Role of the Baryon Exchanges in Generating $\frac{3}{2}^+$ Resonances

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The role of the spin-$\frac{1}{2}$ baryon exchanges in generating the Ω^- is examined in the framework of the N/D method. The resulting integral equations are solved numerically to show that the baryon exchanges lead to an extremely overbound state. A comparison is also made between this and a determinantal-approximation calculation to show that an earlier calculation indicating a resonating Ω^- state is due to a deficiency in the determinantal approximation.

I. INTRODUCTION

THE dynamical relationship between the nucleon-exchange Born diagram in pion-nucleon scattering and the well-known 33 resonance (N^*, 1238 MeV) has been a subject of study for a long time. One of the earliest approaches was the Chew-Low model[1] which has been put on a relativistic dispersion-theoretic footing by Frautschi and Walecka.[2] The essential result of these calculations is that the simple nucleon-exchange Born diagram is primarily responsible for the generation of the N^* resonance. In the context of SU_3 symmetry, however, one has to extend this dynamical relationship between the exchange of the nucleon and the generation of N^*, and consider the relationship between the baryon octet exchange and the generation of the baryon decuplet in an SU_3-symmetric model. It is clear that in such approaches one is faced with the problem of considering many coupled channels and so is forced to make some simple approximation scheme for the dynamics. This was first done by Martin and Wali,[3] who worked in the first-order determinantal approximation and showed in particular that the baryon-exchange force was too weak to produce the Ω^- resonance at the correct position. This result was, however, re-examined by Martin and Uretsky,[4] who gave up the first-order determinantal approximation and showed, by completely solving the dispersion equations, that the nucleon-exchange force in π-N single-channel scattering leads to an enormous overbinding in the 33 state. The nucleon force had to be cut off at a very low value for generating the N^* at the correct position. The only flaw in this calculation is that it is not faithful to the unitary symmetry scheme, for in an SU_3-symmetric model one does not have much reason for neglecting the coupled ΣK channel *a priori*, as has been pointed out earlier.[3] One may feel then that their conclusions are liable to change on consideration of the coupled channels. A "clean" case, on the other hand, is presented by the Ω^- channel which is strongly coupled to only one baryon–pseudoscalar-meson system, the $\Xi\bar{K}$ one. The result of a single-channel calculation in the Ω^- case, is in a certain sense therefore equivalent to considerations of all coupled baryon-pseudoscalar channels for the resonances N^*, Y_1^*, Ξ^*. In this note, we wish to re-examine the conclusions drawn in Ref. 4, by considering the $\Xi\bar{K}$ scattering in $T=0$, $P_{3/2}$ state. The exchanges considered will be the $\frac{1}{2}^+$ baryons and we will solve the resulting integral equations numerically. Our calculations will be SU_3-symmetric, except of course for kinematical factors.

II. THE METHOD

The kinematics of $\Xi\bar{K}$ scattering is very similar to that of π-N scattering which has been discussed in detail by a number of authors. In the following, we follow the work of Martin and Uretsky[4] and modify their equations so as to suit a bound-state problem. We reproduce here some of the essential steps. The partial-wave amplitude,

$$f(W)=e^{i\delta}\sin\delta/q\,, \tag{1}$$

in the $T=0$, $P_{3/2}$ state is related to the partial-wave projections of the invariant functions $A(s,t)$ and $B(s,t)$ by

$$f(W)=(16\pi W^2)^{-1}\{[(W+M)^2-\mu^2][A_1+(W-M)B_1]+[(W-M)^2-\mu^2][-A_2+(W+M)B_2]\}\,, \tag{2}$$

where,

$$q^2=[(W^2-M^2-\mu^2)^2-4M^2\mu^2]/4W^2\,, \tag{3}$$

$$[A_l,B_l]=\tfrac{1}{2}\int_{-1}^{1}dx\,P_l(x)[A(s,t),\,B(s,t)]\,, \tag{4}$$

W is the total c.m. energy, M is the mass of the Ξ, μ is the kaon mass, and A, B are defined by the T-matrix,

$$T=-A+\tfrac{1}{2}i(\mathbf{q}_1+\mathbf{q}_2)B\,; \tag{5}$$

q_1 and q_2 are the initial and final meson momenta. The amplitude $f(W)$ does not have the proper behavior

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[1] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).
[2] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).
[3] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).
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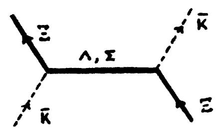

FIG. 1. The $\frac{1}{2}^+$ baryon-exchange forces.

required for a function for which an N/D representation can be written down. From considerations of the behavior of the function at $W=\pm(M+\mu)$ and at $W\to\infty$ we consider the amplitude,

$$h(W)=\rho(W)f(W), \quad (6)$$

in an N/D form. The kinematic factor $\rho(W)$ is given by

$$\rho(W)=W^3/(W+M+\mu)[W^2-(M+\mu)^2]. \quad (7)$$

The baryon-exchange contribution to the functions A and B can be worked out by the usual rules. The result has been given by Choudhury and Pande[5] and by Kane.[6] The contribution to the amplitude $h(W)$ is given by

$$\begin{aligned} h^{\rm Born}(W)=&(W/8(W+M+\mu)^2(W-M-\mu)q^2)\\ &\times[\{(W+M)^2-\mu^2\}\{g_\Sigma^2(W-M_\Sigma)Q_1(a_\Sigma)\\ &+g_\Lambda^2(W-M_\Lambda)Q_l(a_\Lambda)\}+\{(W-M)^2-\mu^2\}\\ &\times\{g_\Sigma^2(W+M_\Sigma)Q_2(a_\Sigma)\\ &+g_\Lambda^2(W+M_\Lambda)Q_2(a_\Lambda)\}], \end{aligned} \quad (8)$$

where the coupling constants g_Λ^2 and g_Σ^2 can be worked out by invoking SU_3 symmetry as

$$g_\Sigma^2=3g_{\pi-N}^2, \quad g_\Lambda^2=-\tfrac{1}{3}(1-4f)^2g_{\pi-N}^2, \quad (9)$$

in terms of the pion-nucleon coupling constant and F-D mixing ratio. The Q_l functions are the usual Legendre functions of the second kind with the arguments

$$a_{\Lambda,\Sigma}=1+(2M^2+2\mu^2-W^2-M_{\Lambda,\Sigma}^2)/2q^2. \quad (10)$$

M_Λ, M_Σ are the masses of the Λ and the Σ particles exchanged in the Born diagram (Fig. 1). We write the amplitude in the form,

$$h(W)=N(W)/D(W), \quad (11)$$

with D having the unitarity cut and N having all the other cuts of $h(W)$. Using the analytic properties of $h(W)$ and unitarity, one gets the integral equations

$$N(W)=h^B(W)+\frac{1}{\pi}\int_{\rm p.c.}\frac{dW'q(W')N(W')}{(W'-W)\rho(W')}\times\left[h^B(W')-\left\{\frac{W-W_0}{W'-W_0}\right\}h^B(W)\right] \quad (12)$$

and

$$D(W)=1-\frac{W-W_0}{\pi}\int_{\rm p.c.}\frac{dW'q(W')N(W')}{(W'-W)(W'-W_0)\rho(W')},$$

where W_0 is the subtraction point and the integral over the physical cuts is defined as

$$\int_{\rm p.c.}\cdots dW'=\int_{-\infty}^{-M-\mu}\cdots dW'+\int_{M+\mu}^{\infty}\cdots dW', \quad (13)$$

with

$$q(-W')=q(W').$$

If one solves the integral equation for the N function numerically, the D function can be evaluated through integration. Since the experimental mass of the Ω^- is 1685 MeV, one looks for a bound-state condition in the $\Xi\bar{K}$ scattering, i.e., $D(W)=0$ for $W<M+\mu$.

In the $T=0$ state, the dependence of $h^B(W)$ on the mixing parameter f is very weak. The plausible values[3,7] of f lie within the range of $0.25\leq f\leq 0.4$. To get a feeling for the dependence on f, let us consider the degenerate case ($M_\Lambda=M_\Sigma$), whence the over-all coupling constant is proportional to

$$g^2\sim 3-\tfrac{1}{3}(1-4f)^2,$$

and this varies from 3 to 2.88 as f is varied from 0.25 to 0.4. We therefore fix the value of f at 0.35.

III. RESULTS AND DISCUSSION

The integral equation for the function $N(W)$ was inverted numerically on the IBM 1620 computer. The results are independent of the subtraction point as they should be. Once the N function was known, the integration for the D function was carried out numerically and a zero was looked for in the D function in the real region corresponding to the Ω^- pole. As expected, the Ω^- comes out to be enormously overbound. To estimate the amount of overbinding, we cut off the baryon-exchange forces and studied the position of the Ω^- pole as a function of the cutoff. As the cutoff was varied from 20 pion masses to about 34 pion masses, the Ω^- changed from a resonating state to a strongly overbound state, the correct Ω^- mass being reproduced for a cutoff of about 26 pion masses as shown in Table I. We thus conclude that the inference drawn by Martin

TABLE I. The Ω^- position as a function of the cutoff.

Cutoff (pion mass)	Exact calculation (MeV)	Determinantal approximation (MeV)
20.0	1850	...
23.5	1785	...
27.0	1625	1832
30.5	1425	1817
34.0	1225	1795

[5] S. Rai Choudhury and L. K. Pande, Phys. Rev. **135**, B1027 (1964).
[6] G. L. Kane, Phys. Rev. **135**, B843 (1964).
[7] A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964); J. M. Cornwall and V. Singh, Phys. Rev. Letters **10**, 551 (1963).

and Uretsky that the nucleon-exchange force leads to strong overbinding holds even when all SU_3-coupled channels are taken into account.

It is interesting to find out the reasons why, in the previous SU_3- symmetric calculations of Martin and Wali, the Ω^- turns out to be a resonating state rather than the overbound state that we are obtaining. We present also in Table I, the results of a first-order determinantal calculation with our amplitude side by side with our exact results for some of the cutoff values. It is clear from an examination of the table that the determinantal approximation (which was used in Ref. 4) grossly underestimates the attraction caused by the baryon exchange. The difference between our results and those of Ref. 3 can thus be ascribed to the deficiencies of the determinantal approximation.

In any complete analysis, one has to consider the exchanges of other resonating states like ρ, N^* etc., in addition to the baryons considered. Any such calculation however, necessarily brings in extra parameters and it is difficult then to make any precise statement about the position of the resonance unlike in the present calculation where only the exchange of spin-$\frac{1}{2}$ baryons is considered.

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Kinematics of Λ-Hyperon β Decay*

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The distributions of several kinematical quantities are presented for the polarized Λ hyperon undergoing β decay. The basic couplings are assumed to be vector and axial-vector, and all the "induced" couplings are neglected. Special emphasis is placed on the kinematically invariant quantities and their asymmetries with respect to the polarization of the Λ hyperon.

THE Λ-hyperon β decay ($\Lambda\rightarrow p+e^-+\bar{\nu}$) has been experimentally studied in hydrogen[1,2] and heavy-liquid[3,4] bubble chambers. At least two counter experiments are currently in progress.[5] The low branching ratio, $\Lambda\rightarrow pe^-\bar{\nu}/\text{all }\Lambda\approx10^{-3}$, makes the spark-chamber counter experiment very attractive, yet it has the drawback that the kinematical reconstruction of the neutral Λ is usually ambiguous or has large experimental errors. The first disadvantage can be overcome by studying the distributions of the transverse components of momenta of the decay particles relative to the Λ direction. Such quantities are kinematically invariant and can be analyzed directly in the laboratory system.[3]

In this note, we present distributions of other kinematically invariant quantities, i.e., the momentum components of decay particles along the spin of the Λ hyperon. The polarization of the Λ hyperons produced by associated production, $\pi^-+p\rightarrow\Lambda+K^\circ$, near the ΣK threshold is known to be large, i.e., $P_\Lambda=-0.91\pm0.10$.[6] Therefore, in practice, these invariant quantities are the momentum components of the decay particles perpendicular to the Λ production plane, i.e., parallel to $(\mathbf{p}_{\pi\,\text{incident}}\times\mathbf{p}_\Lambda)$. To determine these invariant quantities, one needs to know only the Λ production plane. On the other hand, the determination of the transverse components of momenta of the decay particles requires the knowledge of the direction of the Λ momentum. In a typical experimental arrangement, one could measure the Λ production plane more accurately than the direction of the Λ momentum by almost one order of magnitude. We define the momentum components of decay particles along the spin of the Λ hyperon as $p_s=\mathbf{p}\cdot\boldsymbol{\sigma}_\Lambda$, $l_s=\mathbf{l}\cdot\boldsymbol{\sigma}_\Lambda$, and $\nu_s=\boldsymbol{\nu}\cdot\boldsymbol{\sigma}_\Lambda$, where $\mathbf{p}$, $\mathbf{l}$, and $\boldsymbol{\nu}$ are the momenta of proton, electron, and neutrino (either in the Λ center-of-mass system or in the laboratory system), respectively. $\boldsymbol{\sigma}_\Lambda$ is the unit vector for the polarization of Λ, and is normal to the Λ production plane.

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