latter acting as a classical fluid. The superfluid component remains non-interacting until a time of approximately 2000 sec is reached at which time it suddenly interacts, acquiring the usual solid-body angular momentum. If the above ideas are correct, the data permits us to arrive at a value for ρ_n/ρ at the temperature at which the run was made (~2°K). For this situation it is easy to show that

$$\rho_n/\rho = (\omega_2/\omega_3)(\omega_1-\omega_3)/(\omega_1-\omega_2),$$

where $\omega_1 = 0.0814$ rad/sec is the initial angular velocity of the bucket, $\omega_3 = 0.0680$ rad/sec is the velocity at the upper plateau and $\omega_2 = 0.0575$ rad/sec that at the lower plateau. This yields a value of $\rho_n/\rho \cong 0.5$ which, considering the experimental uncertainties, is reasonably concordant with the established value. This behavior is thus in accordance with the above concepts; the normal fluid, of itself, is insufficient to induce interaction in the superfluid fraction.

It appears, therefore, that our non-interacting states are metastable in the sense that they are not in thermodynamic equilibrium. They are, in fact, analogous to the common phenomenon of supercooling which occurs in pure liquids. This view is reinforced by the behavior shown in run 38, Fig. 4. Here no interaction occurred for a period of some 5000 sec and then suddenly full interaction (i.e., leading to solid-body motion) took place. It is possible that this interaction was triggered by an accidental vibration of the apparatus creating macroscopic turbulence in the superfluid.

The above explanation is rendered plausible by a few runs in which we varied the usual experimental regime. Here a short length of No. 40 copper wire was rigidly attached to the inside of the bucket. The volume occupied by the wire was less than 1% of the volume occupied by the helium. During the initial acceleration period, it would be expected that the wire would stir up macroscopic turbulence in the superfluid. It was found (runs 81 and 83 in Fig. 3) that the presence of the wire invariably produced interaction between the bucket and the fluid and, in addition, the time required to reach the equilibrium solid-body rotation of the helium was much shorter. As mentioned previously, these metastable runs were relatively infrequent and not obviously subject to the will of the experimenters; in other words they occurred unexpectedly and at random. It would appear, therefore, if our above explanation is correct, that "vortex-free" helium is a rather rare commodity.

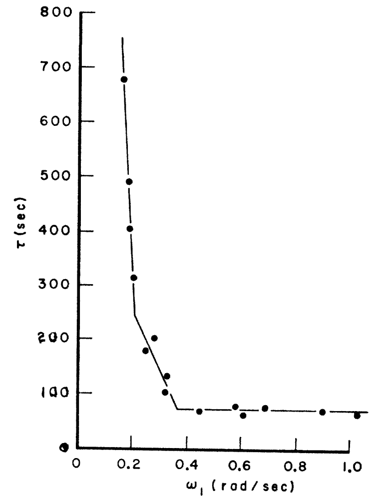

FIG. 6. Helium II at 1.2°K. The relaxation time (defined in text) versus the initial angular velocity of the bucket for those runs where interaction spontaneously takes place.

In Fig. 6 we show the relaxation time (τ) versus the initial angular speed of the bucket, the data being derived from those runs where interaction immediately and spontaneously takes place. We define τ to be the time taken for the angular momentum acquired by the liquid to equal $\frac{1}{2}$ the solid-body value. For speeds in excess of about 0.4 rad/sec (corresponding to a bucket-wall velocity of about 0.44 cm/sec) the relaxation time is constant, and sharply rising for speeds lower than this. For this particular bucket there therefore exists a "critical velocity" of somewhat less than half a cm/sec. The meaning of this would seem to be that, up to this speed, the rate at which vortices are generated in the helium is very low; thereafter an equilibrium array of lines is rapidly generated.

It has been suggested[10] that the reason for the appearance of fluid friction in the case of He II flowing in capillaries is due to the formation of vortex lines in the fluid. The energy required to create them comes from the kinetic energy of the stream, slowing the latter down, and thus giving the appearance of frictional forces. This idea leads to a well-defined critical velocity in order-of-magnitude agreement with experiment. This view is not incompatible with the above results. The precise mechanism by means of which the lines are generated, however, remains obscure. The walls of the bucket or capillary are not microscopically smooth; the increased flow around small protuberances could lead to vortex formation. Effects other than the condition of the walls must, however, come into play, as our metastable experiments show (compare runs 36 and 39). One of these appears to be the existence of macroscopic turbulence in the body of the liquid.