Conductivity of a Degenerate Electron Gas*

MACIEJ SUFFCZYNSKI†
Department of Physics, Michigan State University, East Lansing, Michigan
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The high-frequency, wave-vector-dependent conductivity of a degenerate electron gas near equilibrium is calculated by taking into account the zero-, the first-, and two of the second-order diagrams in the effective interparticle interaction. Approximate formulas are derived for the case when the frequency ω of the electromagnetic wave is high and its wave vector $\mathbf{k}$ is small, i.e., $\omega \gg k p_F/m$, where p_F is the Fermi momentum of the electron gas.

I. INTRODUCTION

THE conductivity of the electron plasma, a system of charged particles interacting through a Coulomb field, has been calculated for the electromagnetic waves whose frequencies are high compared to the electron-collision frequency and whose wavelengths are very long compared to the Bohr radius.[1–10] The wavelength-independent conductivity has been computed for the electron-ion plasma[2,4,5]; it corresponds to the situation of a spatially uniform wave in the plasma.

We now wish to give approximate formulas for the frequency- and wave-vector-dependent conductivity for the case when the frequency is high compared to the electron-collision frequency and the wavelength is sufficiently long. These coefficients are necessary to describe the weakly nonuniform waves in plasma.

The problem has been considered in various approximations. In particular Nakajima and Watabe[5] considered the approximation valid when the frequency is low and the wavelength is not too long. DuBois and Gilinsky[6] have considered the case of a high-frequency and long-wavelength wave in a hot electron gas and did focus attention on the limit of classical statistics. They have been using a modern technique not based on the Kubo formulation of the conductivity coefficients.[11] Here we want to give formulas for the leading terms of the electron-gas conductivity, both frequency- and wave-vector-dependent, for the case when the frequency is high and the wavelength is long.

II. CALCULATION OF THE CONDUCTIVITY

1. General Formulation

We start from the Kubo formula for conductivity[11,12]

$$\sigma_{\mu\nu}(\mathbf{k},\omega)=\frac{1}{V}\int_0^\infty d\tau e^{i\omega\tau} \times\int_0^\beta d\lambda\langle j_\mu(\mathbf{k},\tau-i\hbar\lambda)j_\nu(-\mathbf{k},0)\rangle, \quad (1)$$

where ω is the frequency and $\mathbf{k}$ is the wave vector of the electromagnetic wave. The space Fourier transform of the current operator is, in the Heisenberg representation,

$$j_\mu(\mathbf{k},t)=e^{iHt/\hbar}j_\mu(\mathbf{k},0)e^{-iHt/\hbar} \quad (2)$$

with H the total Hamiltonian and

$$j_\mu(\mathbf{k},0)=(e\hbar/m)\textstyle\sum_p p_\mu a_{p+k/2}^\dagger a_{p-k/2}. \quad (3)$$

The thermal average of any operator O is

$$\langle O\rangle=\mathrm{Tr}\{e^{\beta(\mu N-H)}O\}/\mathrm{Tr}\{e^{\beta(\mu N-H)}\}, \quad (4)$$

where $\beta=1/k_BT$, with k_B the Boltzmann constant and T the absolute temperature; μ is the chemical potential; and N is the number operator. Integrating (1) by parts we can write, following closely Ron and Tzoar,[4]

$$\sigma_{\mu\nu}(\mathbf{k},\omega)=\frac{i}{\omega V}\int_0^\beta d\lambda\langle j_\mu(\mathbf{k},-i\hbar\lambda)j_\nu(-\mathbf{k},0)\rangle + \frac{1}{\hbar\omega V}\int_0^\infty d\tau e^{i\omega\tau}\langle[j_\mu(\mathbf{k},\tau),j_\nu(-\mathbf{k},0)]\rangle = (ie^2n/\omega m)\delta_{\mu\nu}-(i/\omega)M_{\mu\nu}^+(\mathbf{k},\omega). \quad (5)$$

Here n is the average particle density. We write for any function $f(z)$ of the complex z

$$f^\pm(\omega)=\lim_{z\to\omega\pm i\eta} f(z) \quad \eta\to 0^+. \quad (6)$$

In order to compute the function $M_{\mu\nu}^+(\mathbf{k},\omega)$ of Eq. (5) by perturbation expansion we write first the current operator

$$j_\mu(\mathbf{k},u)=e^{uH}j_\mu(\mathbf{k},0)e^{-uH} \quad (7)$$

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† On leave of absence from Warsaw University, Warsaw, Poland.
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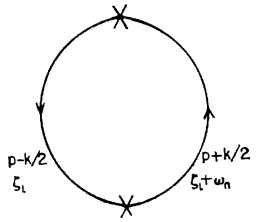

FIG. 1. The zero-order diagram for the calculation of the wave-vector-dependent conductivity.

and introduce the average of the time-ordered product

$$M_{\mu\nu}(\mathbf{k},u)=(1/V)\langle T\{j_\mu(\mathbf{k},u)j_\nu(-\mathbf{k},0)\}\rangle$$
$$=(e^2\hbar^2/m^2V)\sum_{p,p'}p_\mu p_\nu'\langle U(\beta)\rangle_0^{-1}\langle T\{a_{p+k/2}(u)$$
$$\times a_{p-k/2}(u)a_{p'-k/2}^\dagger(0)a_{p'+k/2}(0)U(\beta)\}\rangle_0. \quad (8)$$

Here $\langle\ \rangle_0$ denotes the average with respect to the eigenstate of the Hamiltonian of the noninteracting particles, and

$$U(\beta)=\exp\left\{-\int_0^\beta du\, H_I(u)\right\}. \quad (9)$$

The Coulomb interaction between the electrons has to be renormalized.[2,4] The effective interaction Hamiltonian is

$$H_I(u)=(1/2V)\sum_q U_q(u)\sum_{p,p'}a_{p+q/2}^\dagger(u)$$
$$\times a_{p'-q/2}^\dagger(u)a_{p'+q/2}(u)a_{p-q/2}(u). \quad (10)$$

The Fourier transform of the effective, screened, interaction potential

$$U_q(\alpha_m)=\int_0^\beta du\, e^{u\alpha_m}U_q(u),$$
$$\alpha_m=2\pi im/\beta,\quad m=0,\pm1,\pm2,\cdots \quad (11)$$

is given by[2,4]

$$U_q(\alpha_m)=4\pi e^2q^{-2}[1-4\pi e^2q^{-2}Q_q(\alpha_m)]^{-1}. \quad (12)$$

Here $Q_q(\alpha_m)$ is the polarization operator[13–16]

$$Q_q(\alpha_m)=\frac{1}{V}\sum_p\frac{1}{\beta}\sum_l G_{p+q/2}(\zeta_l+\alpha_m)G_{p-q/2}(\zeta_l)$$
$$=\int\frac{d^3p}{(2\pi)^3}\frac{n_{p+q/2}-n_{p-q/2}}{\epsilon_{p+q/2}-\epsilon_{p-q/2}-\alpha_m} \quad (13)$$

with

$$n_p=[e^{\beta(\epsilon_p-\mu)}+1]^{-1},\quad \epsilon_p=\hbar^2p^2/2m, \quad (14)$$

and $G_p(\zeta_l)$ is the electron propagator

$$G_p(\zeta_l)=(\zeta_l-\epsilon_p)^{-1},\quad \zeta_l=\mu+(2l+1)i\pi/\beta,$$
$$l=0,\pm1,\pm2,\cdots. \quad (15)$$

Following the standard procedure[2,4,12] one now defines $M_{\mu\nu}(\mathbf{k},z)$ as the analytical continuation of the

$$M_{\mu\nu}(\mathbf{k},\omega_n)=\int_0^\beta du\, e^{u\omega_n}M_{\mu\nu}(\mathbf{k},u) \quad (16)$$

from the infinite set of points $\omega_n=2\pi in/\beta(n>0)$ on the positive imaginary axis of z to the upper half-plane of z. This gives the last form of Eq. (5).

2. Calculation of Diagrams

The evaluation of the perturbation expansion of $M_{\mu\nu}(\mathbf{k},\omega_n)$ can be done by using the rules given by Luttinger and Ward[17] and Perel and Eliashberg.[2] In accordance with the latter authors we consider, besides a diagram of zero order in the interparticle potential (Fig. 1), the diagrams 1–5 shown in Fig. 2. We write down the contributions

$$M_{\mu\nu}(\mathbf{k},\omega_n)=\sum_{j=0}^5 M_{\mu\nu}^{(j)}(\mathbf{k},\omega_n)=(e^2\hbar^2/m^2V)\sum_{p,p'}p_\mu p_\nu'\sum_{j=0}^5 K_{pp'}^{(j)}(\mathbf{k},\omega_n), \quad (17)$$

$$K_{pp'}^{(0)}(\mathbf{k},\omega_n)=-\delta_{pp'}(1/\beta)\textstyle\sum_l G_{p-k/2}(\zeta_l)G_{p+k/2}(\zeta_l+\omega_n),$$
$$K_{pp'}^{(1)}(\mathbf{k},\omega_n)=(1/V)(1/\beta)\textstyle\sum_m U_{p-p'}(\alpha_m)(1/\beta)\sum_l G_{p-k/2}(\zeta_l)G_{p+k/2}(\zeta_l+\omega_n)G_{p'-k/2}(\zeta_l+\alpha_m)G_{p'+k/2}(\zeta_l+\alpha_m+\omega_n),$$
$$K_{pp'}^{(2)}(\mathbf{k},\omega_n)=\delta_{pp'}(1/V)\textstyle\sum_q(1/\beta)\sum_m U_q(\alpha_m)(1/\beta)\sum_l[G_{p+k/2}(\zeta_l)]^2G_{p-q+k/2}(\zeta_l-\alpha_m)G_{p-k/2}(\zeta_l-\omega_n),$$
$$K_{pp'}^{(3)}(\mathbf{k},\omega_n)=\delta_{pp'}(1/V)\textstyle\sum_q(1/\beta)\sum_m U_q(\alpha_m)(1/\beta)\sum_l[G_{p-k/2}(\zeta_l)]^2G_{p+q-k/2}(\zeta_l+\alpha_m)G_{p+k/2}(\zeta_l+\omega_n),$$
$$K_{pp'}^{(4)}(\mathbf{k},\omega_n)=(1/V^2)\textstyle\sum_q(1/\beta)\sum_m U_q(\alpha_m)U_{q-k}(\alpha_m-\omega_n)(1/\beta)\sum_l G_{p+k/2}(\zeta_l)G_{p-k/2}(\zeta_l-\omega_n)G_{p-q+k/2}(\zeta_l-\alpha_m)$$
$$\times(1/\beta)\textstyle\sum_{l'}G_{p'+k/2}(\zeta_{l'})G_{p'-k/2}(\zeta_{l'}-\omega_n)G_{p'-q+k/2}(\zeta_{l'}-\alpha_m), \quad (18)$$
$$K_{pp'}^{(5)}(\mathbf{k},\omega_n)=(1/V^2)\textstyle\sum_q(1/\beta)\sum_m U_q(\alpha_m)U_{q-k}(\alpha_m-\omega_n)(1/\beta)\sum_l G_{p+k/2}(\zeta_l)G_{p-k/2}(\zeta_l-\omega_n)G_{p-q+k/2}(\zeta_l-\alpha_m)$$
$$\times(1/\beta)\textstyle\sum_{l'}G_{p'-k/2}(\zeta_{l'})G_{p'+k/2}(\zeta_{l'}+\omega_n)G_{p'+q-k/2}(\zeta_{l'}+\alpha_m).$$

We now perform summations over l and l'.

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The matrix element of zero order in the interparticle potential is

$$M_{\mu\nu}{}^{(0)}(\mathbf{k},\omega_n)=-\frac{e^2\hbar^2}{m^2V}\sum_p p_\mu p_\nu \frac{n_{p+k/2}-n_{p-k/2}}{\epsilon_{p+k/2}-\epsilon_{p-k/2}-\omega_n}. \tag{19}$$

The longitudinal conductivity can be expressed, using Eq. (13), as[16]

$$\sigma_{11}{}^{(0)}(\mathbf{k},\omega)=\frac{ie^2n}{\omega m}-\frac{i}{\omega}M_{11}{}^{(0)+}(\mathbf{k},\omega)=\frac{i\omega}{4\pi}\frac{4\pi e^2}{k^2}Q_k{}^{+}(\hbar\omega). \tag{20}$$

The calculation of the matrix elements corresponding to the diagrams of Fig. 2 requires evaluation of the following sums:

$$M_{\mu\nu}{}^{(1)}(\mathbf{k},\omega_n)$$

$$=\frac{\hbar^2e^2}{m^2V^2}\sum_p\sum_q\frac{1}{\beta}\sum_m U_q(\alpha_m)(p_\mu p_\nu-\tfrac{1}{4}q_\mu q_\nu)\frac{1}{\epsilon_{p+q/2-k/2}-\epsilon_{p+q/2+k/2}+\omega_n}\frac{1}{\epsilon_{p-q/2+k/2}-\epsilon_{p-q/2-k/2}-\omega_n}$$

$$\times\left(\frac{n_{p+q/2-k/2}-n_{p-q/2+k/2}}{\epsilon_{p+q/2-k/2}-\epsilon_{p-q/2+k/2}+\alpha_m+\omega_n}+\frac{n_{p+q/2+k/2}-n_{p-q/2-k/2}}{\epsilon_{p+q/2+k/2}-\epsilon_{p-q/2-k/2}+\alpha_m-\omega_n}\right.$$

$$\left.-\frac{n_{p+q/2-k/2}-n_{p-q/2-k/2}}{\epsilon_{p+q/2-k/2}-\epsilon_{p-q/2-k/2}+\alpha_m}-\frac{n_{p+q/2+k/2}-n_{p-q/2+k/2}}{\epsilon_{p+q/2+k/2}-\epsilon_{p-q/2+k/2}+\alpha_m}\right),$$

$$M_{\mu\nu}{}^{(2)}(\mathbf{k},\omega_n)+M_{\mu\nu}{}^{(3)}(\mathbf{k},\omega_n)$$

$$=\frac{\hbar^2e^2}{m^2V^2}\sum_p\sum_q\frac{1}{\beta}\sum_m U_q(\alpha_m)p_\mu p_\nu\left[\frac{1}{\epsilon_{p+k/2}-\epsilon_{p-k/2}+\omega_n}\left(\frac{\partial n_{p-k/2}/\partial\epsilon_{p-k/2}}{\epsilon_{p-k/2}-\epsilon_{p+q-k/2}+\alpha_m}-\frac{n_{p-k/2}-n_{p+q-k/2}}{(\epsilon_{p-k/2}-\epsilon_{p+q-k/2}+\alpha_m)^2}\right)\right.$$

$$+\frac{1}{\epsilon_{p+k/2}-\epsilon_{p-k/2}-\omega_n}\left(\frac{\partial n_{p+k/2}/\partial\epsilon_{p+k/2}}{\epsilon_{p+k/2}-\epsilon_{p-q+k/2}-\alpha_m}-\frac{n_{p+k/2}-n_{p-q+k/2}}{(\epsilon_{p+k/2}-\epsilon_{p-q+k/2}-\alpha_m)^2}\right) \tag{21}$$

$$+\frac{1}{(\epsilon_{p+k/2}-\epsilon_{p-k/2}+\omega_n)^2}\left(\frac{n_{p+k/2}-n_{p+q-k/2}}{\epsilon_{p+k/2}-\epsilon_{p+q-k/2}+\alpha_m-\omega_n}-\frac{n_{p-k/2}-n_{p+q-k/2}}{\epsilon_{p-k/2}-\epsilon_{p+q-k/2}+\alpha_m}\right)$$

$$\left.+\frac{1}{(\epsilon_{p+k/2}-\epsilon_{p-k/2}-\omega_n)^2}\left(\frac{n_{p-k/2}-n_{p-q+k/2}}{\epsilon_{p-k/2}-\epsilon_{p-q+k/2}-\alpha_m+\omega_n}-\frac{n_{p+k/2}-n_{p-q+k/2}}{\epsilon_{p+k/2}-\epsilon_{p-q+k/2}-\alpha_m}\right)\right],$$

$$M_{\mu\nu}{}^{(4)}(\mathbf{k},\omega_n)+M_{\mu\nu}{}^{(5)}(\mathbf{k},\omega_n)$$

$$=\frac{\hbar^2e^2}{m^2V^3}\sum_{p,p'}\sum_q\frac{1}{\beta}\sum_m U_q(\alpha_m)U_{q-k}(\alpha_m-\omega_n)p_\mu p_\nu'\frac{1}{\epsilon_{p+k/2}-\epsilon_{p-k/2}-\omega_n}\left(\frac{n_{p+k/2}-n_{p-q+k/2}}{\epsilon_{p+k/2}-\epsilon_{p-q+k/2}-\alpha_m}-\frac{n_{p-k/2}-n_{p-q+k/2}}{\epsilon_{p-k/2}-\epsilon_{p-q+k/2}-\alpha_m+\omega_n}\right)$$

$$\times\left[\frac{1}{\epsilon_{p'+k/2}-\epsilon_{p'-k/2}-\omega_n}\left(\frac{n_{p'+k/2}-n_{p'-q+k/2}}{\epsilon_{p'+k/2}-\epsilon_{p'-q+k/2}-\alpha_m}-\frac{n_{p'-k/2}-n_{p'-q+k/2}}{\epsilon_{p'-k/2}-\epsilon_{p'-q+k/2}-\alpha_m+\omega_n}\right)\right.$$

$$\left.+\frac{1}{\epsilon_{p'-k/2}-\epsilon_{p'+k/2}+\omega_n}\left(\frac{n_{p'-k/2}-n_{p'+q-k/2}}{\epsilon_{p'-k/2}-\epsilon_{p'+q-k/2}+\alpha_m}-\frac{n_{p'+k/2}-n_{p'+q-k/2}}{\epsilon_{p'+k/2}-\epsilon_{p'+q-k/2}+\alpha_m-\omega_n}\right)\right].$$

For $k=0$ the matrix element of Eq. (19) vanishes and the sum of corrections given by Eqs. (21) vanishes too, as should be the case for a uniform homogeneous system. It is this cancellation of contributions from diagrams 1, 2, 3 on one hand and 4, 5 on the other hand, which led to the choice of the diagrams 4 and 5 from among the second-order diagrams.[2,4] By the same reasoning, if one took into account all the second-order diagrams, one would have also to take some from among the third-order diagrams to achieve cancellation at $k=0$.

3. Simplifying Approximation

In Eqs. (21) the integrations over $\mathbf{p}$ and $\mathbf{p}'$ are difficult (see Refs. 1, 6, 8–10, 13, 18, 19). There have been several attempts to calculate the multidenominator expressions with distribution function in the numerator, and various approximations have been tried. We will follow here the simplest approximation used by Tzoar and Klein,[1] which eliminates the multiple denominators altogether. We will approximate all the denominators whose frequency is ω_n, and which do not contain $\alpha_m+\omega_n$, by ω_n. For ω_n different from zero and $\mathbf{k}$ sufficiently small, and for any finite $\mathbf{p}$ and $\mathbf{q}$, these energy denominators can be developed into a power series in $\mathbf{k}/\omega_n$:

$$[\epsilon_{p-q+k/2}-\epsilon_{p-q-k/2}\pm\omega_n]^{-1}=[(\mathbf{p}-\mathbf{q})\mathbf{k}\hbar^2m^{-1}\pm\omega_n]^{-1}$$
$$=\pm\frac{1}{\omega_n}-\frac{(\mathbf{p}-\mathbf{q})\mathbf{k}\hbar^2}{m\omega_n^2}\pm\frac{[(\mathbf{p}-\mathbf{q})\mathbf{k}\hbar^2]^2}{m^2\omega_n^3}-\cdots. \quad (22)$$

Using this development, one drops out all the poles corresponding to zeros of these denominators. Therefore the resulting formulas will be useful only for frequency ω sufficiently high and wave number k sufficiently small. For a degenerate electron gas with the Fermi momentum p_F, the inequality $\omega\gg kp_F/m$ ensures that the contribution of the neglected poles will be small, since n_p goes to zero for large $\mathbf{p}$ and also the interparticle potential $U_q(\alpha_m)$ goes to zero for large momentum transfer $\mathbf{q}$, thus making small any contribution from regions of large $\mathbf{q}$'s in sums of Eqs. (21).

We will not write down the terms arising from the first and second powers of $\mathbf{k}$ in the development in $\mathbf{k}/\omega_n$ of the denominators exhibited in Eq. (22): these terms can be written down straightforwardly,[6] and we omit them for reasons of brevity only. Thus our final formula will be incomplete. But the enumerated omissions are our only approximation.

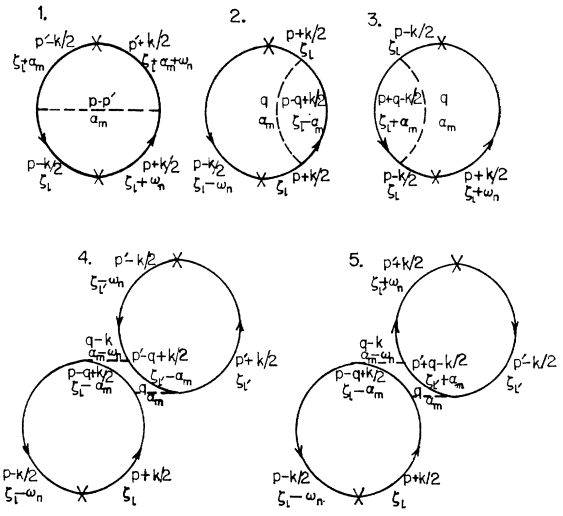

FIG. 2. The first- and second-order diagrams contributing to the high-frequency conductivity.

4. The Final Formula

In Eqs. (21) the summations over the index m can be rewritten in terms of a principal value integration following the device described by Perel and Eliashberg.[2] The sum of contributions given by Eqs. (21), in the approximation described in the preceding paragraph, can be written in the form

$$\Delta\sigma_{\mu\nu}(\mathbf{k},\omega)=\frac{1}{i\omega}\sum_{j=1}^{5}M_{\mu\nu}{}^{(j)+}(\mathbf{k},\omega)=\frac{e^2\hbar^2}{\omega^3m^2}\frac{\mathcal{P}}{2\pi}\int_{-\infty}^{+\infty}dx\tfrac{1}{2}\coth(\beta x/2)\int\frac{d^3q}{(2\pi)^3}\{q_\mu q_\nu[U_q{}^+(x)Q_q{}^+(x)-U_q{}^-(x)Q_q{}^-(x)$$
$$+U_q{}^+(x+\hbar\omega)[Q_{q+k}{}^+(x)-Q_{q+k}{}^-(x)]-[U_q{}^+(x)-U_q{}^-(x)]Q_{q+k}{}^+(x+\hbar\omega)]$$
$$+(\tfrac{1}{2}q_\mu k_\nu+\tfrac{1}{2}q_\nu k_\mu+q_\mu q_\nu)[U_{q+k}{}^+(x+\hbar\omega)Q_{q+k}{}^+(x+\hbar\omega)[U_q{}^+(x)Q_q{}^+(x)-U_q{}^-(x)Q_q{}^-(x)]$$
$$-U_q{}^+(x+\hbar\omega)Q_q{}^+(x+\hbar\omega)[U_{q+k}{}^+(x)Q_{q+k}{}^+(x)-U_{q+k}{}^-(x)Q_{q+k}{}^-(x)]]$$
$$-(q_\mu+k_\mu)(q_\nu+k_\nu)[U_{q+k}{}^+(x+\hbar\omega)[U_q{}^+(x)(Q_q{}^+(x))^2-U_q{}^-(x)(Q_q{}^-(x))^2]$$
$$-U_q{}^+(x+\hbar\omega)(Q_q{}^+(x+\hbar\omega))^2[U_{q+k}{}^+(x)-U_{q+k}{}^-(x)]]\}. \quad (23)$$

Thus in our approximation the corrections to conductivity are described in terms of quadratures over the same functions $U_q(x)$ and $Q_q(x)$ as in the case of a uniform electron-ion plasma.[2,4] In Eq. (23) the screened interaction potential U_{q+k} is not developed into powers of $\mathbf{k}$. If one approximates screening by the Thomas-Fermi screening constant, one can develop U_{q+k} into powers of $\mathbf{k}$ at every $\mathbf{q}$. The corresponding development of the function Q_{q+k} contains the first derivative which has a logarithmic, and thus integrable, discontinuity.

Evaluation of the formula (23) requires a fourfold integration which can be done numerically only.

The above formulation of the corrections due to interparticle interactions gives a first approximation for the conductivity coefficients in the case of a spatially weakly nonuniform wave in a degenerate interacting electron gas, whenever the zero-wave-vector conductivity alone

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does not suffice for description of the wave propagation. The skin effect in metals is an instance of a situation in which the formulas derived here are needed to take account of the electron interaction.

A generalization of the above calculations to more realistic and more complicated systems of interacting particles can be carried out, in principle, along the same lines, though this appears to involve considerable labor. One should first consider systems of different species of particles,[2,4] the electron system with impurities present,[8,20,21] electrons interacting with phonons,[18,22–24] and the electron propagators with the damping included in the electron self-energy.[25,26]

Concluding we can say that the equations presented here constitute an initial step in the effort to take into account spatial nonuniformities in the calculation of conductivity of an interacting electron gas.

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Ranges of C^{11} in Aluminum*

J. A. PANONTIN,† L. L. SCHWARTZ,‡ A. F. STEHNEY, E. P. STEINBERG, AND L. WINSBERG§
Argonne National Laboratory, Argonne, Illinois
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The ranges of 0.66- to 1.64-MeV C^{11} atoms in aluminum have been determined by the stacked-foil catcher technique. Monoenergetic C^{11} recoils were produced from the interaction of 3.94- to 6.96-MeV protons with thin B^{11} targets in the reaction $B^{11}(p,n)C^{11}$. The results are compared with previous data and theoretical calculations, and are in agreement with the semiempirical calculations of Northcliffe.

INTRODUCTION

THE stopping of heavy ions has recently been the subject of renewed theoretical interest.[1] The accumulation of reliable experimental data is essential for continued progress in this field. Furthermore, this information is required in the analysis of data from the recoil-range type of experiment for investigating the mechanisms of nuclear reactions.

The values of the recoil ranges of low-energy C^{11} nuclei were necessary for the analysis of an investigation of the mechanism of the $C^{12}(p,pn)C^{11}$ reaction. Although theoretical and semiempirical range-energy curves are available,[1,2] there have been no direct experimental checks of the data in the energy region of interest (0.5–1.5 MeV). Moreover, a dependence of the observed range on the crystalline nature of the absorber has been noted,[3] and it was felt advisable to calibrate the commonly used aluminum-leaf catcher foils.

In the present study, C^{11} ions of known energy were produced in the reaction $B^{11}(p,n)C^{11}$. Protons with energies between 4 and 7 MeV from a tandem Van de Graaff generator initiated the reaction, and the C^{11} ions recoiling in the forward direction were caught in thin aluminum foils. The range of the C^{11} ions was determined from the distribution of 20.5-min C^{11} activity in these foils. The energy of the recoiling C^{11} ions is readily calculable from the kinematics of the reaction.

EXPERIMENTAL PROCEDURE AND DATA

The target used in this work consisted of B^{11} evaporated onto a gold foil by means of electron bombardment. The boron deposit weighed 0.3 μg and was spread over an area of 5 cm^2. The thickness of the gold foil was 37 μin. corresponding to a surface density of 1.8 mg/cm^2. The target was supported on an aluminum frame perpendicular to the beam direction, with the boron deposit facing downstream (see Fig. 1).

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† Present address: Carnegie Institute of Technology, Pittsburgh, Pennsylvania.
‡ Present address: Lawrence Radiation Laboratory, University of California, Livermore, California.
§ Present address: University of Illinois at Chicago Circle, Chicago, Illinois.
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