Scattering of Electromagnetic Radiation by a Nonequilibrium Electron-Phonon System

Y. C. LEE AND N. TZOAR
Bell Telephone Laboratories, Whippany, New Jersey
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The scattering of electromagnetic radiation from an electron-phonon system not in thermal equilibrium has been studied. In particular, an enhancement of the cross section at the phonon line is found when the electrons are forced to drift relative to the ions. For some semiconductors for which the phonon frequency is approximately equal to the plasma frequency, such a scattering experiment would provide information on the phonon line and the plasma line simultaneously.

I. INTRODUCTION

RECENTLY there has been an increased interest in incoherent scattering of light from solids. Such experiments can provide at the same time information about the plasma line as well as the phonon line. An interesting case would be when the phonon frequency and the plasma frequency are very close to one another, as in some semiconductors.

The scattered radiation is determined by the spectrum of the electron-density fluctuation. In a solid, when the incident wavelength is much greater than the lattice spacing, the scattering arises predominantly from the conduction electrons. The density fluctuations of these conduction electrons are coupled by the electron-phonon interaction to the motion of the ions. In general, the scattering cross section would show two resonance lines, one due to the collective oscillation of the electrons at the plasma frequency, the other due to the collective oscillation of the ions at the phonon frequency. In materials with an average electron density greater than 10^{18}/cc, as in metals, these two lines lie far apart.[1] However, in semiconductors it is possible to have the two frequencies approximately the same. In such cases the two resonance lines affect one another strongly. In an experiment, one can vary the electron plasma frequency almost continuously and can therefore study the coupling between the two lines. The cross section is typically of the order of the Thomson cross section (10^{-25} cm^2) which is very small. An enhancement of the cross section at the phonon line can be obtained either by exciting phonons mechanically or by forcing the electrons to drift relative to the ions to induce large phonon fluctuations.

We start with the expression for the scattering cross section given in terms of the Fourier transform of the density-density correlation function. This correlation function is calculated using the test-particle method.[2] It is assumed that the incident wave frequency is above the plasma frequency and that the sample is optically thin enough to be transparent.

[1] K. L. Bowles, Phys. Rev. Letters **1**, 454 (1958); J. P. Dougherty and D. T. Farley, Proc. Roy. Soc. (London) **A259**, 79 (1960); M. N. Rosenbluth and N. Rostoker, Phys. Fluids **5**, 776 (1962).

[2] N. Rostoker and M. N. Rosenbluth, Phys. Fluids **8**, 1 (1960).

II. EVALUATION OF THE CROSS SECTION

The incoherent scattering cross section can be easily obtained in the Born approximation and is given by

$$\frac{d\sigma}{d\omega d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}}\frac{1}{2\pi}S(\mathbf{k},\omega)\,, \qquad (1)$$

where

$$S(\mathbf{k},\omega)=\int_{-\infty}^{\infty}dte^{-i\omega t}\langle n_{\mathbf{k}}(t)n_{-\mathbf{k}}(0)\rangle \qquad (2)$$

and

$$(d\sigma/d\Omega)_{\mathrm{Th}}=r_0^2\tfrac{1}{2}(1+\cos^2\theta)\,. \qquad (3)$$

Here $n_{\mathbf{k}}$ is the electron-density operator, $(d\sigma/d\Omega)_{\mathrm{Th}}$ is the Thomson cross section, $r_0=(e^2/mc^2)$ is the classical electron radius, and ω represents the difference in frequency between the outgoing and incoming light beams.

The spectral function $S(\mathbf{k},\omega)$ in thermal equilibrium is related to the response to an external field by[3]

$$S(\mathbf{k},\omega)=\frac{-1}{e^{\beta\omega}-1}\,\mathrm{Im}\int_0^{\infty}dte^{-i\omega t}\langle[n_{\mathbf{k}}(t),n_{-\mathbf{k}}(0)]\rangle\,, \qquad (4)$$

which is the Nyquist theorem. However, in nonthermal-equilibrium conditions, the Nyquist theorem as given by Eq. (4) no longer applies. Therefore, for systems not in thermal equilibrium one has to solve for the density-density correlation function directly. The easiest method is the "test-particle approach"[2] which we shall use here. Our system is described by the Hamiltonian,

$$H=\sum_{\mathbf{p}}\epsilon_{\mathbf{p}}a_{\mathbf{p}}^{\dagger}a_{\mathbf{p}}+\tfrac{1}{2}\sum_{\mathbf{q}}\varphi_q\sum_{\mathbf{p}}\sum_{\mathbf{p}'}a_{\mathbf{p}}^{\dagger}a_{\mathbf{p}'}^{\dagger}a_{\mathbf{p}'-\mathbf{q}}a_{\mathbf{p}+\mathbf{q}}$$
$$+\tfrac{1}{2}\sum_{\mathbf{k}}[P_{\mathbf{k}}^{\dagger}P_{\mathbf{k}}+\Omega_{\mathbf{k}}^2Q_{\mathbf{k}}^{\dagger}Q_{\mathbf{k}}]+\sum_{\mathbf{k}}v_{\mathbf{k}}Q_{\mathbf{k}}\sum_{\mathbf{p}}a_{\mathbf{p}}^{\dagger}a_{\mathbf{p}-\mathbf{k}}\,, \qquad (5)$$

where $a_{\mathbf{p}}^{\dagger}$, $a_{\mathbf{p}}$ are, respectively, the creation and destruction operators for the electron, $Q_{\mathbf{k}}$ and $P_{\mathbf{k}}$ represent, respectively, the phonon coordinate and its conjugate momentum. In Eq. (5), $\epsilon_{\mathbf{p}}=(p^2/2m)$ is the kinetic energy of the electron of momentum p, and $\Omega_{\mathbf{k}}$ is the bare phonon frequency of wavenumber $\mathbf{k}$. $\varphi_{\mathbf{k}}$ and $v_{\mathbf{k}}$ represent, respectively, the Coulomb and the electron-phonon interactions.

[3] See, for example, A. I. Larkin, Zh. Eksperim. i Teor. Fiz. **37**, 264 (1959) [English transl.: Soviet Phys.—JETP **10**, 186 (1960)].

In the test-particle approach one follows a particular electron or phonon in a definite state and asks for the charge-density fluctuation induced by this particle (the "test particle").

For the test electron in the $\mathbf{p}_0$ state one adds to the Hamiltonian of the system given by Eq. (5) another term H' representing the interaction of the test electron with the rest of the system.

$$H'=\sum_{\mathbf{q}}\varphi_{\mathbf{q}}\sum_{\mathbf{p}}a_{\mathbf{p}}^{\dagger}a_{\mathbf{p+q}}\alpha_{\mathbf{p}_0}^{\dagger}\alpha_{\mathbf{p}_0-\mathbf{q}}+\sum_{\mathbf{k}'}v_{\mathbf{k}'}Q_{\mathbf{k}'}\alpha_{\mathbf{p}_0}^{\dagger}\alpha_{\mathbf{p}_0-\mathbf{k}'}\exp[i(\epsilon_{\mathbf{p}_0}-\epsilon_{\mathbf{p}_0-\mathbf{k}'})t],\quad(6)$$

where $\alpha_{\mathbf{p}_0}^{\dagger}$, $\alpha_{\mathbf{p}_0}$ are, respectively, the creation and destruction operators for the test electron.

We now define the "distribution function" $F(\mathbf{p}+\mathbf{k},\mathbf{p},t)$ in the Heisenberg representation to be

$$F(\mathbf{p}+\mathbf{k},\mathbf{p},t)=\mathrm{Tr}\{\rho a_{\mathbf{p}}^{\dagger}(t)a_{\mathbf{p+k}}(t)\}\,,\quad =\langle a_{\mathbf{p}}^{\dagger}(t)a_{\mathbf{p+k}}(t)\rangle\,,\quad(7)$$

where ρ is the density matrix determined by $H+H'$. The response of the system to the test electron is assumed to be a small perturbation $F^{(1)}(\mathbf{p}+\mathbf{k},\mathbf{p},t)$ on the zeroth-order solution $F^{(0)}(\mathbf{p}+\mathbf{k},\mathbf{p},t)$ given by

$$F^{(0)}(\mathbf{p}+\mathbf{k},\mathbf{p},t)=\delta_{\mathbf{k},0}\tilde{f}_{\mathbf{p}}\,,\quad(8)$$

where

$$\tilde{f}_{\mathbf{p}}=\frac{1}{\exp\beta[(\mathbf{p}-\mathbf{p}_D)^2/2m-\mu]+1}\quad(9)$$

is the shifted Fermi-Dirac distribution function, and μ is the chemical potential. Here $\mathbf{p}_D=m\mathbf{v}_D$, where $\mathbf{v}_D$ is the drift velocity of the electrons relative to the lattice. In Eq. (9), $\tilde{f}_{\mathbf{p}}$ reduces to the usual Fermi-Dirac distribution function when $\mathbf{v}_D=0$. The equation of motion for $F^{(1)}(\mathbf{p}+\mathbf{k},\mathbf{p},t)$ in the random-phase approximation reads

$$\begin{aligned}i(\partial/\partial t)F^{(1)}(\mathbf{p}+\mathbf{k},\mathbf{p},t)&=(\epsilon_{\mathbf{p+k}}-\epsilon_{\mathbf{p}})F^{(1)}(\mathbf{p}+\mathbf{k},\mathbf{p},t)+\varphi_{\mathbf{k}}(\tilde{f}_{\mathbf{p}}-\tilde{f}_{\mathbf{p+k}})\sum_{\mathbf{p}'}F^{(1)}(\mathbf{p}'+\mathbf{k},\mathbf{p}',t)\\&+v_{\mathbf{k}}G^{(1)}(\mathbf{k},t)(\tilde{f}_{\mathbf{p}}-\tilde{f}_{\mathbf{p+k}})+\varphi_{\mathbf{k}}(\tilde{f}_{\mathbf{p}}-\tilde{f}_{\mathbf{p+k}})\alpha_{\mathbf{p}_0}^{\dagger}\alpha_{\mathbf{p}_0+\mathbf{k}}e^{i\Omega t},\quad(10)\end{aligned}$$

where $\Omega\equiv\epsilon_{\mathbf{p}_0}-\epsilon_{\mathbf{p}_0+\mathbf{k}}$ and $G^{(1)}(\mathbf{k},t)=\langle Q(\mathbf{k},t)\rangle$, $\mathbf{k}$ being the momentum transfer from the test electron to the system. Similarly, the equation of motion for $G^{(1)}(\mathbf{k},t)$ reads

$$(\partial^2/\partial t^2)G^{(1)}(\mathbf{k},t)+\Omega_k^2G^{(1)}(\mathbf{k},t)=-v_{\mathbf{k}}^*(\sum_{\mathbf{p}'}F^{(1)}(\mathbf{p}'+\mathbf{k},\mathbf{p}',t)+\alpha_{\mathbf{p}_0}^{\dagger}\alpha_{\mathbf{p}_0+\mathbf{k}}e^{i\Omega t}).\quad(11)$$

The solution to the coupled Eqs. (10) and (11) is given by

$$\delta n_{\mathrm{el}}(\mathbf{k},t)=\sum_{\mathbf{p}'}F^{(1)}(\mathbf{p}'+\mathbf{k},\mathbf{p}',t)=\frac{(\varphi_{\mathbf{k}}+|v_{\mathbf{k}}|^2D_{\mathbf{k}}(\Omega))\tilde{Q}_e(\mathbf{k},\Omega)}{\mathcal{E}(\mathbf{k},\Omega)}\alpha_{\mathbf{p}_0}^{\dagger}\alpha_{\mathbf{k}_0+\mathbf{k}}e^{i\Omega t},\quad(12)$$

where

$$\tilde{Q}_e(\mathbf{k},\omega)=\sum_{\mathbf{p}}\frac{\tilde{f}_{\mathbf{p+k}}-\tilde{f}_{\mathbf{p}}}{\epsilon_{\mathbf{p+k}}-\epsilon_{\mathbf{p}}+\omega-i\delta}\,,\quad(13)$$

$$\mathcal{E}(\mathbf{k},\omega)=1-\varphi_{\mathbf{k}}\tilde{Q}_e(\mathbf{k},\omega)-|v_{\mathbf{k}}|^2\tilde{Q}_e(\mathbf{k},\omega)D_k(\omega)\,,\quad(14)$$

and $D_{\mathbf{k}}(\omega)=[(\omega-i\delta)^2-\Omega_k^2]^{-1}$ is the "bare phonon" propagator. In Eq. (12), $\delta n_{\mathrm{el}}(\mathbf{k},t)$ represents the charge cloud surrounding the test electron. Thus the test electron becomes "dressed." Correspondingly, its density operator is renormalized to be

$$n_{\mathrm{el}}(\mathbf{k},t)=\alpha_{\mathbf{p}_0}^{\dagger}\alpha_{\mathbf{p}_0+\mathbf{k}}e^{i\Omega t}+\delta n_{\mathrm{el}}(\mathbf{k},t)=(1/\mathcal{E}(\mathbf{k},\Omega))\alpha_{\mathbf{p}_0}^{\dagger}\alpha_{\mathbf{p}_0+\mathbf{k}}e^{i\Omega t}.\quad(15)$$

This befits our labeling of $\mathcal{E}(\mathbf{k},\omega)$, since Eq. (15) says that the total charge density is related to the test-charge density by $\mathcal{E}(\mathbf{k},\omega)$, the dielectric function of the medium.

When one substitutes Eq. (15) into Eq. (2) and evaluates the ensemble average by considering that the electrons in the system are uncorrelated fully dressed test particles, one obtains

$$\begin{aligned}S_{\mathrm{elec}}(\mathbf{k},\omega)&=\sum_{\mathbf{p}_0,\mathbf{p}_0'}\frac{1}{\mathcal{E}(k,\epsilon_{\mathbf{p}_0}-\epsilon_{\mathbf{p}_0+\mathbf{k}})\mathcal{E}(-\mathbf{k},\epsilon_{\mathbf{p}_0'}-\epsilon_{\mathbf{p}_0-\mathbf{k}})}\int_{-\infty}^{\infty}\langle a_{\mathbf{p}_0}^{\dagger}a_{\mathbf{p}_0+\mathbf{k}}a_{\mathbf{p}_0'}^{\dagger}a_{\mathbf{p}_0'-k}\rangle e^{i(\Omega-\omega)t}dt\\&=2\pi\frac{1}{\mathcal{E}(\mathbf{k},\omega)\mathcal{E}(-\mathbf{k},-\omega)}\sum_{\mathbf{p}}\delta(\omega-\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{p+k}})\tilde{f}_{\mathbf{p}}(1-\tilde{f}_{\mathbf{p+k}}).\end{aligned}\quad(16)$$

After some algebra one finds the contribution of the dressed electrons to the spectral function to be

$$S_{\mathrm{elec}}(\mathbf{k},\omega)=\frac{2}{(e^{\beta(\omega+\mathbf{k}\cdot\mathbf{v}_D)}-1)}\frac{1}{\mathcal{E}(\mathbf{k},\omega)\mathcal{E}(-\mathbf{k},-\omega)}\mathrm{Im}\tilde{Q}_e(\mathbf{k},\omega).\quad(17)$$

In order to express our result in terms of the equilibrium Fermi-Dirac distribution function we notice from Eqs. (9) and (13) that

$$\tilde{Q}_e(\mathbf{k},\omega)=Q_e(\mathbf{k},\,\omega+\mathbf{k}\cdot\mathbf{v}_D)\,, \tag{18}$$

where

$$Q_e(\mathbf{k},\omega)=\sum_{\mathbf{p}}\frac{f_{\mathbf{p}+\mathbf{k}}-f_{\mathbf{p}}}{\epsilon_{\mathbf{p}+\mathbf{k}}-\epsilon_{\mathbf{p}}+\omega-i\delta}\,. \tag{19}$$

On substituting Eq. (18) into Eq. (17), S_{elec} now reads

$$S_{\mathrm{elec}}(\mathbf{k},\omega)=\frac{2}{(e^{\beta(\omega+\mathbf{k}\cdot\mathbf{v}_D)}-1)}\frac{\mathrm{Im}Q_e(\mathbf{k},\,\omega+\mathbf{k}\cdot\mathbf{v}_D)}{|\mathcal{E}_D(\mathbf{k},\omega)|^2}\,, \tag{20}$$

where we have rewritten $\mathcal{E}(\mathbf{k},\omega)$ as $\mathcal{E}_D(\mathbf{k},\omega)$ to exhibit the explicit dependence of the dielectric function on the drift velocity:

$$\mathcal{E}_D(\mathbf{k},\omega)=1-\varphi_{\mathbf{k}}Q_e(\mathbf{k},\,\omega+\mathbf{k}\cdot\mathbf{v}_D)-|v_{\mathbf{k}}|^2Q_e(\mathbf{k},\,\omega+\mathbf{k}\cdot\mathbf{v}_D)D_{\mathbf{k}}(\omega)\,. \tag{21}$$

The physical interpretation is made clear by this method of superposition of test particles. $S_{\mathrm{elec}}(\mathbf{k},\omega)$ as given by Eq. (20) is simply the scattering from the screening cloud and the core of the test electrons, i.e., the fully dressed electrons.

The contribution of the dressed phonons to the spectral function can be obtained in a similar way. The interaction of the test phonon in the $\mathbf{k}$ state with the system is given by

$$H''=v_{\mathbf{k}}\chi(\mathbf{k},t)\sum_{\mathbf{p}'}a_{\mathbf{p}'}(t)a_{\mathbf{p}'-\mathbf{k}}^{\dagger}(t)\,, \tag{22}$$

where $\chi(\mathbf{k},t)$ is the test-phonon coordinate. In the language of second quantization,

$$\chi(\mathbf{k},t)=(2\Omega_k)^{-1/2}\{b_{\mathbf{k}}e^{-i\Omega_{\mathbf{k}}t}+b_{-\mathbf{k}}^{\dagger}e^{i\Omega_{\mathbf{k}}t}\}\,. \tag{23}$$

Corresponding to Eq. (12), one obtains, on perturbing the system by H'', an analogous expression for the density of the charge cloud surrounding the test phonon,

$$\delta n_{\mathrm{ph}}(\mathbf{k},t)=\frac{v_k}{(2\Omega_k)^{1/2}}\left\{\frac{\tilde{Q}_e(\mathbf{k},\,-\Omega_k)}{\mathcal{E}(\mathbf{k},\,-\Omega_k)}b_{\mathbf{k}}e^{-i\Omega_{\mathbf{k}}t}+\frac{\tilde{Q}_e(-\mathbf{k},\,-\Omega_k)}{\mathcal{E}(-\mathbf{k},\,-\Omega_k)}b_{-\mathbf{k}}^{\dagger}e^{i\Omega_{\mathbf{k}}t}\right\}\,, \tag{24}$$

where $b_{\mathbf{k}}^{\dagger}$ and $b_{\mathbf{k}}$ are, respectively, the creation and destruction operators for the phonon. By considering the phonons in the system as uncorrelated but dressed by the charge cloud $\delta n_{\mathrm{ph}}(\mathbf{k},t)$ in Eq. (24) we obtain the contribution of the dressed phonons to the spectral function

$$S_{\mathrm{ph}}(\mathbf{k},\omega)=\frac{2}{e^{\beta\omega}-1}\left|\frac{v_{\mathbf{k}}Q_e(\mathbf{k},\,\omega+\mathbf{k}\cdot\mathbf{v}_D)}{\mathcal{E}_D(\mathbf{k},\omega)}\right|^2\mathrm{Im}D_{\mathbf{k}}(\omega)\,. \tag{25}$$

This $S_{\mathrm{ph}}(\mathbf{k},\omega)$ represents the scattering from the electron cloud associated with the dressed phonons. The total spectral function is obtained by combining Eqs. (20) and (25),

$$S(\mathbf{k},\omega)=S_{\mathrm{elec}}(\mathbf{k},\omega)+S_{\mathrm{ph}}(\mathbf{k},\omega)\,, \tag{26}$$

and the scattering cross section is gotten by substituting Eq. (26) into Eq. (1).

III. DISCUSSION

In thermal equilibrium, i.e., when the drift velocity $\mathbf{v}_D=0$, the spectral function is reduced to the following form:

$$S(\mathbf{k},\omega)=[2/(e^{\beta\omega}-1)]\,\mathrm{Im}[Q_e(\mathbf{k},\omega)/\mathcal{E}(\mathbf{k},\omega)]\,. \tag{27}$$

This can be easily obtained by using the Green's-function technique which is valid for thermal equilibrium.[4] The same result can also be obtained by considering test electrons and renormalized test phonons inducing charge-density fluctuation in the electron plasma, as in Ref. 4, instead of in the electron-phonon system. However, this way of applying the test-particle method can not be directly generalized to systems not in thermal equilibrium.

We next look at our result given by Eq. (25). Physically one does not anticipate that $S_{\mathrm{ph}}(\mathbf{k},\omega)$ would contribute to the scattering except possibly in the vicinity of the bare phonon frequency. Furthermore, since the bare phonon frequency is no longer a resonance frequency of the coupled electron-phonon system, one does not expect $S_{\mathrm{ph}}(\mathbf{k},\omega)$ to show any singularity at Ω_k. These points are born out by inspection of Eq. (25). In the limit of very long phonon lifetime, $\mathrm{Im}D_{\mathbf{k}}(\omega)$ is nonvanishing only around $\omega=\pm\Omega_k$, where it has an amplitude proportional to the phonon lifetime. However, at Ω_k, the screening described by $|\mathcal{E}_D(\mathbf{k},\omega)|^2$ becomes large, proportional to the square of the phonon lifetime, so that it completely nullifies the resonance effect of $\mathrm{Im}D_{\mathbf{k}}(\omega)$.

The scattering cross section is therefore obtained by substituting Eq. (20) into Eq. (1) to give

$$\frac{d\sigma}{d\omega d\Omega}=\left(\frac{d\sigma}{d\omega}\right)_{\mathrm{Th}}\frac{1}{\pi}\frac{1}{(e^{\beta(\omega+\mathbf{k}\cdot\mathbf{v}_D)}-1)}\times\frac{\mathrm{Im}Q_e(\mathbf{k},\,\omega+\mathbf{k}\cdot\mathbf{v}_D)}{|\mathcal{E}_D(\mathbf{k},\omega)|^2}\,, \tag{28}$$

where $\mathcal{E}_D(\mathbf{k},\omega)$ is defined by Eq. (21).

We note that for systems with electrons drifting relative to the lattice one has only to modify the corresponding thermal-equilibrium scattering cross section by replacing ω by $\omega+\mathbf{k}\cdot\mathbf{v}_D$ in all the electronic functions $Q_e(\mathbf{k},\omega)$.

For the purpose of discussion, we rewrite Eq. (28) in the classical limit, assuming that the bare phonon frequency is no longer a resonance frequency of the coupled

[4] A. Ron, Phys. Rev. 132, 978 (1963).

system:

$$\frac{d\sigma}{d\omega d\Omega}=n\left(\frac{d\sigma}{d\Omega}\right)_{\text{Th}} \times\frac{1}{\pi^{1/2}}\frac{\exp[-(\omega+\mathbf{k}\cdot\mathbf{v}_D)^2/2k^2v_{\text{th}}^2]}{\sqrt{2}kv_{\text{th}}}\times\frac{1}{X^2+Y^2}, \quad (29)$$

where

$$X=\text{Re}\,\mathcal{E}_D(\mathbf{k},\omega),$$

and

$$Y=\text{Im}\,\mathcal{E}_D(\mathbf{k},\omega) =\pi^{1/2}\left(1+\frac{|v_k|^2}{\varphi_k}\text{Re}D_k(\omega)\right)^2 \times\frac{k_D^2}{k^2}\frac{(\omega+\mathbf{k}\cdot\mathbf{v}_D)}{\sqrt{2}kv_{\text{th}}}\exp\left[-\frac{(\omega+\mathbf{k}\cdot\mathbf{v}_D)^2}{2k^2v_{\text{th}}^2}\right].$$

In Eq. (29), n is the average electron density; $k_D=(4\pi ne^2\beta)^{1/2}$ is the Debye wavenumber and $v_{\text{th}}=(\beta m)^{-1/2}$ is the electron thermal velocity. The resonances are determined by the zeros of X. To exhibit the plasma line and the phonon line, we can convert Eq. (29) into a more transparent form:

$$\frac{d\sigma}{d\Omega d\bar{\omega}}\Big/n\left(\frac{d\sigma}{d\Omega}\right)_{\text{Th}}=\frac{1}{\sqrt{\pi}}\frac{e^{-(+\bar{\omega}_D\bar{\omega})^2}}{|\epsilon_e(\mathbf{k},\bar{\omega}+\bar{\omega}_D)|^2} \times\frac{(\bar{\omega}^2-\bar{\Omega}_k^2)^2}{|\bar{\omega}^2-\bar{\Omega}_k^2+\alpha\bar{\Omega}_k^2[1-1/\epsilon_e(\mathbf{k},\omega+\omega_D)]|^2}, \quad (30)$$

where

$$\epsilon_e(\mathbf{k},\omega)=1-\varphi_kQ_e(\mathbf{k},\omega),$$

$$\bar{\omega}=\frac{\omega}{\sqrt{2}kv_{\text{th}}},\quad \bar{\omega}_D=\frac{\mathbf{k}\cdot\mathbf{v}_D}{\sqrt{2}kv_{\text{th}}},\quad \bar{\Omega}_k=\frac{\Omega_k}{\sqrt{2}kv_{\text{th}}},$$

and

$$\alpha=|v_k|^2/(\varphi_k\Omega_P),$$

Ω_P being the ion plasma frequency. We have specifically chosen the optical branch of the phonon spectrum. In the right-hand side of Eq. (30), the first factor corresponds to the plasma resonance line, with the exponential factor arising from the Landau damping. The second factor corresponds to the phonon resonance line. When the electron-phonon coupling strength α approaches zero, the second factor becomes unity and only the plasma line remains as expected.

For the purpose of illustration, let us assume that the plasma frequency is much higher than the phonon frequency. In this case, the position of the phonon line depends only weakly on the drift velocity $\mathbf{v}_D$. However, one can enhance the cross section by choosing $\mathbf{v}_D$ to be such that $\mathbf{k}\cdot\mathbf{v}_D+\omega_{\text{ph}}=0$, where ω_{ph} is the root of $X=0$ corresponding to the dressed phonon frequency. By examining the phonon-line factor in Eq. (30), one can immediately show that this occurs when $\omega_{\text{ph}}=-\omega_D$ $=\Omega_k[1-(\alpha k_D^2)/(k^2+k_D^2)]^{1/2}$, at which the denominator actually vanishes for long-lived bare phonons. Thus, at this critical value of the drift velocity, we obtain a phonon instability in which case the analytic expression for the cross section is no longer applicable. The physical reason for this enhancement is clear. When the electrons are drifting with a velocity $\mathbf{v}_D$ approximately equal to the phonon phase velocity, a large phonon excitation is expected due to the "coherent" interaction between the electrons and the phonons. This in turn excites large electron-density fluctuations at the phonon frequency ω_{ph} which strongly scatter the electromagnetic wave. The corresponding phonon frequency shift, $\omega_{\text{ph}}-\Omega_k$, caused by this static screening of the electrons, is then rather insensitive to changes in electron density as long as the electron-density-fluctuation wavelength is large compared to the Debye length. Therefore, in this case, the phonon instability peak stays almost constant as the electron density is varied.

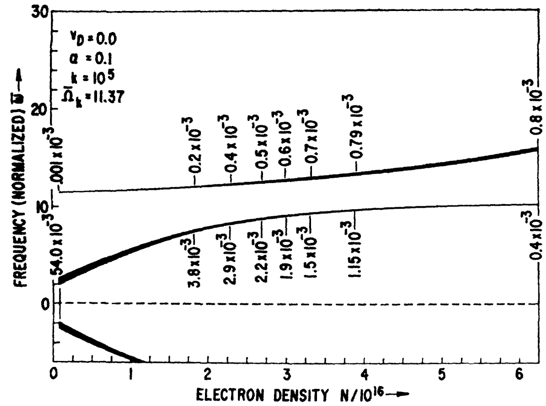

FIG. 1. Locations and intensities of resonance lines as a function of electron density when $V_D=0$.

IV. NUMERICAL RESULTS

The scattering cross section has been computed for different values of the electron density and drift velocity. The electron-phonon coupling-strength parameter α is taken to be 0.1, which is a typical value for semiconductors which are ionic to a small degree such as InSb, InP, GaP. The wavenumbers difference k is taken to be 10^5 cm^{-1}. This is less than the Debye wavenumber which is in the neighborhood of 10^6 cm^{-1} for room temperature and an electron density of 3×10^{16}/cc. The phonon frequency is assumed to be 10^{13} cps which is equivalent to a normalized $\bar{\Omega}_k$ of 11.4. For simplicity, the bare-electron mass and charge, instead of the effective mass and charge of an electron embedded in a lattice, are used. Figure 1 shows the location of the resonances when we vary the electron density N in the absence of drift velocity. The numbers alongside the curves represent the integrated cross section at the resonance when normalized with respect to the Thomson cross section.

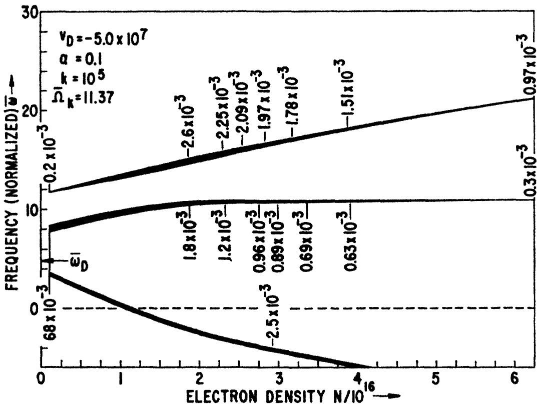

FIG. 2. Locations and intensities of resonance lines as a function of electron density when V_D is finite.

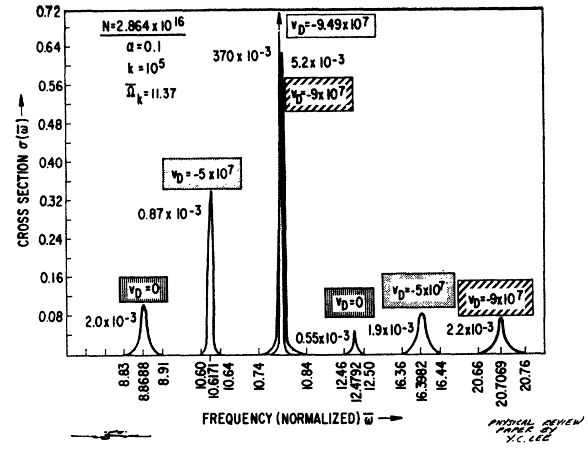

FIG. 4. The scattering cross section for different drift velocities.

The thickness of these curves are roughly proportional to these numbers to aid visualization. Starting from the right, the top curve depicts the plasma resonance which weakens gradually in intensity and decreases in slope as it approaches lower densities. It finally becomes flat and behaves like a phonon resonance. Starting from the left, the lower curve represents the plasma resonance which gradually weakens and behaves like a phonon resonance toward the right side (high densities) of the plot. At intermediate densities or in the central part of the plot, these two resonances strongly interefere with and repel each other due to the electron-phonon interaction. Since $v_D=0$, the plot is symmetric with respect to positive and negative frequencies. Figure 2 shows the same resonances in the presence of finite drift velocity. We note that the central curve is becoming flattened and correspondingly the top curve is repelled by it, becoming straightened. When the drift velocity is close to the critical value (the dressed phonon phase velocity), static screening rather than dynamic screening governs the phonon resonance as discussed in the last section. This is why in Fig. 3 the central curve which represents the phonon resonance becomes so flat. The upper curve which depicts the plasma resonance is further repelled and begins to arch up. In Fig. 4, the scattering cross section is plotted against the normalized frequency $\bar{\omega}$ for different values of the drift velocity but for a fixed electron density. The two resonance peaks for each value of the drift velocity are clearly shown here. The area under each peak is also indicated. We observe that while nothing drastic happens to the plasma peak, the phonon peak gets sharper and higher as the drift velocity is increased. At $v_D=-9.49\times10^7$ cm/sec, the phonon resonance shoots up, with the corresponding area increased almost by two orders of magnitude, indicating the onset of an instability.

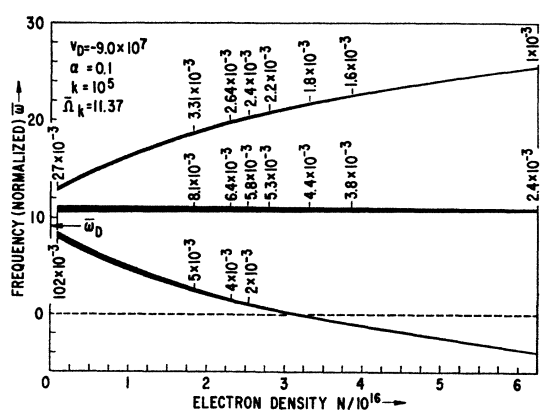

FIG. 3. Locations and intensities of resonance lines as a function of electron density when V_D is close to the phonon phase velocity.

V. CONCLUSION

In an electron-phonon system it is known that there exist two types of collective oscillations, the plasmon and the phonon. These two oscillations are coupled to one another by electron-phonon interaction. We have demonstrated in this calculation that by shining electromagnetic waves on such a system one can observe from the scattered radiation how the two collective oscillations exhibit themselves as two resonance lines, how they interfere with each other as the electron density is varied and how the cross section at the phonon line can be enhanced by imparting a drift velocity to the electrons so as to induce a phonon instability. With present-day lasers, it is therefore possible to use electromagnetic scattering as a tool for the study of the optical properties of solids.

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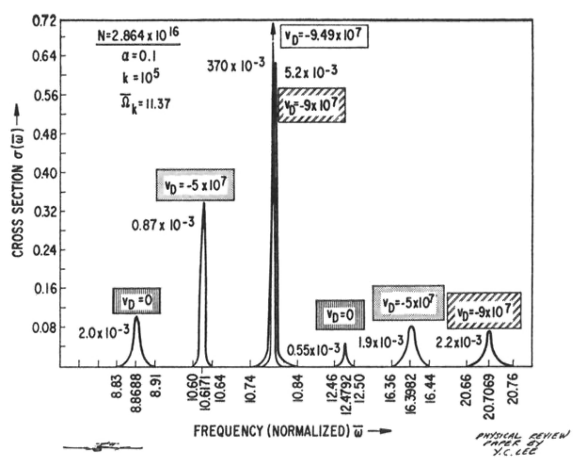

FIG. 4. The scattering cross section for different drift velocities.