Theory of Muon Capture with Initial and Final Nuclei Treated as "Elementary" Particles*

C. W. KIM AND H. PRIMAKOFF
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania
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The theory of the capture of muons by nuclei ($\mu^-+N_a\rightarrow\nu_\mu+N_b$) is developed, the nuclei N_a and N_b being treated as "elementary" particles. Form factors are introduced to describe the $N_a\rightarrow N_b$ transition matrix elements of the hadron weak currents; these form factors are then evaluated on the basis of the conserved-vector-current and partially-conserved-axial-vector-current hypotheses and with use of appropriate experimental data regarding the corresponding electromagnetic and beta-decay transitions. The reactions $\mu^-+{}_2He_1{}^3\rightarrow\nu_\mu+{}_1H_2{}^3$, $\mu^-+{}_6C_6{}^{12}\rightarrow\nu_\mu+{}_5B_7{}^{12}$, and $\mu^-+{}_3Li_3{}^6\rightarrow\nu_\mu+{}_2He_4{}^6$ are discussed explicitly and the calculated rates are compared with available measured values.

I. INTRODUCTION

In the present paper we apply to the theory of muon capture the methods developed in a previous paper[1] for the treatment of beta decay. In brief, we treat the nuclei N_a and N_b, which participate in the muon capture process $\mu^-+N_a\rightarrow\nu_\mu+N_b$, as "elementary" particles and apply the hypothesis of the conserved polar-vector hadron weak current (CVC) and the hypothesis of the partially conserved axial-vector hadron weak current (PCAC). The CVC hypothesis, which permits identification of the polar-vector hadron weak current with the isospin current, relates the polar-vector and weak-magnetism $N_a\rightarrow N_b$ form factors with the Dirac and Pauli electromagnetic form factors of N_a and N_b; the PCAC hypothesis, which together with a suitable pion-pole-dominance assumption implies the Goldberger-Treiman (G-T) relation, connects the axial-vector and induced-pseudoscalar $N_a\rightarrow N_b$ weak form factors; finally, the axial-vector $N_a\rightarrow N_b$ form factor is known at zero momentum transfer from the observed $N_b\rightarrow N_a+e^-+\bar{\nu}_e$ beta-decay rate while its dependence on momentum transfer can be found from an analysis of suitable empirical nuclear-structure data (see below). In this way, we avoid all recourse to the use of nuclear models and of the impulse approximation in order to calculate the $N_a\leftrightarrow N_b$ transition matrix elements of $j_\lambda{}^{(V)}$ and $j_\lambda{}^{(A)}$ and are able to give theoretical expressions for at least several of the muon capture rates which are substantially free of the uncertainties of nuclear physics.[2]

II. CALCULATIONS

The relation between the rate of the muon-capture reaction: $\Gamma(\mu^-+N_a\rightarrow\nu_\mu+N_b)$ and the rate of the corresponding beta-decay reaction: $\Gamma(N_b\rightarrow N_a+e^-+\bar{\nu}_e)$, is given by[3]

$$\frac{\Gamma(\mu^-+N_a\rightarrow\nu_\mu+N_b)}{\Gamma(N_b\rightarrow N_a+e^-+\bar{\nu}_e)}=\left[\frac{4\pi}{(2\pi)^3}\right]E_\nu{}^2\left(1-\frac{E_\nu}{m_\mu+m_b}\right)\left[\frac{C(N_a)}{\pi}\left(\frac{Z(N_a)}{137}\frac{m_\mu m_a}{m_\mu+m_a}\right)^3\right]\int\frac{d\hat{p}_\nu}{4\pi}\left(\tfrac{1}{2}\mathfrak{L}_{\kappa\lambda}{}^{(\mu)}\mathfrak{N}_{\kappa\lambda}{}^{(\mu)}(N_a\rightarrow N_b)\right)\Big/$$
$$\left[\frac{4\pi}{(2\pi)^3}\right]^2\int_{m_e}^{m_b-m_a}dE_eE_e(E_e{}^2-m_e{}^2)^{1/2}(m_b-m_a-E_e)^2F(Z(N_a),E_e)\int\frac{d\hat{p}_e}{4\pi}\int\frac{d\hat{p}_{\bar{\nu}}}{4\pi}\left(\mathfrak{L}_{\kappa\lambda}{}^{(e)}\mathfrak{N}_{\kappa\lambda}{}^{(e)}(N_b\rightarrow N_a)\right),$$

$$\mathfrak{L}_{\kappa\lambda}{}^{(\mu)}=\mathrm{Tr}[(1+\gamma_5)\gamma_\kappa\gamma_4(\gamma\cdot p_\nu\gamma_4/2iE_\nu)\gamma_4\gamma_\lambda(1+\gamma_5)((\gamma\cdot p_\mu+im_\mu)\gamma_4/2iE_\mu)]$$
$$=(E_\nu E_\mu)^{-1}[(p_\nu)_\kappa(p_\mu)_\lambda+(p_\nu)_\lambda(p_\mu)_\kappa-\delta_{\kappa\lambda}p_\nu\cdot p_\mu+\epsilon_{\kappa\lambda\rho\sigma}(p_\nu)_\rho(p_\mu)_\sigma](-1)^{\delta_{\kappa4}},$$

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[1] C. W. Kim and H. Primakoff, Phys. Rev. **139**, B1447 (1965). This paper will be referred to as I.

[2] We recall that the equations of the impulse approximation are

$$|N_a\rangle\cong\Psi(N_a;\cdots,\mathbf{r}^{(k)},\sigma_3{}^{(k)},\tau_3{}^{(k)},\cdots),$$
$$|N_b\rangle\cong\Psi(N_b;\cdots,\mathbf{r}^{(k)},\sigma_3{}^{(k)},\tau_3{}^{(k)},\cdots),$$

and

$$j_\lambda{}^{(V)}\cong\sum_{k=1}^{A}[j_\lambda{}^{(V)}]_k\exp[+i\mathbf{q}\cdot\mathbf{r}^{(k)}],$$

$$j_\lambda{}^{(A)}\cong\sum_{k=1}^{A}[j_\lambda{}^{(A)}]_k\exp[+i\mathbf{q}\cdot\mathbf{r}^{(k)}];$$

$$[j_\lambda{}^{(V)}]_k=[\tau_-\gamma_4(\gamma_\lambda F_V(q^2;p\rightarrow n)-(\sigma_{\lambda\rho}q_\rho/2m_p)F_M(q^2;p\rightarrow n))]_k,$$
$$[j_\lambda{}^{(A)}]_k=[\tau_-\gamma_4(\gamma_\lambda\gamma_5F_A(q^2;p\rightarrow n)+(iq_\lambda(m_n+m_p)/m_\pi{}^2)\gamma_5F_P(q^2;p\rightarrow n))]_k,$$
$$q_\lambda\equiv(p_\mu-p_\nu)_\lambda=(p_b-p_a)_\lambda.$$

See I for notation.

[3] A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959); H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959).

$$\mathfrak{N}_{\kappa\lambda}^{(\mu)}(N_a\to N_b)=(2J_a+1)^{-1}\sum_{M_a=-J_a}^{J_a}\sum_{M_b=-J_b}^{J_b}\{\langle N_b;\cdots M_b\cdots|(j_\kappa^{(V)}+j_\kappa^{(A)})|N_a;\cdots M_a\cdots\rangle^* \times\langle N_b;\cdots M_b\cdots|(j_\lambda^{(V)}+j_\lambda^{(A)})|N_a;\cdots M_a\cdots\rangle\},\quad (1)$$

$$q^2=(p_b-p_a)^2=(p_\mu-p_\nu)^2,$$

$$\mathfrak{L}_{\kappa\lambda}^{(e)}=\mathrm{Tr}[(1+\gamma_5)\gamma_\kappa\gamma_4((\gamma\cdot p_e+im_e)\gamma_4/2iE_e)\gamma_4\gamma_\lambda(1+\gamma_5)(\gamma\cdot p_{\bar\nu}\gamma_4/2iE_{\bar\nu})]$$
$$=(E_eE_{\bar\nu})^{-1}[(p_e)_\kappa(p_{\bar\nu})_\lambda+(p_e)_\lambda(p_{\bar\nu})_\kappa-\delta_{\kappa\lambda}p_e\cdot p_{\bar\nu}+\epsilon_{\kappa\lambda\rho\sigma}(p_e)_\rho(p_{\bar\nu})_\sigma](-1)^{\delta_{\kappa4}},$$

$$\mathfrak{N}_{\kappa\lambda}^{(e)}(N_b\to N_a)=(2J_b+1)^{-1}\sum_{M_b=-J_b}^{J_b}\sum_{M_a=-J_a}^{J_a}\{\langle N_a;\cdots M_a\cdots|(j_\kappa^{(V)}+j_\kappa^{(A)})^\dagger|N_b;\cdots M_b\cdots\rangle^* \times\langle N_a;\cdots M_a\cdots|(j_\lambda^{(V)}+j_\lambda^{(A)})^\dagger|N_b;\cdots M_b\cdots\rangle\},$$

$$q^2=(p_a-p_b)^2=(p_e+p_{\bar\nu})^2.$$

$C(N_a)$ is a correction factor arising from the effect of the nonpoint character of the charge distribution of N_a (see Appendix) and $F(Z(N_a),E_e)$ is the Fermi function of beta-decay theory

$$[F(Z(N_a),E_e)\cong[2\pi Z(N_a)/137v_e]\{1-\exp[-2\pi Z(N_a)/137v_e]\};\ v_e=|\mathbf{p}_e|/E_e=(E_e^2-m_e^2)^{1/2}/E_e].$$

In obtaining Eq. (1) we have of course supposed, in accordance with the basic assumption of muon-electron symmetry, that the coupling between (N_b,N_a) and (μ,ν_μ) is identical with the coupling (N_b,N_a) and (e,ν_e). In the case where the $N_b\to N_a$ beta-decay reaction is "allowed"

$$\int\frac{d\hat{p}_e}{4\pi}\int\frac{d\hat{p}_{\bar\nu}}{4\pi}(\mathfrak{L}_{\kappa\lambda}^{(e)}\mathfrak{N}_{\kappa\lambda}^{(e)}(N_b\to N_a))$$

is effectively independent of E_e and Eq. (1) can be written as

$$\frac{\Gamma(\mu^-+N_a\to\nu_\mu+N_b)}{\Gamma(N_b\to N_a+e^-+\bar\nu_e)}=\left\{\pi E_\nu^2\left(1-\frac{E_\nu}{m_\mu+m_b}\right)\left[C(N_a)\left(\frac{Z(N_a)}{137}\frac{m_\mu m_a}{m_\mu+m_a}\right)^3\right]\Big/f(N_b\to N_a)\right\}R(N_a,N_b)$$

$$f(N_b\to N_a)\equiv\int_{m_e}^{m_b-m_a}dE_eE_e(E_e^2-m_e^2)^{1/2}(m_b-m_a-E_e)^2F(Z(N_a),E_e)\quad (2)$$

$$R(N_a,N_b)\equiv\int\frac{d\hat{p}_\nu}{4\pi}\mathfrak{L}_{\kappa\lambda}^{(\mu)}\mathfrak{N}_{\kappa\lambda}^{(\mu)}(N_a\to N_b)\Big/\!\!\Big/\int\frac{d\hat{p}_e}{4\pi}\int\frac{d\hat{p}_{\bar\nu}}{4\pi}\mathfrak{L}_{\kappa\lambda}^{(e)}\mathfrak{N}_{\kappa\lambda}^{(e)}(N_b\to N_a),$$

where $f(N_b\to N_a)$ is the f function of beta-decay theory appropriate to the $N_b\to N_a$ "allowed" electron-energy spectrum. It thus remains to express the $N_a\leftrightarrow N_b$ transition matrix elements of $j_\kappa^{(V)}$ and $j_\kappa^{(A)}$ in terms of appropriate form factors, to evaluate these form factors in the manner described in the Introduction, and to use the results to calculate $R(N_a,N_b)$ on the basis of Eqs. (1) and (2).

$\mu^-+{}_2He_1^3\to\nu_\mu+{}_1H_2^3$

We proceed to apply the above procedure to the muon capture reaction $\mu^-+{}_2\mathrm{He}_1^3\to\nu_\mu+{}_1\mathrm{H}_2^3$.[4] We have from Eq. (13) of I

$$\langle \mathrm{H}^3;\cdots M_b\cdots|j_\lambda^{(V)}|\mathrm{He}^3;\cdots M_a\cdots\rangle$$
$$=\{u^\dagger(\mathrm{H}^3;\cdots M_b\cdots)\gamma_4[\gamma_\lambda F_V(q^2;\mathrm{He}^3\to\mathrm{H}^3)-(\sigma_{\lambda\rho}q_\rho/2m_p)F_M(q^2;\mathrm{He}^3\to\mathrm{H}^3)]u(\mathrm{He}^3;\cdots M_a\cdots)\}$$

$$\langle \mathrm{H}^3;\cdots M_b\cdots|j_\lambda^{(A)}|\mathrm{He}^3;\cdots M_a\cdots\rangle$$
$$=\{u^\dagger(\mathrm{H}^3;\cdots M_b\cdots)\gamma_4[\gamma_\lambda\gamma_5F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)+(iq_\lambda(m_b+m_a)/m_\pi^2)\gamma_5F_P(q^2;\mathrm{He}^3\to\mathrm{H}^3)]u(\mathrm{He}^3;\cdots M_a\cdots)\};$$

$$\sigma_{23,31,12}=(i)^{-1}\gamma_4\gamma_{1,2,3}\gamma_5=\sigma_{1,2,3},\quad \sigma_{14,24,34}=(i)^{-1}\gamma_{1,2,3}\gamma_4;$$
$$F_{V,M,A,P}(q^2;\mathrm{He}^3\to\mathrm{H}^3)=F_{V,M,A,P}(q^2;\mathrm{H}^3\to\mathrm{He}^3);\quad (3)$$
$$q^2=(p_b-p_a)^2=(p_\mu-p_\nu)^2=-m_\mu^2+2m_\mu E_\nu\cong m_\mu^2+2m_\mu(m_a-m_b-m_\mu^2/2m_a)=0.96m_\mu^2;$$

$$\langle \mathrm{He}^3;\cdots M_a\cdots|j_\lambda^{(V)}|\mathrm{H}^3;\cdots M_b\cdots\rangle\cong[u^\dagger(\mathrm{He}^3;\cdots M_a\cdots)\delta_{\lambda4}u(\mathrm{H}^3;\cdots M_b\cdots)]F_V(0;\mathrm{H}^3\to\mathrm{He}^3),$$
$$\langle \mathrm{He}^3;\cdots M_a\cdots|j_\lambda^{(A)}|\mathrm{H}^3;\cdots M_b\cdots\rangle\cong[u^\dagger(\mathrm{He}^3;\cdots M_a\cdots)i\sigma_\lambda(1-\delta_{\lambda4})u(\mathrm{H}^3;\cdots M_b\cdots)]F_A(0;\mathrm{H}^3\to\mathrm{He}^3),$$
$$q^2=(p_a-p_b)^2=(p_e+p_{\bar\nu})^2=-m_e^2+2(\mathbf{p}_e\cdot\mathbf{p}_{\bar\nu}-E_eE_{\bar\nu})\cong-m_e^2\ll1/\langle r^2\rangle_b,\ 1/\langle r^2\rangle_a$$

[4] For previous accounts of application of the above procedure to $\mu^-+\mathrm{He}^3\to\nu_\mu+\mathrm{H}^3$ see A. Fujii and Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **31**, 107 (1964); W. Drechsler and B. Stech, Z. Physik **178**, 1 (1964); H. Primakoff, *Weak Interactions and High Energy Neutrino Physics*, edited by T. D. Lee (Academic Press Inc., New York, to be published).

so that, carrying out the indicated sums over M_a, M_b, κ, λ in Eqs. (1) and (2), we have[5]

$$
\begin{aligned}
&R(\mathrm{He}^3,\mathrm{H}^3)=N/D\\
&N\equiv[G_V(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)]^2+3\{[G_A(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)]^2\\
&\quad-\tfrac{2}{3}[G_A(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)][G_P(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)]+\tfrac{1}{3}[G_P(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)]^2\}\\
&D\equiv[F_V(0;\mathrm{H}^3\to\mathrm{He}^3)]^2+3[F_A(0;\mathrm{H}^3\to\mathrm{He}^3)]^2;\\
&G_V(q^2;\mathrm{He}^3\to\mathrm{H}^3)\equiv F_V(q^2;\mathrm{He}^3\to\mathrm{H}^3)(1+E_\nu/2m_a)\\
&G_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)\equiv -F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)-F_M(q^2;\mathrm{He}^3\to\mathrm{H}^3)(E_\nu/2m_p)-F_V(q^2;\mathrm{He}^3\to\mathrm{H}^3)(E_\nu/2m_a)\\
&G_P(q^2;\mathrm{He}^3\to\mathrm{H}^3)\equiv[(m_\mu(m_b+m_a)/m_\pi^2)F_P(q^2;\mathrm{He}^3\to\mathrm{H}^3)+F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)-F_V(q^2;\mathrm{He}^3\to\mathrm{H}^3)](E_\nu/2m_a)\\
&\quad-F_M(q^2;\mathrm{He}^3\to\mathrm{H}^3)(E_\nu/2m_p).
\end{aligned}
\tag{4}
$$

We now must specify the numerical values of the form factors in Eq. (4). On the basis of the CVC hypothesis, and with use of appropriate electron $-\mathrm{He}^3$ and electron $-\mathrm{H}^3$ scattering data for $F_{\mathrm{Dirac}}(q^2;\mathrm{He}^3)$, $F_{\mathrm{Dirac}}(q^2;\mathrm{H}^3)$, $F_{\mathrm{Pauli}}(q^2;\mathrm{He}^3)$, $F_{\mathrm{Pauli}}(q^2;\mathrm{H}^3)$,[6] we have

$$
\begin{aligned}
&F_V(q^2;\mathrm{He}^3\to\mathrm{H}^3)=F_V(q^2;\mathrm{H}^3\to\mathrm{He}^3)=F_{\mathrm{Dirac}}(q^2;\mathrm{He}^3)-F_{\mathrm{Dirac}}(q^2;\mathrm{H}^3);\\
&F_V(0;\mathrm{H}^3\to\mathrm{He}^3)=F_{\mathrm{Dirac}}(0;\mathrm{He}^3)-F_{\mathrm{Dirac}}(0;\mathrm{H}^3)=2-1=1;\\
&F_V(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)=F_{\mathrm{Dirac}}(0.96m_\mu^2;\mathrm{He}^3)-F_{\mathrm{Dirac}}(0.96m_\mu^2;\mathrm{H}^3)=0.82;\\
&F_M(q^2;\mathrm{He}^3\to\mathrm{H}^3)=F_M(q^2;\mathrm{H}^3\to\mathrm{He}^3)=F_{\mathrm{Pauli}}(q^2;\mathrm{He}^3)-F_{\mathrm{Pauli}}(q^2;\mathrm{H}^3);\\
&F_M(0;\mathrm{H}^3\to\mathrm{He}^3)=F_{\mathrm{Pauli}}(0;\mathrm{He}^3)-F_{\mathrm{Pauli}}(0;\mathrm{H}^3)=[\mu(\mathrm{He}^3)-\tfrac{2}{3}]-[\mu(\mathrm{H}^3)-\tfrac{1}{3}]\\
&\qquad=(-2.13-\tfrac{2}{3})-(2.98-\tfrac{1}{3})=-5.44;\\
&F_M(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)=F_{\mathrm{Pauli}}(0.96m_\mu^2;\mathrm{He}^3)-F_{\mathrm{Pauli}}(0.96m_\mu^2;\mathrm{H}^3)=-4.73=(0.87)\times(-5.44).
\end{aligned}
\tag{5}
$$

Also, with the value of $F_V(0;\mathrm{H}^3\to\mathrm{He}^3)$ known, the value of $F_A(0;\mathrm{H}^3\to\mathrm{He}^3)$ can be calculated [see Eqs. (25)–(27) of I] from the measured rate of the beta-decay reaction ${}_1\mathrm{H}_2{}^3\to{}_2\mathrm{He}_1{}^3+e^-+\bar{\nu}_e$, viz.[7];

$$F_A(0;\mathrm{H}^3\to\mathrm{He}^3)=-|F_A(0;\mathrm{H}^3\to\mathrm{He}^3)|=-1.22,\tag{6}$$

where the minus sign is chosen on the basis of an impulse-approximation calculation[4] of

$$F_A(0;\mathrm{H}^3\to\mathrm{He}^3)\cong\frac{F_A(0;n\to p)\langle\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3{}^{(k)},\tau_3{}^{(k)},\cdots)|\sum_{k=1}^{3}\tau_+{}^{(k)}\boldsymbol{\sigma}^{(k)}|\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3{}^{(k)},\tau_3{}^{(k)},\cdots)\rangle}{[u^\dagger(\mathrm{He}^3)\boldsymbol{\sigma}u(\mathrm{H}^3)]},\tag{7}$$

while the variation of $F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)$ with q^2 is assumed given by

$$\frac{F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_A(0;\mathrm{He}^3\to\mathrm{H}^3)}\cong\frac{F_M(q^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_M(0;\mathrm{He}^3\to\mathrm{H}^3)}\tag{8}$$

so that

$$F_A(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)\cong\left[\frac{F_M(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_M(0;\mathrm{H}^3\to\mathrm{He}^3)}\right]\times F_A(0;\mathrm{H}^3\to\mathrm{He}^3)=(0.87)\times(-1.22)=-1.06.\tag{9}$$

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[5] See the analogous calculations for $\mu^-+p\to n+\nu_\mu$ in Ref. 3. Equation (4) is correct within neglect of terms $\approx E_\nu^2/4m_p^2$, $E_\nu^2/4m_a^2$, $E_\nu^2/4m_pm_a$, $\cdots$. Also see Ref. 4.

[6] H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. **138**, B57 (1965).

[7] See Refs. 3 and 4.

The assumption in Eq. (8) is suggested by the impulse-approximation results[8]

$$\frac{F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_A(0;\mathrm{He}^3\to\mathrm{H}^3)} \cong \frac{F_A(q^2;p\to n)\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\boldsymbol{\sigma}^{(k)}\exp(i\mathbf{q}\cdot\mathbf{r}^{(k)})|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle}{F_A(0;p\to n)\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\boldsymbol{\sigma}^{(k)}|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle}$$

$$\cong\left[\frac{F_V(q^2;p\to n)}{F_V(0;p\to n)}\right]\frac{\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\boldsymbol{\sigma}^{(k)}\exp(i\mathbf{q}\cdot\mathbf{r}^{(k)})|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle}{\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\boldsymbol{\sigma}^{(k)}|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle} \quad (10)$$

and

$$\frac{F_M(q^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_M(0;\mathrm{He}^3\to\mathrm{H}^3)} \cong \frac{F_M(q^2;p\to n)\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\gamma_4^{(k)}\boldsymbol{\sigma}^{(k)}\exp(i\mathbf{q}\cdot\mathbf{r}^{(k)})|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle}{F_M(0;p\to n)\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\gamma_4^{(k)}\boldsymbol{\sigma}^{(k)}|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle}$$

$$\cong\left[\frac{F_V(q^2;p\to n)}{F_V(0;p\to n)}\right]\frac{\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\gamma_4^{(k)}\boldsymbol{\sigma}^{(k)}\exp(i\mathbf{q}\cdot\mathbf{r}^{(k)})|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle}{\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\gamma_4^{(k)}\boldsymbol{\sigma}^{(k)}|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle} \quad (11)$$

since $\gamma_4^{(k)}\cong 1$ in nonrelativistic approximation for nucleon motion. Further, we postulate the general validity of the PCAC hypothesis and of an associated pion-pole-dominance assumption whence follows the general validity of the G-T relation [see Eqs. (13)–(18) of I]

$$F_P(0;\mathrm{H}^3\to\mathrm{He}^3)\cong -F_A(0;\mathrm{H}^3\to\mathrm{He}^3)\,, \quad (12)$$

so that, using Eq. (6),

$$F_P(0;\mathrm{H}^3\to\mathrm{He}^3)\cong 1.22\,. \quad (13)$$

As regards the variation of $F_P(q^2;\mathrm{He}^3\to\mathrm{H}^3)$ with q^2, we can write on the basis of the impulse approximation

$$\frac{F_P(q^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_P(0;\mathrm{He}^3\to\mathrm{H}^3)} \cong \frac{F_P(q^2;p\to n)\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\gamma_4^{(k)}\gamma_5^{(k)}\exp(i\mathbf{q}\cdot\mathbf{r}^{(k)})|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle}{F_P(0;p\to n)\langle\Psi(\mathrm{H}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)|\sum_{k=1}^{3}\tau_-^{(k)}\gamma_4^{(k)}\gamma_5^{(k)}|\Psi(\mathrm{He}^3;\cdots,\mathbf{r}^{(k)},\sigma_3^{(k)},\tau_3^{(k)},\cdots)\rangle}$$

$$\cong\left[\left(\frac{F_P(q^2;p\to n)}{F_P(0;p\to n)}\right)\bigg/\left(\frac{F_A(q^2;p\to n)}{F_A(0;p\to n)}\right)\right]\frac{F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_A(0;\mathrm{He}^3\to\mathrm{H}^3)}\,, \quad (14)$$

[8] See the last of the Refs. 4. The CVC hypothesis together with electron-proton and electron-neutron scattering data for $F_{\mathrm{Dirac}}(q^2;\text{prot})$, $F_{\mathrm{Dirac}}(q^2;\text{neut})$, $F_{\mathrm{Pauli}}(q^2;\text{prot})$, $F_{\mathrm{Pauli}}(q^2;\text{neut})$ indicates that $(F_M(q^2;p\to n)/F_M(0;p\to n))\cong(F_V(q^2;p\to n)/(F_V(0;p\to n))$ while the CERN $\nu_\mu+n\to\mu^-+p$ experiments are consistent with $(F_A(q^2;p\to n)/F_A(0;p\to n))\cong(F_V(q^2;p\to n)/F_V(0;p\to n))$.

since nonrelativistically $\gamma_4^{(k)}\gamma_5^{(k)}\cong\boldsymbol{\sigma}^{(k)}\cdot(\mathbf{q}/2m_p)$. Furthermore, since $(F_P(q^2;p\to n)/F_P(0;p\to n))\cong m_\pi^2/(m_\pi^2+q^2)$ and $(F_A(q^2;p\to n)/F_A(0;p\to n))\cong 1$ for $0\leq q^2\lesssim m_\mu^2$ [see Eqs. (11)-(16) of I] we have

$$\frac{F_P(q^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_P(0;\mathrm{He}^3\to\mathrm{H}^3)}\cong\left(\frac{m_\pi^2}{m_\pi^2+q^2}\right)\frac{F_A(q^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_A(0;\mathrm{He}^3\to\mathrm{H}^3)}\tag{15}$$

so that, using also Eqs. (12) and (9),[9]

$$F_P(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)\cong\left(\frac{m_\pi^2}{m_\pi^2+0.96m_\mu^2}\right)\times(-F_A(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3))$$
$$=0.635\times(1.06)=0.67.\tag{16}$$

Substitution of Eqs. (5), (6), (9), and (16) into Eq. (4) yields

$$R(\mathrm{He}^3,\mathrm{H}^3)=0.834\tag{17}$$

and this, together with the value $C(\mathrm{He}^3)=0.965$ (see Appendix) and with

$$(ft)_{\mathrm{exper}}=\{[f(\mathrm{H}^3\to\mathrm{He}^3)/\Gamma({}_1\mathrm{H}_2{}^3\to{}_2\mathrm{He}_1{}^3+e^-+\bar{\nu}_e)]_{\mathrm{exper}}\}\ln 2=1137\pm 20,$$

gives, upon use of Eq. (2),[10]

$$[\Gamma(\mu^-+{}_2\mathrm{He}_1{}^3\to\nu_\mu+{}_1\mathrm{H}_2{}^3)]_{\mathrm{theor}}=(1.51\pm0.04)\times10^3\ \mathrm{sec}^{-1}\tag{18}$$

in excellent agreement with the most recent experimental value[11]

$$[\Gamma(\mu^-+{}_2\mathrm{He}_1{}^3\to\nu_\mu+{}_1\mathrm{H}_2{}^3)]_{\mathrm{exper}}=(1.505\pm0.046)\times10^3\ \mathrm{sec}^{-1}.\tag{19}$$

It is worth emphasizing explicitly that this excellent agreement is a strong argument in favor of the CVC-implied weak magnetism term and, to some extent, of the PCAC-implied G-T relation for the induced-pseudoscalar term. Thus, for example, if the numerical value of $-F_M(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)$ is decreased from 4.73 [Eq. (5)] to 3, $[\Gamma(\mu^-+\mathrm{He}^3\to\nu_\mu+\mathrm{H}^3)]_{\mathrm{theor}}$ is decreased from $(1.51\pm0.04)\times10^3\ \mathrm{sec}^{-1}$ to $(1.36\pm0.04)\times10^3\ \mathrm{sec}^{-1}$, while if the numerical value of $F_P(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)$ is increased from 0.67 [Eq. (16)] to 1, $[\Gamma(\mu^-+\mathrm{He}^3\to\nu_\mu+\mathrm{H}^3)]_{\mathrm{theor}}$ is decreased from $(1.51\pm0.04)\times10^3\ \mathrm{sec}^{-1}$ to $(1.44\pm0.04)\times10^3\ \mathrm{sec}^{-1}$.

$\mu^-+{}_6\mathrm{C}_6{}^{12}\to\nu_\mu+{}_5\mathrm{B}_7{}^{12}$ and $\mu^-+{}_3\mathrm{Li}_3{}^6\to\nu_\mu+{}_2\mathrm{He}_4{}^6$

In the case of the muon-capture reactions $\mu^-+{}_6\mathrm{C}_6{}^{12}\to\nu_\mu+{}_5\mathrm{B}_7{}^{12}$ and $\mu^-+{}_3\mathrm{Li}_3{}^6\to\nu_\mu+{}_2\mathrm{He}_4{}^6$ we have from Eq. (31) of I

$$\begin{aligned}
&\langle\mathrm{B}^{12};\cdots M_b\cdots|j_\lambda^{(V)}|\mathrm{C}^{12};\cdots M_a\cdots\rangle=\{[Q_\lambda+(m_b^2-m_a^2)(q_\lambda/q^2)](F_{\mathrm{Ch}}(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})/2m_a)\\
&\qquad-\epsilon_{\lambda\rho\kappa}S_\kappa^*(q_\rho/2m_p)F_M(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})\}\\
&\langle\mathrm{B}^{12};\cdots M_b\cdots|j_\lambda^{(A)}|\mathrm{C}^{12};\cdots M_a\cdots\rangle=\{iS_\lambda^*F_A(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})+(iq_\lambda S^*\cdot q/m_\pi^2)F_P(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})\};\\
&F_{M,A,P,\mathrm{Ch}}(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})=F_{M,A,P,\mathrm{Ch}}(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12});\\
&Q_\lambda\equiv(p_a+p_b)_\lambda;\quad \lim_{q^2\to0}[F_{\mathrm{Ch}}(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})/q^2]=\text{finite constant};\\
&S_\lambda\equiv[S(S+1)]^{1/2}\xi_\lambda(M_b)=\sqrt{2}\xi_\lambda(M_b);\quad \xi_\lambda(M_b)\equiv\text{spin-one type polarization four-vector};\quad \xi(M_b)\cdot p_b=0;\\
&q^2=(p_b-p_a)^2=(p_\mu-p_\nu)^2=-m_\mu^2+2m_\mu E_\nu\cong m_\mu^2+2m_\mu(m_a-m_b-(m_\mu^2/2m_b))=0.73m_\mu^2;\\
&\langle\mathrm{C}^{12};\cdots M_a\cdots|j_\lambda^{(V)}|\mathrm{B}^{12};\cdots M_b\cdots\rangle\cong0,\\
&\langle\mathrm{C}^{12};\cdots M_a\cdots|j_\lambda^{(A)}|\mathrm{B}^{12};\cdots M_b\cdots\rangle\cong iS_\lambda(1-\delta_{\lambda4})F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12});\\
&q^2=(p_a-p_b)^2=(p_e+p_{\bar\nu})^2=-m_e^2+2(\mathbf{p}_e\cdot\mathbf{p}_{\bar\nu}-E_eE_{\bar\nu})\cong-\tfrac{1}{2}(m_b-m_a)^2\ll1/\langle r^2\rangle_b,\quad 1/\langle r^2\rangle_a.
\end{aligned}\tag{20}$$

[9] The G-T value of $F_P(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)$ in Eq. (16):

$$\left[\frac{m_\mu(m_a+m_b)}{m_\pi^2}\right]F_P(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)\cong\left[\frac{m_\mu(m_a+m_b)}{m_\pi^2}\right]\left(\frac{m_\pi^2}{m_\pi^2+0.96m_\mu^2}\right)(-F_A(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3))$$
$$\cong\left[\frac{m_\mu(m_a+m_b)}{m_\pi^2}\right]\left(\frac{m_\pi^2}{m_\pi^2+0.96m_\mu^2}\right)\left[\frac{F_M(0.96m_\mu^2;\mathrm{He}^3\to\mathrm{H}^3)}{F_M(0;\mathrm{H}^3\to\mathrm{He}^3)}\right](-F_A(0;\mathrm{H}^3\to\mathrm{He}^3))=16.9(-F_A(0;\mathrm{H}^3\to\mathrm{He}^3))$$

corresponds to the familiar G-T value of $F_P(0.88m_\mu^2;p\to n)$ (see the last of the Refs. 4):

$$\left[\frac{m_\mu(m_p+m_n)}{m_\pi^2}\right]F_P(0.88m_\mu^2;p\to n)=\left[\frac{m_\mu(m_p+m_n)}{m_\pi^2}\right]\left(\frac{m_\pi^2}{m_\pi^2+0.88m_\mu^2}\right)[-F_A(0;n\to p)]=6.7[-F_A(0;n\to p)].$$

[10] The uncertainty of ±0.04 in Eq. (18) arises largely from the uncertainty in the experimental values of $F_{\mathrm{Dirac}}(q^2;\mathrm{He}^3)$, $F_{\mathrm{Dirac}}(q^2;\mathrm{H}^3)$, $F_{\mathrm{Pauli}}(q^2;\mathrm{He}^3)$, $F_{\mathrm{Pauli}}(q^2;\mathrm{H}^3)$ (see Ref. 6), and from the uncertainty in $(ft)_{\mathrm{exper}}$ for ${}_1\mathrm{H}_2{}^3\to{}_2\mathrm{He}_1{}^3+e^-+\bar\nu_e$. We wish to thank Dr. R. J. Esterling for a helpful comment on this point.

[11] L. B. Auerbach, R. J. Esterling, R. E. Hill, D. A. Jenkins, J. T. Lach, and N. H. Lipman, Phys. Rev. **138**, B127 (1965). Full references to all previous experimental and theoretical work on the $\mu^-+\mathrm{He}^3\to\nu_\mu+\mathrm{H}^3$ reaction are given in this paper. *Note added in proof.* See also D. R. Clay, J. W. Keuffel, R. L. Wagner, Jr., and R. M. Edelstein (to be published), who find $[\Gamma(\mu^-+{}_2\mathrm{He}_1{}^3\to\nu_\mu+{}_1\mathrm{H}_2{}^3)]_{\mathrm{exper}}=(1.465\pm0.067)\times10^3\ \mathrm{sec}^{-1}$.

Thus, carrying out the indicated sums over M_a, M_b, κ, λ in Eqs. (1) and (2), we have[12]

$$R(\mathrm{C}^{12},\mathrm{B}^{12})=\frac{3\times2\{[G_A(0.73m_\mu^2;\mathrm{C}^{12}\to\mathrm{B}^{12})]^2-\frac{2}{3}[G_A(0.73m_\mu^2;\mathrm{C}^{12}\to\mathrm{B}^{12})][G_P(0.73m_\mu^2;\mathrm{C}^{12}\to\mathrm{B}^{12})]+\frac{1}{3}[G_P(0.73m_\mu^2;\mathrm{C}^{12}\to\mathrm{B}^{12})]^2\}}{2[F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12})]^2}; \tag{21}$$

$$G_A(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})\equiv-F_A(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})-F_M(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})(E_\nu/2m_p),$$

$$G_P(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})\equiv F_P(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})(E_\nu^2/m_\pi^2)-F_M(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})(E_\nu/2m_p).$$

In an entirely similar way

$$R(\mathrm{Li}^6,\mathrm{He}^6)=\frac{2\{[G_A(0.91m_\mu^2;\mathrm{Li}^6\to\mathrm{He}^6)]^2-\frac{2}{3}[G_A(0.91m_\mu^2;\mathrm{Li}^6\to\mathrm{He}^6)][G_P(0.91m_\mu^2;\mathrm{Li}^6\to\mathrm{He}^6)]+\frac{1}{3}[G_P(0.91m_\mu^2;\mathrm{Li}^6\to\mathrm{He}^6)]^2\}}{3\times2[F_A(0;\mathrm{He}^6\to\mathrm{Li}^6)]^2}; \tag{22}$$

$$G_A(q^2;\mathrm{Li}^6\to\mathrm{He}^6)\equiv-F_A(q^2;\mathrm{Li}^6\to\mathrm{He}^6)-F_M(q^2;\mathrm{Li}^6\to\mathrm{He}^6)(E_\nu/2m_p),$$

$$G_P(q^2;\mathrm{Li}^6\to\mathrm{He}^6)\equiv F_P(q^2;\mathrm{Li}^6\to\mathrm{He}^6)(E_\nu^2/m_\pi^2)-F_M(q^2;\mathrm{Li}^6\to\mathrm{He}^6)(E_\nu/2m_p).$$

We must now specify the numerical values of the form factors in Eqs. (21), (22). First of all, Eqs. (20) and (1) show that the value of $|F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12})|$ can be calculated from the observed rate of the beta-decay reaction ${}_5\mathrm{B}_7{}^{12}\to{}_6\mathrm{C}_6{}^{12}+e^-+\bar{\nu}_e$, viz.;

$$|F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12})|=0.515 \tag{23}$$

and similarly

$$|F_A(0;\mathrm{He}^6\to\mathrm{Li}^6)|=1.13. \tag{24}$$

Further, we again assume the general validity of the PCAC hypothesis and of the associated pion-pole-dominance assumption, and so, of the G-T relation, viz. [see Eqs. (31)–(33) of I]

$$F_P(0;\mathrm{B}^{12}\to\mathrm{C}^{12})\cong-F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12});\quad F_P(0;\mathrm{He}^6\to\mathrm{Li}^6)\cong-F_A(0;\mathrm{He}^6\to\mathrm{Li}^6), \tag{25}$$

and, again suppose, analogously to Eqs. (8), (15), and (12),

$$\begin{aligned}F_A(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})&=F_A(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12})\cong[F_M(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12})/F_M(0;\mathrm{B}^{12}\to\mathrm{C}^{12})]\cdot F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12})\\&\equiv\mathfrak{F}_M(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12})\cdot F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12});\\F_A(q^2;\mathrm{Li}^6\to\mathrm{He}^6)&=F_A(q^2;\mathrm{He}^6\to\mathrm{Li}^6)\cong[F_M(q^2;\mathrm{He}^6\to\mathrm{Li}^6)/F_M(0;\mathrm{He}^6\to\mathrm{Li}^6)]\cdot F_A(0;\mathrm{He}^6\to\mathrm{Li}^6)\\&\equiv\mathfrak{F}_M(q^2;\mathrm{He}^6\to\mathrm{Li}^6)\cdot F_A(0;\mathrm{He}^6\to\mathrm{Li}^6)\end{aligned} \tag{26}$$

and

$$\begin{aligned}F_P(q^2;\mathrm{C}^{12}\to\mathrm{B}^{12})&=F_P(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12})\\&\cong(m_\pi^2/(m_\pi^2+q^2))[F_A(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12})/F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12})]F_P(0;\mathrm{B}^{12}\to\mathrm{C}^{12})\\&\cong(m_\pi^2/(m_\pi^2+q^2))(-F_A(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12}))=(m_\pi^2/(m_\pi^2+q^2))\mathfrak{F}_M(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12})(-F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12}));\\F_P(q^2;\mathrm{Li}^6\to\mathrm{He}^6)&=F_P(q^2;\mathrm{He}^6\to\mathrm{Li}^6)\\&\cong(m_\pi^2/(m_\pi^2+q^2))[F_A(q^2;\mathrm{He}^6\to\mathrm{Li}^6)/F_A(0;\mathrm{He}^6\to\mathrm{Li}^6)]F_P(0;\mathrm{He}^6\to\mathrm{Li}^6)\\&\cong(m_\pi^2/(m_\pi^2+q^2))(-F_A(q^2;\mathrm{He}^6\to\mathrm{Li}^6))=(m_\pi^2/(m_\pi^2+q^2))\mathfrak{F}_M(q^2;\mathrm{He}^6\to\mathrm{Li}^6)(-F_A(0;\mathrm{He}^6\to\mathrm{Li}^6)).\end{aligned} \tag{27}$$

It thus remains to find $F_M(0;\mathrm{B}^{12}\to\mathrm{C}^{12})$ and $\mathfrak{F}_M(q^2;\mathrm{B}^{12}\to\mathrm{C}^{12})$, and, $F_M(0;\mathrm{He}^6\to\mathrm{Li}^6)$ and $\mathfrak{F}_M(q^2;\mathrm{He}^6\to\mathrm{Li}^6)$.

To find $F_M(0;\mathrm{B}^{12}\to\mathrm{C}^{12})$ we consider the weak-magnetism correction factor to the otherwise "allowed" ${}_5\mathrm{B}_7{}^{12}\to{}_6\mathrm{C}_6{}^{12}+e^-+\bar{\nu}_e$ electron-energy spectrum; this is calculated from Eq. (1) with use of the $\mathrm{C}^{12}\leftrightarrow\mathrm{B}^{12}$ transition matrix element in Eq. (20) as[13]

$$\begin{aligned}dE_eN(E_e)&=dE_e[E_e(E_e^2-m_e^2)^{1/2}(m_b-m_a-E_e)^2F(Z(\mathrm{C}^{12}),E_e)](1+aE_e),\\a&\equiv\frac{8}{3}\left(\frac{1}{2m_p}\right)\left[\frac{F_M(0;\mathrm{B}^{12}\to\mathrm{C}^{12})}{\sqrt{2}F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12})}\right]-\frac{16}{9}\frac{Z(\mathrm{C}^{12})}{137}R(\mathrm{C}^{12}),\\&=\frac{8}{3}\left(\frac{1}{2m_p}\right)\left[\frac{F_M(0;\mathrm{B}^{12}\to\mathrm{C}^{12})}{\sqrt{2}F_A(0;\mathrm{B}^{12}\to\mathrm{C}^{12})}\right]-1.1\times10^{-3}/\mathrm{MeV},\end{aligned} \tag{28}$$

[12] Equation (21) is correct within neglect of terms $\approx E_\nu/2m_a$, $E_\nu^2/4m_p^2$, $\cdots$.
[13] The second term in a is a Coulomb correction; $R(\mathrm{C}^{12})=(12)^{1/3}(5.7/m_p)$ is the radius of the C^{12} nucleus.

while experimentally[14]

$$a=(5.5\pm1.0)\times10^{-3}/\text{MeV}. \tag{29}$$

Equations (28), (29), and (23) yield

$$F_M(0;B^{12}\to C^{12})=(6.57\pm1.0)F_A(0;B^{12}\to C^{12})=\pm(3.4\pm0.5). \tag{30}$$

Alternatively, the CVC hypothesis predicts

$$F_M(q^2;B^{12}\to C^{12})=\sqrt{2}\mu(q^2;C^{12*}\to C^{12}), \tag{31}$$

where $\mu(q^2;C^{12*}\to C^{12})$ is the transition magnetic moment from the C^{12} excited state which is in the same isotriplet as the B^{12} ground state to the C^{12} ground state; $\mu(q^2=(p_a-p_a')^2=p_\gamma^2=0;C^{12*}\to C^{12})$ is determined from the observed value of the corresponding transition rate for $C^{12*}\to C^{12}+\gamma(N_a'\to N_a+\gamma)$ photon emission as[15]

$$\begin{aligned}&\Gamma(C^{12*}\to C^{12}+\gamma)=\tfrac{1}{3}(1/137)[\mu(0;C^{12*}\to C^{12})]^2(E_\gamma^3/m_p^2)=(50\pm5)\text{ eV},\\&|\mu(0;C^{12*}\to C^{12})|=2.30\pm0.10,\end{aligned} \tag{32}$$

so that, using Eq. (31),

$$|F_M(0;B^{12}\to C^{12})|=3.24\pm0.15 \tag{33}$$

in excellent agreement with Eq. (30). As regards the value of $|F_M(0;He^6\to Li^6)|$ no study of the weak-magnetism correction factor has been made and only the CVC-based method of Eqs. (31)–(33) is available; this yields

$$F_M(q^2;He^6\to Li^6)=\sqrt{2}\mu(q^2;Li^{6*}\to Li^6) \tag{34}$$

with[16]

$$\begin{aligned}&\Gamma(Li^{6*}\to Li^6)=(1/137)[\mu(0;Li^{6*}\to Li^6)]^2E_\gamma^3/m_p^2=6.4\pm0.6\text{ eV};\\&|F_M(0;He^6\to Li^6)|=\sqrt{2}|\mu(0;Li^{6*}\to Li^6)|=\sqrt{2}(4.14\pm0.20)=5.83\pm0.30,\end{aligned} \tag{35}$$

where $\mu(q^2;Li^{6*}\to Li^6)$ is the transition magnetic moment from the Li^6 excited state which is in the same isotriplet as the He^6 ground state to the Li^6 ground state. Thus, substitution of Eqs. (23), (24), (26), (27), (30), (33), and (35) into Eqs. (21), (22) gives

$$\begin{aligned}
G_A(0.73m_\mu^2;C^{12}\to B^{12})&=-[F_A(0;B^{12}\to C^{12})+F_M(0;B^{12}\to C^{12})(0.87m_\mu/2m_p)]\mathfrak{F}_M(0.73m_\mu^2;B^{12}\to C^{12})\\
&=\mp(0.672)\mathfrak{F}_M(0.73m_\mu^2;B^{12}\to C^{12}),\\
G_P(0.73m_\mu^2;C^{12}\to B^{12})&=-\left[\left(\frac{m_\pi^2}{m_\pi^2+0.73m_\mu^2}\right)F_A(0;B^{12}\to C^{12})\left(\frac{0.87m_\mu}{m_\pi}\right)^2+F_M(0;B^{12}\to C^{12})(0.87m_\mu/2m_p)\right]\\
&\quad\times\mathfrak{F}_M(0.73m_\mu^2;B^{12}\to C^{12})=\mp(0.316)\mathfrak{F}_M(0.73m_\mu^2;B^{12}\to C^{12});\\
R(C^{12},B^{12})&=3\times1.30[\mathfrak{F}_M(0.73m_\mu^2;B^{12}\to C^{12})]^2;\\
G_A(0.91m_\mu^2;Li^6\to He^6)&=-[F_A(0;He^6\to Li^6)+F_M(0;He^6\to Li^6)(0.96m_\mu/2m_p)]\mathfrak{F}_M(0.91m_\mu^2;He^6\to Li^6)\\
&=\mp(1.44)\mathfrak{F}_M(0.91m_\mu^2;He^6\to Li^6),\\
G_P(0.91m_\mu^2;Li^6\to He^6)&=-\left[\left(\frac{m_\pi^2}{m_\pi^2+0.91m_\mu^2}\right)F_A(0;He^6\to Li^6)\left(\frac{0.96m_\mu}{m_\pi}\right)^2+F_M(0;He^6\to Li^6)(0.96m_\mu/2m_p)\right]\\
&\quad\times\mathfrak{F}_M(0.91m_\mu^2;He^6\to Li^6)=\mp(0.703)\mathfrak{F}_M(0.91m_\mu^2;He^6\to Li^6);\\
R(Li^6,He^6)&=\tfrac{1}{3}\times1.23[\mathfrak{F}_M(0.91m_\mu^2;He^6\to Li^6)]^2;
\end{aligned} \tag{36}$$

and it only remains to determine the numerical values of $\mathfrak{F}_M(0.73m_\mu^2;B^{12}\to C^{12})$ and $\mathfrak{F}_M(0.91m_\mu^2;He^6\to Li^6)$.

[14] See C. S. Wu, Rev. Mod. Phys. **36**, 618 (1964).

[15] The "observed" value of $\Gamma(C^{12*}\to C^{12}+\gamma)$ given in Eq. (32) is a weighted average of four recent measurements as quoted in T. Mayer-Kuckuk and F. Michel, Phys. Rev. **127**, 547 (1962).

[16] The "observed" value of $\Gamma(Li^{6*}\to Li^6+\gamma)$ given in Eq. (35) is the weighted average of a measurement by W. Barber, F. Berthold, G. Fricke, and F. E. Gudden, Phys. Rev. **120**, 2081 (1960) [$\Gamma(Li^{6*}\to Li^6+\gamma)=6.2\pm0.6$ eV] and of a measurement by L. Cohen and R. Tobin, Nucl. Phys. **14**, 243 (1959) [$\Gamma(Li^{6*}\to Li^6+\gamma)=9.1\pm2.0$ eV].

The numerical values of $\mathfrak{F}_M(0.73m_\mu^2; B^{12}\to C^{12})$ and $\mathfrak{F}_M(0.91m_\mu^2; He^6\to Li^6)$ are given, on the basis of Eqs. (31), (32), (34), (35) which are implied by the CVC hypothesis and the defining Eq. (26), by

$$\begin{aligned}\mathfrak{F}_M(q^2; B^{12}\to C^{12}) &= [\mu(q^2; C^{12*}\to C^{12})/\mu(0; C^{12*}\to C^{12})] = [\mu(q^2; C^{12*}\to C^{12})/2.30];\\ \mathfrak{F}_M(q^2; He^6\to Li^6) &= [\mu(q^2; Li^{6*}\to Li^6)/\mu(0; Li^{6*}\to Li^6)] = [\mu(q^2; Li^{6*}\to Li^6)/4.14]\end{aligned} \tag{37}$$

with $\mu(q^2; C^{12*}\to C^{12})$ and $\mu(q^2; Li^{6*}\to Li^6)$ determined by the differential cross sections for the inelastic scattering processes: $e^-+C^{12}\to e^-+C^{12*}$ and $e^-+Li^6\to e^-+Li^{6*}$. At an electron scattering angle of 180°, these differential cross sections are of the form

$$\begin{aligned}
\frac{d\sigma(e^-+C^{12}\to e^-+C^{12*}; E_e, 180°)}{d\Omega} &= \left(\frac{e^4}{2m_p^2}\right)\left(\frac{|\mathbf{q}|}{|\mathbf{q}|+(m_a'-m_a)}\right)^2[\mu(q^2; C^{12*}\to C^{12})]^2\\
&= \left(\frac{e^4}{2m_p^2}\right)\left(\frac{|\mathbf{q}|}{|\mathbf{q}|+(m_a'-m_a)}\right)^2[\mu(0; C^{12*}\to C^{12})]^2[\mathfrak{F}_M(q^2; C^{12*}\to C^{12})]^2\\
&= (1.15\times10^{-32})\times(0.855)^2\times(2.30)^2[\mathfrak{F}_M(q^2; C^{12*}\to C^{12})]^2 \text{ cm}^2/\text{sr}\\
&= 4.46\times10^{-32}[\mathfrak{F}_M(q^2; C^{12*}\to C^{12})]^2 \text{ cm}^2/\text{sr};\\
q^2 &= |\mathbf{q}|^2-(i^{-1}q_4)^2 = |\mathbf{p}_a'-\mathbf{p}_a|^2-(E_a'-E_a)^2 = |\mathbf{p}_e-\mathbf{p}_e'|^2-(E_e-E_e')^2\\
&\cong [(E_e+E_e')^2-2E_eE_e'(1+\cos180°)]-(E_e-E_e')^2\\
&\cong [2E_e-(m_a'-m_a)]^2-(m_a'-m_a)^2; \quad m_a'-m_a=15.1\text{ MeV};\\
\frac{d\sigma(e^-+Li^6\to e^-+Li^{6*}; E_e, 180°)}{d\Omega} &= \frac{1}{3}\left(\frac{e^4}{2m_p^2}\right)\left(\frac{|\mathbf{q}|}{|\mathbf{q}|+(m_a'-m_a)}\right)^2[\mu(q^2; Li^{6*}\to Li^6)]^2\\
&= \frac{1}{3}\left(\frac{e^4}{2m_p^2}\right)\left(\frac{|\mathbf{q}|}{|\mathbf{q}|+(m_a'-m_a)}\right)^2[\mu(0; Li^{6*}\to Li^6)]^2[\mathfrak{F}_M(q^2; Li^{6*}\to Li^6)]^2\\
&= \tfrac{1}{3}(1.15\times10^{-32})\times(0.966)^2\times(4.14)^2[\mathfrak{F}_M(q^2; Li^{6*}\to Li^6)]^2 \text{ cm}^2/\text{sr};\\
&= 6.13\times10^{-32}[\mathfrak{F}_M(q^2; Li^{6*}\to Li^6)]^2 \text{ cm}^2/\text{sr};\\
q^2 &= |\mathbf{q}|^2-(i^{-1}q_4)^2 = |\mathbf{p}_a'-\mathbf{p}_a|^2-(E_a'-E_a)^2 = |\mathbf{p}_e-\mathbf{p}_e'|^2-(E_e-E_e')^2\\
&\cong [(E_e+E_e')^2-2E_eE_e'(1+\cos180°)]-(E_e-E_e')^2\\
&\cong [2E_e-(m_a'-m_a)]^2-(m_a'-m_a)^2; \quad m_a'-m_a=3.56\text{ MeV};
\end{aligned} \tag{38}$$

with $q^2=0.73m_\mu^2=(91\text{ MeV})^2$ and $q^2=0.91m_\mu^2=(101\text{ MeV})^2$ corresponding to $|\mathbf{q}|=92$ MeV, $E_e=53.5$ MeV and $|\mathbf{q}|=101$ MeV, $E_e=57$ MeV, respectively. Then, with the interpolated experimental values[17]

$$\begin{aligned}d\sigma(e^-+C^{12}\to e^-+C^{12*}; E_e=53.5\text{ MeV}, 180°)/d\Omega &= (2.00\pm0.30)\times10^{-32}\text{ cm}^2/\text{sr},\\ d\sigma(e^-+Li^6\to e^-+Li^{6*}; E_e=57\text{ MeV}, 180°)/d\Omega &= (1.75\pm0.30)\times10^{-32}\text{ cm}^2/\text{sr},\end{aligned} \tag{39}$$

we have

$$\begin{aligned}[\mathfrak{F}_M(0.73m_\mu^2; C^{12*}\to C^{12})]^2 &= 0.448\pm0.070,\\ [\mathfrak{F}_M(0.91m_\mu^2; Li^{6*}\to Li^6)]^2 &= 0.286\pm0.040,\end{aligned} \tag{40}$$

whence, using Eq. (36),

$$\begin{aligned}R(C^{12},B^{12}) &= 1.75\pm0.20,\\ R(Li^6,He^6) &= 0.117\pm0.018.\end{aligned} \tag{41}$$

Thus, substituting Eq. (41) into Eq. (2), and with $C(C^{12})=0.885$, $C(Li^6)=0.928$ (see Appendix), $(ft)_{\text{exper}}$

[17] J. Goldemberg, W. C. Barber, F. H. Lewis, Jr., and J. D. Walecka, Phys. Rev. **134**, B1022 (1964).

$=11\,700\pm300$ for $B^{12}\rightarrow C^{12}$, $(ft)_{\text{exper}}=808\pm30$ for $He^6\rightarrow Li^6$, we finally obtain[18]

$$[\Gamma(\mu^-+{}_6C_6{}^{12}\rightarrow \nu_\mu+{}_5B_7{}^{12})]_{\text{theor}}=(6.6\pm1.0)\times10^3\ \text{sec}^{-1} \tag{42}$$

and

$$[\Gamma(\mu^-+{}_3Li_3{}^6\rightarrow \nu_\mu+{}_2He_4{}^6)]_{\text{theor}}=(0.98\pm0.15)\times10^3\ \text{sec}^{-1}. \tag{43}$$

These theoretical predictions for the indicated muon capture rates can be compared with experiment in the case of C^{12} where the most recent measurements yield[19,20]

$$\begin{aligned}[\Gamma(\mu^-+{}_6C_6{}^{12}\rightarrow \nu_\mu+{}_5B_7{}^{12})]_{\text{exper}}&=(6.7\pm0.9)\times10^3\ \text{sec}^{-1}\\ &=(6.75_{-0.75}{}^{+0.30})\times10^3\ \text{sec}^{-1}.\end{aligned} \tag{44}$$

In the case of Li^6 no measurements are as yet available but previous nuclear model-impulse approximation calculations have given values ranging from 0.4×10^3 sec^{-1} to 2.1×10^3 sec^{-1}.[21]

APPENDIX

In this Appendix we describe the calculation of $C(N_a)$, the correction factor arising from the nonpoint character of the charge distribution of N_a. We have

$$\begin{aligned}&\left\{-\frac{1}{2\mu}\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)+V(r)-E\right\}\Psi_E(r)=0;\\ &V(r)=(2\mu aR)^{-1}[-3+r^2/R^2]:\quad 0\leqq r\leqq R,\\ &V(r)=(\mu a^2)^{-1}[-a/r]:\qquad r\geqq R;\\ &\mu\equiv m_\mu m_a/(m_\mu+m_a),\quad a\equiv137/Z\mu,\end{aligned} \tag{A1}$$

where E and $\Psi_E(r)$ are, respectively, the muon energy-eigenvalue and muon energy-eigenfunction appropriate to a muon $1s$ orbit about a nucleus of charge $Z(N_a)\equiv Z$ and radius $R(N_a)\equiv R=(5/3)^{1/2}\times$[root-mean-square charge radius]. In terms of $\Psi_E(r)$, $C(N_a)$ is given by

$$C(N_a)=\pi a^3\left[\int_0^R \Psi_E(r)r^2dr\bigg/\int_0^R r^2dr\right]^2. \tag{A2}$$

To calculate $\Psi_E(r)$, and so $C(N_a)$, we note that Eq. (A1) yields

$$\begin{aligned}\Psi_E(r)=&\{[N(\pi a^3)^{-1/2}G(-\eta+1,2;2R/\eta a)\exp(-R/\eta a)][\exp(\tfrac{1}{2}(R/a)^{1/2})/F(\tfrac{1}{2}(\tfrac{3}{2}-\epsilon),\tfrac{3}{2};(R/a)^{1/2})]\}\\ &\times F(\tfrac{1}{2}(\tfrac{3}{2}-\epsilon),\tfrac{3}{2};(r^2/a^2)(R/a)^{1/2})\exp(-\tfrac{1}{2}(r^2/R^2)(R/a)^{1/2}):\quad 0\leqq r\leqq R,\\ \Psi_E(r)=&\{N(\pi a^3)^{-1/2}\}G(-\eta+1,2;2r/\eta a)\exp(-r/\eta a):\quad r\geqq R;\\ &\eta\equiv\left[\frac{(\tfrac{1}{2}(Z/137))/a}{-E}\right]^{1/2},\quad \epsilon\equiv-\frac{1}{2\eta^2}\left(\frac{R^3}{a^3}\right)^{1/2}+\frac{3}{2}\left(\frac{R}{a}\right)^{1/2};\end{aligned} \tag{A3}$$

where $F(\alpha,\beta;x)$ and $G(\alpha,\beta;x)$ are solutions of the confluent hypergeometric differential equation

$$\left[x\frac{d^2}{dx^2}+(\beta-x)\frac{d}{dx}-\alpha\right]\begin{Bmatrix}F(\alpha,\beta;x)\\ G(\alpha,\beta;x)\end{Bmatrix}=0 \tag{A4}$$

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18 See also L. L. Foldy and J. D. Walecka, Nuovo Cimento **35**, 1026 (1964), footnote on p. 1058. *Note added in proof.* Foldy and Walecka (to be published) have very recently given a comprehensive study of the $C^{12}\rightarrow B^{12}$ case on the basis of the impulse approximation but in an essentially nuclear-model-independent way and find a result for the muon capture rate which is in substantial agreement with that in Eq. (42).

19 G. T. Reynolds, D. B. Scarl, R. A. Swanson, J. R. Waters, and R. A. Zdanis, Phys. Rev. **129**, 1790 (1963).

20 E. J. Maier, R. M. Edelstein, and R. T. Siegel, Phys. Rev. **133**, B663 (1964).

21 See A. Lodder and C. C. Jonker, Phys. Letters **15**, 245 (1965).

which have, respectively, uniformly convergent and asymptotically convergent expansions

$$F(\alpha,\beta;x)=1+\frac{\alpha}{\beta}\left(\frac{x}{1!}\right)+\frac{\alpha(\alpha+1)}{\beta(\beta+1)}\left(\frac{x^2}{2!}\right)+\cdots,$$
$$G(\alpha,\beta;x)=\frac{(\beta-1)!}{(\beta-1-\alpha)!}(-x)^{-\alpha}\left\{1-\alpha(\alpha-\beta+1)\left(\frac{1}{1!x}\right)+\alpha(\alpha+1)(\alpha-\beta+1)(\alpha-\beta+2)\left(\frac{1}{2!x^2}\right)+\cdots\right\}. \tag{A5}$$

Also, E or η is determined by the continuity of $(d/dr)\ln\Psi_E(r)$ at $r=R$ and N is fixed on the basis of the normalization condition: $\int[\Psi_E(r)]^2 4\pi r^2 dr=1$.

We now confine ourselves to the case of low-Z nuclei, where $R/a\ll 1$. Here

$$\eta\cong 1;\qquad \epsilon\cong\tfrac{3}{2}(R/a)^{1/2};$$
$$G(-\eta+1,2;2r/\eta a)\cong G(0,2;2r/\eta a)=1:\quad r\geqq R;\ N\cong 1. \tag{A6}$$

Substitution of Eqs. (A6) and (A3) into Eq. (A2) then gives

$$C(N_a)\cong\left[\exp\left(-\frac{2R}{a}\right)\right]\left\{\frac{3}{R^3}\int_0^R\frac{F(\frac{1}{2}(\frac{3}{2}-\epsilon),\frac{3}{2};(R/a)^{1/2}(r^2/a^2))}{F(\frac{1}{2}(\frac{3}{2}-\epsilon),\frac{3}{2};(R/a)^{1/2})}\exp\left[-\frac{1}{2}\left(\frac{R}{a}\right)^{1/2}\left(\frac{r^2-R^2}{R^2}\right)\right]r^2dr\right\}^2$$
$$=\begin{cases}0.965: & He^3(Z=2)\\ 0.928: & Li^6(Z=3)\\ 0.885: & C^{12}(Z=6)\end{cases}. \tag{A7}$$

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Proton Total Reaction Cross Sections at 16.4 MeV†

ROBERT E. POLLOCK AND G. SCHRANK*
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
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Total reaction cross sections for protons of a laboratory energy of 16.4 MeV at the center of foil targets of C, Mg, Al, Ni, Cu, and Pb have been measured by a beam attenuation method. The technique differs from other measurements with intermediate energy protons in that a double-focusing magnetic spectrometer is contained within the scintillation counter telescope which precedes the target. The magnet selects a beam free from slit-scattered protons, with a precisely determined momentum, while the focusing compensates for the beam divergence in the first detector so that all detectors see comparable counting rates. Solid-state circuitry with controlled recovery characteristics was developed to permit instantaneous rates in excess of 10^6 protons/sec and to circumvent the problem of a low duty cycle. The measurements require several major corrections, and continuing effort to improve the evaluation of these corrections since this measurement was first described has led to the following values for reaction cross sections:

Target	C	Mg	Al	Ni	Cu	Pb
σ_R(mb)	368	712	701	898	955	1330
Standard deviation	30	56	34	53	64	180

Total reaction cross sections have been predicted by optical-model analyses of proton elastic scattering at this energy with a variety of optical potentials. The measured values for Ni and Cu lie somewhat lower than the predictions of the optical model, while the values for Pb and C are higher than the predictions.

I. INTRODUCTION

TOTAL reaction cross sections determined by experiment can restrict the choice of scattering potential used to describe the nucleon-nucleus interaction. Early in the development of a suitable optical potential, the need for realistic reaction cross sections led to diffuse-edged potentials much as realistic polarizations required the added spin-orbit interactions. With the many-parameter potentials now in common use, it is misleading to speak of one experiment as determining one or another parameter since all are effective to varying extents. A helpful description of the way in

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* Present address: Department of Physics, University of California, Santa Barbara, California.