and Uretsky that the nucleon-exchange force leads to strong overbinding holds even when all SU_3-coupled channels are taken into account.

It is interesting to find out the reasons why, in the previous SU_3- symmetric calculations of Martin and Wali, the Ω^- turns out to be a resonating state rather than the overbound state that we are obtaining. We present also in Table I, the results of a first-order determinantal calculation with our amplitude side by side with our exact results for some of the cutoff values. It is clear from an examination of the table that the determinantal approximation (which was used in Ref. 4) grossly underestimates the attraction caused by the baryon exchange. The difference between our results and those of Ref. 3 can thus be ascribed to the deficiencies of the determinantal approximation.

In any complete analysis, one has to consider the exchanges of other resonating states like ρ, N^* etc., in addition to the baryons considered. Any such calculation however, necessarily brings in extra parameters and it is difficult then to make any precise statement about the position of the resonance unlike in the present calculation where only the exchange of spin-$\frac{1}{2}$ baryons is considered.

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Kinematics of Λ-Hyperon β Decay*

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The distributions of several kinematical quantities are presented for the polarized Λ hyperon undergoing β decay. The basic couplings are assumed to be vector and axial-vector, and all the "induced" couplings are neglected. Special emphasis is placed on the kinematically invariant quantities and their asymmetries with respect to the polarization of the Λ hyperon.

THE Λ-hyperon β decay ($\Lambda \to p+e^-+\bar{\nu}$) has been experimentally studied in hydrogen[1,2] and heavy-liquid[3,4] bubble chambers. At least two counter experiments are currently in progress.[5] The low branching ratio, $\Lambda \to pe^-\bar{\nu}/\text{all } \Lambda \approx 10^{-3}$, makes the spark-chamber counter experiment very attractive, yet it has the drawback that the kinematical reconstruction of the neutral Λ is usually ambiguous or has large experimental errors. The first disadvantage can be overcome by studying the distributions of the transverse components of momenta of the decay particles relative to the Λ direction. Such quantities are kinematically invariant and can be analyzed directly in the laboratory system.[3]

In this note, we present distributions of other kinematically invariant quantities, i.e., the momentum components of decay particles along the spin of the Λ hyperon. The polarization of the Λ hyperons produced by associated production, $\pi^-+p \to \Lambda+K^\circ$, near the ΣK threshold is known to be large, i.e., $P_\Lambda = -0.91 \pm 0.10$.[6] Therefore, in practice, these invariant quantities are the momentum components of the decay particles perpendicular to the Λ production plane, i.e., parallel to $(\mathbf{p}_{\pi\,\mathrm{incident}} \times \mathbf{p}_\Lambda)$. To determine these invariant quantities, one needs to know only the Λ production plane. On the other hand, the determination of the transverse components of momenta of the decay particles requires the knowledge of the direction of the Λ momentum. In a typical experimental arrangement, one could measure the Λ production plane more accurately than the direction of the Λ momentum by almost one order of magnitude. We define the momentum components of decay particles along the spin of the Λ hyperon as $p_s = \mathbf{p}\cdot\boldsymbol{\sigma}_\Lambda$, $l_s = \mathbf{l}\cdot\boldsymbol{\sigma}_\Lambda$, and $\nu_s = \boldsymbol{\nu}\cdot\boldsymbol{\sigma}_\Lambda$, where $\mathbf{p}$, $\mathbf{l}$, and $\boldsymbol{\nu}$ are the momenta of proton, electron, and neutrino (either in the Λ center-of-mass system or in the laboratory system), respectively. $\boldsymbol{\sigma}_\Lambda$ is the unit vector for the polarization of Λ, and is normal to the Λ production plane.

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[1] F. S. Crawford, Jr., M. Cresti, M. L. Good, G. R. Kalbfleish, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters 1, 377 (1958).

[2] V. G. Lind, T. O. Binford, M. L. Good, and D. Stern, Phys. Rev. 135, B1483 (1964).

[3] C. Baglin, V. Brisson, A. Rousset, J. Six, H. H. Bingham, M. Nikolic, K. Schultze, C. Henderson, D. J. Miller, F. R. Stannard, R. T. Elliot, L. K. Rangan, A. Haatuft, and K. Myklebost, Nuovo Cimento 35, 977 (1965).

[4] R. P. Ely, G. Gidal, G. E. Kalmus, W. M. Powell, W. J. Singleton, C. Henderson, D. J. Miller, and F. R. Stannard, Phys. Rev. 137, B1302 (1965).

[5] S. Frankel and W. Wales; C. Rubbia and H. Sens, quoted in Ref. 3.

[6] Average value used in Ref. 2.

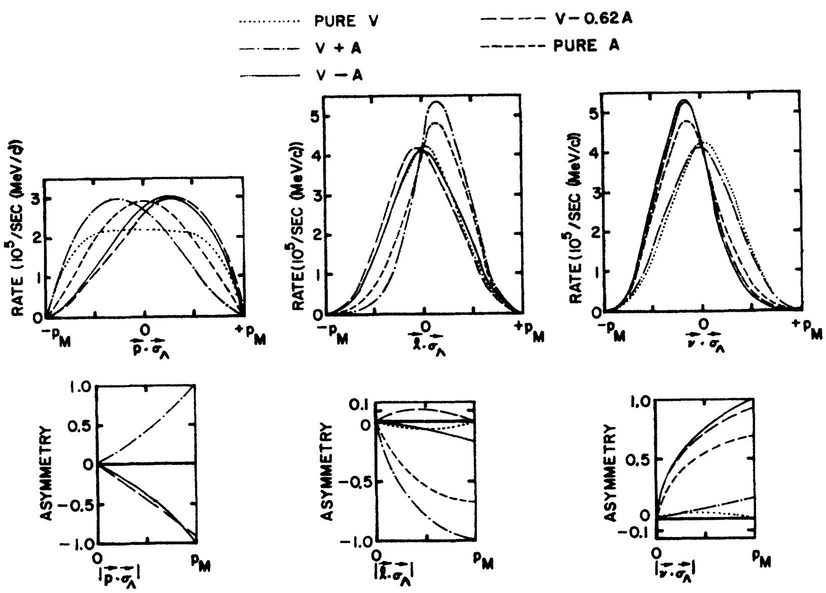

FIG. 1. The Λ β-decay spectra as functions of invariant momenta, $\mathbf{p}\cdot\boldsymbol{\sigma}_\Lambda$, $\mathbf{l}\cdot\boldsymbol{\sigma}_\Lambda$, and $\boldsymbol{\nu}\cdot\boldsymbol{\sigma}_\Lambda$, are shown for the cases of pure V, $V+A$, $V-A$, $V-0.64A$ (see footnote 12), and pure A. The polarization of Λ, P_Λ, is set at -1.0. The abscissa is in units of the maximum attainable value, $p_M=(M^2-m^2)/2M\approx163$ MeV/c. All areas are normalized to 5.7×10^7 sec^{-1}, the decay rate computed under the universal Fermi interaction hypothesis for the case when $|c_A|=|c_V|$. The dependences of the "up-down" asymmetry coefficients on the invariant momenta are also shown.

The β decay of the Λ has been studied theoretically by many authors.[7–10] To obtain the decay distributions in terms of p_s, l_s, and ν_s, we follow the expressions of Albright. We work within the framework of a V and A theory and assume that all "induced" terms can be neglected at this stage of calculation. The electron mass is ignored because of the large Q value. The decay probability then is given by

$$d\omega=\frac{1}{(2\pi)^5}\frac{2}{p_0}\frac{\nu}{lp}p^2\,dp\,d\Omega_p l^2\,dl\,d\Omega_l \times\Big\{A+B(\hat{l}\cdot\hat{\nu})+\langle\boldsymbol{\sigma}_\Lambda\rangle\cdot\Big[F\hat{l}+G\hat{\nu}+\Big(\frac{K}{m}\mathbf{l}+\frac{L}{m}\boldsymbol{\nu}\Big)(\hat{l}\cdot\hat{\nu})\Big]+T\langle\boldsymbol{\sigma}_\Lambda\rangle\cdot(\hat{l}\times\hat{\nu})\Big\} \times\delta\Big(\cos\alpha-\frac{M^2-m^2-2mp_0-2Ml+2p_0l}{2lp}\Big),\quad(1)$$

where $M=\Lambda$ mass, $m=$ proton mass, $p_0=$ proton energy, $\langle\boldsymbol{\sigma}_\Lambda\rangle$ is the polarization of the Λ, i.e., $\langle\boldsymbol{\sigma}_\Lambda\rangle=\boldsymbol{\sigma}_\Lambda P_\Lambda$, and $\cos\alpha=\hat{p}\cdot\hat{l}$. Since we keep only c_V and c_A terms, the coefficients A, B, F, G, K, L, and T are functions of c_V, c_A, M, m, p, and l only, as defined in Ref. (7). The decay probabilities as functions of the invariant momenta are readily obtained. The distribution for p_s is found to be[11]

$$\begin{aligned}d\omega/dp_s=&(2\pi)^{-3}\{[(M+m)^2|c_V|^2+(M-m)^2|c_A|^2]\\&\times\tfrac{1}{2}(p_M^2-p_s^2)-m[(M+m)^2|c_V|^2\\&-(M-m)^2|c_A|^2][E_M-E(p_s)]\\&-\tfrac{4}{3}M(|c_V|^2+|c_A|^2)[\tfrac{1}{3}(E_M^3-E^3(p_s))\\&-m^2(E_M-E(p_s))]\\&+\tfrac{2}{3}\langle\sigma_\Lambda\rangle\mathrm{Re}(c_Vc_A^*)p_s[2M(p_M^2-p_s^2)\\&-(M^2+3m^2)(E_M-E(p_s))]\},\quad(2)\end{aligned}$$

where $p_M=(M^2-m^2)/2M$, $E_M=(M^2+m^2)/2M$, and $E(p_s)=(m^2+p_s^2)^{1/2}$.

Analogous expressions for the electron and the neutrino, $d\omega/dl_s$, and $d\omega/d\nu_s$, are also obtained. The results are shown below in the form of simple integrals:

$$\begin{aligned}\frac{d\omega}{dl_s}=&\frac{2M}{(2\pi)^3}\int_{\pm l_s}^{p_M}dv\Big(\frac{p_M-v}{M-2v}\Big)^2\\&\times\{(c_V+c_A)^2[(2M-3v)v-\langle\sigma_\Lambda\rangle(M-v)l_s]\\&-(c_V+c_A)(c_V-c_A)m(v-\langle\sigma_\Lambda\rangle l_s)\\&-4\,\mathrm{Re}(c_Vc_A^*)[(M-v)v-\langle\sigma_\Lambda\rangle vl_s]\\&-((p_M-v)/3(M-2v))(c_V-c_A)^2\\&\times[(3M-2v)v-\langle\sigma_\Lambda\rangle(M+2v)l_s]\},\quad(3)\end{aligned}$$

where the lower limits $+l_s$ and $-l_s$ apply for the positive and the negative values of l_s. The neutrino distribution is obtained by replacing l_s by ν_s and reversing all the signs of c_A and $\langle\sigma_\Lambda\rangle$ in (3).

The results are shown in Fig. 1, where the abscissa is expressed in units of $p_M\approx163$ MeV/c. The "up-down"

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[7] C. H. Albright, Phys. Rev. **115**, 750 (1959).
[8] D. R. Harrington, Carnegie Institute of Technology Report No. NYO-9282, 1961 (unpublished); Phys. Rev. **120**, 1482 (1960).
[9] L. Egardt, Nuovo Cimento **27**, 357 (1962); **27**, 368 (1962); Arkiv Fysik **24**, 123 (1963).
[10] V. M. Shekhter, Zh. Eksperim. i Teor. Fiz. **35**, 458 (1958) [English transl.: Soviet Phys.—JETP **35**, 316 (1959)].

[11] In (1) the electron variables are integrated out, and the proton variables are changed according to $dp\,dx=[\partial(p,x)/\partial(p_s,v)]dp_s\,dv$, where $x=\hat{p}\cdot\boldsymbol{\sigma}_\Lambda$ and we choose $v=p$.

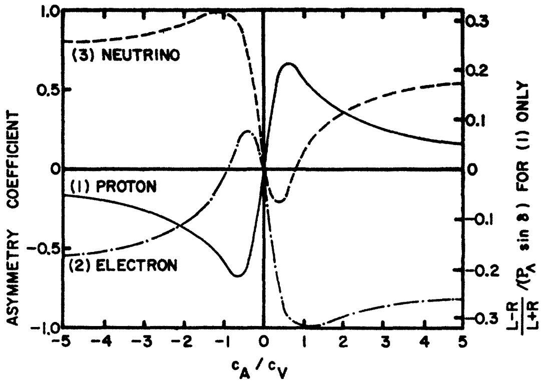

FIG. 2. The "up-down" asymmetry coefficients versus the ratio c_A/c_V for (1) proton, (2) electron, and (3) neutrino in Λ β decay (c_V and c_A are assumed to be real). The curve (1) read on the right-hand ordinate shows "left-right" asymmetry which appears if time-reversal invariance is violated ($c_A/c_V=|c_A|/|c_V|e^{i\delta}$).

asymmetry coefficients as functions of p_s, l_s, and ν_s are also shown in Fig. 1. Note the changes in the functional dependence of the asymmetry on p_s, l_s, and ν_s as the ratio[12] c_A/c_V is changed. The energy dependences of the asymmetry coefficients for leptons have been computed previously.[10] The asymmetry coefficients change in sign as functions of the lepton energies, whereas, as functions of l_s and ν_s, they do not. The asymmetry coefficients for the proton, electron, and neutrino versus the ratio c_A/c_V with real c_V and c_A are shown in Fig. 2.

The term that violates the time-reversal invariance T in (1) has also been computed with

$$c_A=(|c_A|/|c_V|)c_Ve^{i\delta}.$$

For example, for $V-A$ the maximum violation of time-reversal invariance, i.e., $\delta=\pm\pi/2$, corresponds to only $\mp 18\%$ "left-right" asymmetry. The "left-right" asymmetry is defined as $(L-R)/(L+R)$, where R and L are probabilities of decay particles to have positive and negative values of $\boldsymbol{\sigma}_\Lambda\cdot\mathbf{p}\times\mathbf{l}$. The magnitude and sign of δ can be determined experimentally by observing the ratio c_A/c_V and the "left-right" asymmetry.

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[12] The Cabibbo octet hypothesis for leptonic decays gives two solutions of c_A/c_V, viz., -0.64 ± 0.05 and -1.0 ± 0.05, of which the former is regarded as the preferred solution. See, N. Cabibbo, Phys. Rev. Letters **12**, 531 (1963); and R. H. Dalitz, in *Proceedings of The International School of Physics "Enrico Fermi"* (Villa Monastero, Varenna, Como, Italy, to be published).

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Test of the Nonrelativistic Quark Model for "Elementary" Particles: Radiative Decays of Vector Mesons

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An experimental test of the nonrelativistic quark model proposed by one of the authors (G.M.) to decribe the internal dynamics of elementary particles is suggested and discussed. The idea is the following: In the nonrelativistic quark model mentioned above, one can obtain not only the ratio $-\frac{3}{2}$ of the magnetic moment of the proton to that of the neutron, but also the absolute value of the magnetic moment of the proton in terms of the quark magnetic moment. By using the value of the quark magnetic moments determined in this way, we calculate the rates of the $M1$ radiative transitions $V\to P+\gamma$, where V is a vector meson and P a pseudoscalar meson. The following results for the widths are obtained (in MeV): $\omega\to\pi^0\gamma$ (1.17); $\omega\to\eta\gamma$ (6.4×10^{-3}); $\rho\to\pi\gamma$ (1.2×10^{-1}); $\rho_0\to\eta\gamma$ (4.4×10^{-2}); $K^{*+}\to K^+\gamma$ (7×10^{-2}); $K^{*0}\to K^0\gamma$ (2.8×10^{-1}); $\varphi\to\eta\gamma$ (3.04×10^{-1}). The result for the ω agrees with the present experimental data, assuming that the $\pi^0\gamma$ decay dominates the neutral decay rate of the ω; no experimental data are available for the other decays and it is suggested that the $K^{*0}\to K^*\gamma$ and $\varphi\to\eta\gamma$ decay rates are sufficiently large to deserve a measurement. Two possible reasons for uncertainty are discussed in detail: (a) dependence of the vertex function upon the masses, and (b) choice of the ω and η unitary spin functions.

1. THE PROBLEM

IT has been remarked by one of us[1] that if quarks do exist as real massive particles, the internal dynamics of "elementary" particles might be nonrelativistic. In particular it could be possible, at least as a convenient approximation, to write an Hamiltonian for an "elementary" particle in terms of the quark coordinates only in the same way in which one writes an Hamiltonian for a nucleus in terms of the nucleon coordinates only.

For instance, the proton and the neutron might be conceived as being three-quark structures in much the same way as He^3 and H^3 are three-nucleon systems. If,

[1] G. Morpurgo, report presented at the Frascati meeting of the Istituto Nazionale di Fisica Nucleare (to be published).