does not suffice for description of the wave propagation. The skin effect in metals is an instance of a situation in which the formulas derived here are needed to take account of the electron interaction.

A generalization of the above calculations to more realistic and more complicated systems of interacting particles can be carried out, in principle, along the same lines, though this appears to involve considerable labor. One should first consider systems of different species of particles,[2,4] the electron system with impurities present,[8,20,21] electrons interacting with phonons,[18,22–24] and the electron propagators with the damping included in the electron self-energy.[25,26]

Concluding we can say that the equations presented here constitute an initial step in the effort to take into account spatial nonuniformities in the calculation of conductivity of an interacting electron gas.

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Ranges of C^{11} in Aluminum*

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The ranges of 0.66- to 1.64-MeV C^{11} atoms in aluminum have been determined by the stacked-foil catcher technique. Monoenergetic C^{11} recoils were produced from the interaction of 3.94- to 6.96-MeV protons with thin B^{11} targets in the reaction $B^{11}(p,n)C^{11}$. The results are compared with previous data and theoretical calculations, and are in agreement with the semiempirical calculations of Northcliffe.

INTRODUCTION

The stopping of heavy ions has recently been the subject of renewed theoretical interest.[1] The accumulation of reliable experimental data is essential for continued progress in this field. Furthermore, this information is required in the analysis of data from the recoil-range type of experiment for investigating the mechanisms of nuclear reactions.

The values of the recoil ranges of low-energy C^{11} nuclei were necessary for the analysis of an investigation of the mechanism of the $C^{12}(p,pn)C^{11}$ reaction. Although theoretical and semiempirical range-energy curves are available,[1,2] there have been no direct experimental checks of the data in the energy region of interest (0.5–1.5 MeV). Moreover, a dependence of the observed range on the crystalline nature of the absorber has been noted,[3] and it was felt advisable to calibrate the commonly used aluminum-leaf catcher foils.

In the present study, C^{11} ions of known energy were produced in the reaction $B^{11}(p,n)C^{11}$. Protons with energies between 4 and 7 MeV from a tandem Van de Graaff generator initiated the reaction, and the C^{11} ions recoiling in the forward direction were caught in thin aluminum foils. The range of the C^{11} ions was determined from the distribution of 20.5-min C^{11} activity in these foils. The energy of the recoiling C^{11} ions is readily calculable from the kinematics of the reaction.

EXPERIMENTAL PROCEDURE AND DATA

The target used in this work consisted of B^{11} evaporated onto a gold foil by means of electron bombardment. The boron deposit weighed 0.3 μg and was spread over an area of 5 cm^2. The thickness of the gold foil was 37 μin. corresponding to a surface density of 1.8 mg/cm^2. The target was supported on an aluminum frame perpendicular to the beam direction, with the boron deposit facing downstream (see Fig. 1).

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TABLE I. Fraction $F(t)$ of the C^{11} activity recoiling into an absorber of thickness Δt (in μg Al/cm^2).

E_p (MeV) \ Foil No.	1 $F(t)$	1 Δt	2 $F(t)$	2 Δt	3 $F(t)$	3 Δt	4 $F(t)$	4 Δt	5 $F(t)$	5 Δt	6 $F(t)$	6 Δt	7 $F(t)$	7 Δt
3.94	0.108	122	0.260	119	0.561	124	0.061	118	0.010	127				
	0.119	122	0.350	121	0.487	124	0.044	119	0.000	116				
4.44	0.086	117	0.141	122	0.511	128	0.147	156	0.051	156	0.064	~200		
4.95	0.055	151	0.186	126	0.609	170	0.059	150	0.009	160	0.082	~200		
	0.045	117	0.101	120	0.554	129	0.270	117	0.014	131	0.016	117		
5.45	0.081	127	0.206	115	0.283	122	0.319	123	0.074	156	0.037	156		
5.95	0.070	122	0.160	141	0.331	127	0.338	122	0.058	128	0.043	~200		
	0.099	112	0.072	116	0.170	117	0.410	122	0.221	118	0.028	117		
6.45	0.068	139	0.140	146	0.134	116	0.380	144	0.100	142	0.119	139	0.059	131
6.96	0.089	137	0.089	133	0.177	146	0.262	130	0.145	115	0.136	138	0.102	133
	0.097	114	0.227	148	0.157	157	0.252	156	0.099	116	0.099	137	0.069	124

The proton beam was focused to a circular spot of $\frac{1}{16}$ to $\frac{1}{8}$ in. diam. At a distance of 4 in. from the B^{11} deposit and in line with the proton beam was located a stack of aluminum foils parallel to the target foil. This stack consisted of a 21-mil aluminum foil with a $\frac{9}{16}$ in. hole cut out of the middle, followed by a stack of 7 or 8 sheets of aluminum leaf, each with a surface density of $\sim$150 μg/cm^2. The aluminum leaves were mounted on aluminum frames for support. The 21-mil aluminum foil served to collimate the C^{11} recoils since its thickness far exceeded the range of the recoiling ions. The entire assembly was inside an evacuated chamber directly coupled to the proton beam tube.

After the proton bombardment the aluminum leaves were separately counted on beta proportional counters. Since the proton beam also passed through these foils, an appreciable amount of radioactivity due to direct activation was present. It was, of course, necessary to correct for the presence of this extraneous activity. At the proton energies in this experiment the possible contaminating activities are 10-min N^{13}, 111-min F^{18}, 15-h Na^{24}, and 9.5-min Mg^{27} as well as C^{11} itself from other reactions or from boron impurity in the catcher foils.

The counting data were analyzed for 10-min, 20.5-min (C^{11}), 111-min, and 15-h components by the method of least squares.[4] The range of C^{11} was determined from the distribution of the 20.5-min component in the aluminum leaves. The presence of C^{11} activity from contaminating reactions was estimated from the activity of the foils beyond the C^{11} range and from irradiations in which the B^{11} deposit was absent. The value of the straggling parameter (Table III) was sensitive to this correction. The value of the range, on the other hand, was not changed significantly when this correction was made except at the higher bombarding energies.

Two typical examples of the recoil distribution of the C^{11} activity are shown in Fig. 2. In the cases shown here, the reactions were induced by 3.94- and 5.95-MeV protons. The ordinate $F(t)/\Delta t$ is the fraction $F(t)$ of the total activity found in a given foil divided by the thickness Δt of the foil. The abscissa is the total thickness t.

The results of all the experiments from which it was possible to extract values for the C^{11} ranges are given in Table I in terms of $F(t)$ and Δt. Duplicate experiments were performed at 3.94, 4.95, 5.95, and 6.96 MeV. The proton energies listed in the table are corrected for loss of energy in the gold foil on which the boron was deposited. Experiments were also performed with 8-, 9-,

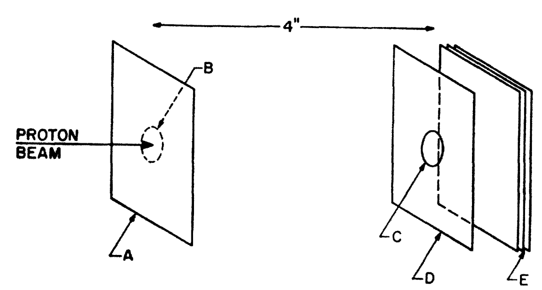

FIG. 1. Arrangement of foils for proton irradiations. A. Gold foil; B. Deposit of B^{11} on "downstream" side; C. $\frac{9}{16}$-in. diam hole in D (21-mil aluminum masking absorber); E. Stack of aluminum leaves ($\approx$150 μg Al/cm^2 per leaf).

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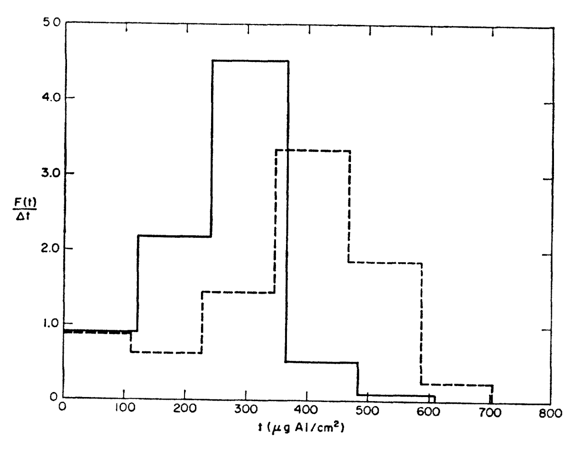

FIG. 2. The distribution of C^{11} activity from the bombardment by 4-MeV protons (solid curve) and 6-MeV protons (dashed curve). The ordinate $F(t)/\Delta t$ is the fraction $F(t)$ of the total activity found in a given foil divided by the thickness Δt of the foil. The abscissa is the total thickness t.

10-, and 11-MeV protons. However, the C^{11} component at these latter proton energies was too weak relative to the contaminating radioactivities to be determined accurately.

METHOD OF ANALYSIS AND RANGE VALUES

The recoil energy (E_C) of C^{11} from the reaction

$$p+B^{11}\rightarrow C^{11}+n \quad (1)$$

depends on the energy of the incident proton (E), on the angle of recoil of the C^{11} (θ), and on the Q value of the reaction. Near the reaction threshold, C^{11} can be produced only in its ground state. In this case, the value of Q is 2.765 MeV. As the proton energy increases, it becomes possible to produce C^{11} both in its ground state ($E_0=0$ MeV) and in its first excited state ($E_1=1.99$ MeV). In the latter case $Q=2.765\text{ MeV}+E_1$. As the proton energy increases still further, more excited states can be populated. At proton energies below 7 MeV, which include all of our successful experiments, the second and higher excited states are not formed.

By applying the laws of energy and momentum conservation, the following nonrelativistic relation is obtained:

$$(m_C+m_n)E_C-2(m_Cm_pE)^{1/2}E_C{}^{1/2}\cos\theta-(m_n-m_p)E+m_nQ=0, \quad (2)$$

where m_C is the mass of C^{11}, m_n is the mass of the neutron, and m_p is the mass of the proton. The other quantities are defined above. The angle of recoil θ is 0° at the center of the hole in the aluminum masking absorber and 2° at the edge of the hole (see Fig. 1). The variation of recoil energy (E_C) as a function of θ is slight ($\sim$0.2% over the area of this hole).

Equation (2) has two solutions for E_C for each value of θ and Q. These two values of E_C correspond to the recoil of C^{11} in the forward and backward directions in the center-of-mass system. In both cases the actual direction of recoil in the laboratory system is forward. The values of E_C, given in Table II, are averages of the values at $\theta=0°$ and $\theta=2°$, as obtained from Eq. (2). They are listed in the table as a function of bombarding energy E and the energy state, E_0 or E_1, in which the C^{11} is formed.

TABLE II. The value of the recoil energy E_C in the forward direction as a function of the proton energy E and the energy E_0 or E_1 of the C^{11} state.

E (MeV)	E_0 or E_1 (MeV)	E_C(MeV)	
3.94	0	0.081	0.666
4.44	0	0.064	0.836
4.95	0	0.054	1.003
5.45	0	0.046	1.164
	1.99	0.256	0.620
5.95	0	0.041	1.323
	1.99	0.189	0.841
6.45	0	0.036	1.481
	1.99	0.154	1.029
6.96	0	0.033	1.641
	1.99	0.131	1.209

TABLE III. Range-energy results with corresponding values of ϵ and the straggling parameter ρ.

E_C (MeV)	ϵ	R_0 (μg Al/cm^2)	ρ
0.666	66.5	258; 273	0.26; 0.23
0.836	83.4	304	0.23
1.003	100.1	328; 334	0.22; 0.22
1.164	116.2	373	0.23
1.323	132.0	391; 416	0.23[a]
1.481	147.8	451	0.23[a]
1.641	163.8	495; 499	0.23[a]

[a] Value for ρ assumed to be 0.23 for all components.

At the bombarding energies, 3.94, 4.44, and 4.95 MeV only the ground state of C^{11} is formed. In each case there was no difficulty in determining the average range and the range distribution of the more energetic recoil component, since all the activity of the less energetic component was found in the first aluminum leaf. The distribution of the activity (Fig. 2) in the remaining foils was found to be essentially Gaussian. By fitting the data to the function

$$P(R)dR=\frac{1}{R_0\rho(2\pi)^{1/2}}\exp\left[-\left[\frac{R-R_0}{\sqrt{2}R_0\rho}\right]^2\right]dR \quad (3)$$

it was possible to determine the average range R_0 and the straggling parameter ρ.[5] These values are given in Table III as a function of E_C.

At the higher bombarding energies, the presence of recoils from the production of the first excited state, in addition to the ground state, caused the distribution of activity after the first foil to deviate from a pure Gaussian. In order to extract the value of R_0 for the most energetic component, the range distributions of all components were assumed to be Gaussian with the straggling parameter $\rho=0.23$, the value obtained above. The values of R_0 of the three weakest components were determined from the portion of the range-energy curve already obtained, either directly or by extrapolation to lower energies. The final value of R_0 for the most energetic component was then obtained by a least-squares analysis[4] of the experimental data given in Table I, based on the assumed value for ρ and on the values of the ranges of the three weakest recoil components obtained as described above. These values of R_0 for the maximum recoil energy of C^{11}, given in Table III and Fig. 3, are insensitive to the precise values of the two lowest ranges that had to be obtained from the extrapolated portion of the range-energy curve and to the value of ρ.

An accurate estimate of experimental errors is not possible. A major source of error is the effect of the nonuniformity of the thin catcher foils. In order to deter-

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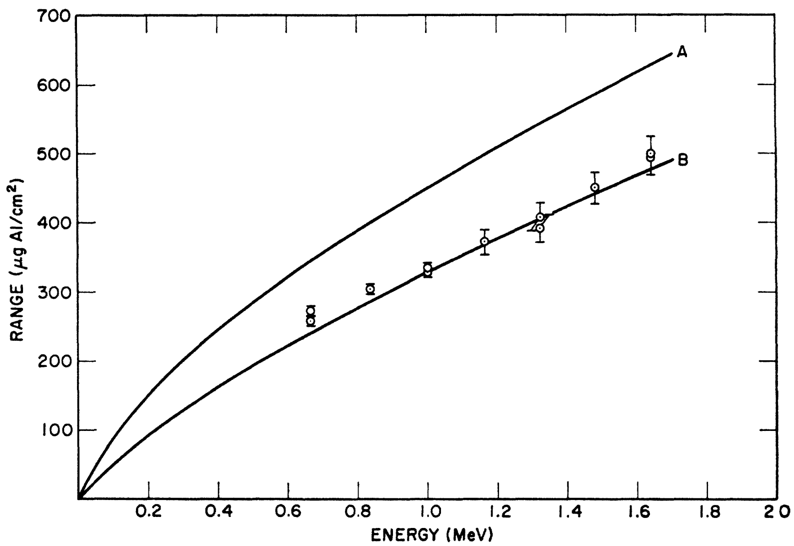

FIG. 3. Comparison of ranges of C^{11} determined in present work with theoretical and semiempirical range-energy curves. Curve A represents theoretical projected ranges from Lindhard, Scharff, and Schiott (Ref. 1); curve B, the semiempirical curve of Northcliffe (Ref. 2).

mine the amount of this error, several of these foils were cut into small pieces and weighed. The mean deviation from the mean value of the surface density was approximately 3% for a given foil. Since the recoiling atoms were caught over a large fraction of the catcher foil, there is some cancellation of errors. The error in the mean range from this source is thus estimated to be 2% or less. Additional errors occurred in the counting of the samples and in the analysis of the data. The effect of these errors on the value of the mean range is difficult to assess but appears to be of the order of several percent.

At the three lower proton energies the values of R_0 given in Table III are thus accurate to approximately 3%. At the higher proton energies the errors are probably nearer to 5% because of the presence of four recoil components.

DISCUSSION

The values of the C^{11} range as a function of energy can be compared with the theoretical predictions of Lindhard, Scharff, and Schiott.[1] These authors define the dimensionless quantities

$$\varrho = RNM_2 4\pi a^2 \frac{M_1}{(M_1+M_2)^2} \tag{4}$$

and

$$\epsilon = E_C \frac{aM_2}{Z_1 Z_2 e^2 (M_1+M_2)}, \tag{5}$$

where R and E_C are the range and recoil energy, respectively, and can be obtained from Table III; N is the number of stopping atoms per unit volume; M_1 and M_2 are the masses of the penetrating particle and stopping atoms, respectively; Z_1 and Z_2 are the respective atomic numbers; e is the electronic charge; and $a = a_0\ 0.8853(Z_1^{2/3}+Z_2^{2/3})^{-1/2}$, a_0 being the Bohr radius of the hydrogen atom.

The stopping of ions may be taken as the sum of nuclear and electronic effects, i.e.,

$$\frac{d\epsilon}{d\varrho} = \left(\frac{d\epsilon}{d\varrho}\right)_n + k\epsilon^{1/2}, \tag{6}$$

where k is a function of the atomic and mass numbers of the penetrating particle and the atoms of the stopping medium. Integration of Eq. (6) gives ϱ-versus-ϵ curves for various values of k. These curves are convenient to use for ϵ values of the order of 10 or less. For larger values of ϵ, where electronic stopping dominates, it is more convenient to correct a pure electronic range $\varrho_e = (2/k)\epsilon^{1/2}$, for the effects of nuclear collisions, $\Delta(k,\epsilon)$. Thus,

$$\varrho(\epsilon) = (2/k)\epsilon^{1/2} - \Delta(k,\epsilon)\,. \tag{7}$$

In the present experiment, ϵ values of 66–164 are applicable, and Eq. (7) was utilized with Δ values obtained from Ref. 1. The quantity measured here is the projection of the total path length on the beam direction. The values of $\varrho(\epsilon)$ were corrected to projected range values from curves given in Ref. 1 for comparison with the experimental data. This correction was the order of 10%.

The range data of Table III are compared in Fig. 3 with the theoretical projected ranges (curve A). The discrepancy may be an indication of difficulties with the theory in the energy region where electronic and nuclear stopping are comparable in magnitude. In the present case, nuclear stopping represents about 25% of the total.

It is interesting to compare the present data for C^{11} ranges in aluminum with data on fission fragment ranges in various stopping media. This is done in Fig. 4 where the quantity $\frac{1}{2}k(\varrho+\Delta)$ is plotted against ϵ. The figure is taken from Ref. 1, and presents the ranges with nuclear stopping eliminated. The straight

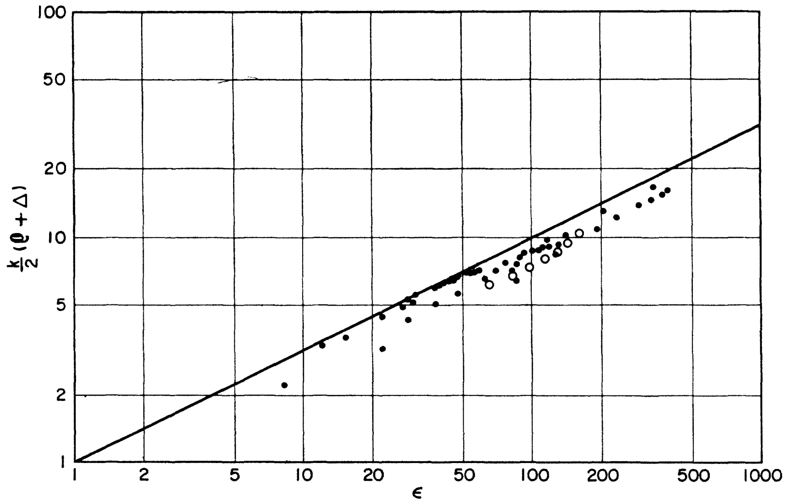

FIG. 4. Comparison of ranges of C^{11} determined in present work (open circles) with range measurements for fission fragments (solid points) with nuclear stopping eliminated. Straight line represents theoretical curve for pure electronic stopping (see Ref. 1, Fig. 14).

line represents the theoretical curve for pure electronic stopping, $\frac{1}{2}k\varrho_e=\epsilon^{1/2}$. The data of the present experiment are plotted as open circles and fit well with the fission fragment range data.

Ranges of C^{11} ions in aluminum can also be obtained from Northcliffe's integration of experimental stopping-power data for C^{12} in aluminum.[2] The energy-loss measurements of 0.36 to 3.2 MeV C^{12} ions in aluminum by Porat and Ramavataram[6] were included in this treatment. The resulting range-energy curve, corrected to C^{11}, is shown in Fig. 3 as curve B. This curve includes the corrections for nuclear stopping and projected range, and is in essential agreement with the experimental data.

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Intensity Fluctuations in GaAs Laser Emission

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The intensity fluctuations in the emission from various lasing and nonlasing modes of a cw GaAs laser have been measured. The measurements were made by two techniques: the coincidence-counting version of the Hanbury Brown–Twiss intensity interferometer, and the single-detector excess-photon-noise technique. The two independent methods give excellent quantitative agreement. The intensity noise in the single lasing mode was studied as the laser was taken continuously through the threshold region; this has permitted observation of the gradual change in the statistical nature of the photon noise which occurs at laser threshold. Observations have also been made of correlations between the intensity fluctuations in the emission from different modes of the laser. The experimental observations of intensity fluctuations and correlations and their dependences on injection current can be understood in terms of the response of single or of coupled van der Pol oscillators to random-noise excitation.

I. INTRODUCTION

THE subject of noise in laser oscillators has received considerable attention in recent years. The earliest experimental work[1] involved the determination of the linewidth of the laser output well above threshold. This width is due to random fluctuations in the *phase* of the oscillator. More recently experiments have been performed[2–5] which have detected and meas-

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