FIG. 2. The "up-down" asymmetry coefficients versus the ratio c_A/c_V for (1) proton, (2) electron, and (3) neutrino in $\Lambda\,\beta$ decay (c_V and c_A are assumed to be real). The curve (1) read on the right-hand ordinate shows "left-right" asymmetry which appears if time-reversal invariance is violated $(c_A/c_V = |c_A|/|c_V|e^{i\delta})$.

asymmetry coefficients as functions of p_s, l_s, and ν_s are also shown in Fig. 1. Note the changes in the functional dependence of the asymmetry on p_s, l_s, and ν_s as the ratio[12] c_A/c_V is changed. The energy dependences of the

asymmetry coefficients for leptons have been computed previously.[10] The asymmetry coefficients change in sign as functions of the lepton energies, whereas, as functions of l_s and ν_s, they do not. The asymmetry coefficients for the proton, electron, and neutrino versus the ratio c_A/c_V with real c_V and c_A are shown in Fig. 2.

The term that violates the time-reversal invariance T in (1) has also been computed with

$$c_A = (|c_A|/|c_V|)c_V e^{i\delta}.$$

For example, for $V-A$ the maximum violation of time-reversal invariance, i.e., $\delta = \pm\pi/2$, corresponds to only $\mp 18\%$ "left-right" asymmetry. The "left-right" asymmetry is defined as $(L-R)/(L+R)$, where R and L are probabilities of decay particles to have positive and negative values of $\sigma_\Lambda \cdot p \times l$. The magnitude and sign of δ can be determined experimentally by observing the ratio c_A/c_V and the "left-right" asymmetry.

ACKNOWLEDGMENTS

[12] The Cabibbo octet hypothesis for leptonic decays gives two solutions of c_A/c_V, viz., -0.64 ± 0.05 and -1.0 ± 0.05, of which the former is regarded as the preferred solution. See, N. Cabibbo, Phys. Rev. Letters **12**, 531 (1963); and R. H. Dalitz, in *Proceedings of The International School of Physics "Enrico Fermi"* (Villa Monastero, Varenna, Como, Italy, to be published).

Test of the Nonrelativistic Quark Model for "Elementary" Particles: Radiative Decays of Vector Mesons

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An experimental test of the nonrelativistic quark model proposed by one of the authors (G.M.) to decribe the internal dynamics of elementary particles is suggested and discussed. The idea is the following: In the nonrelativistic quark model mentioned above, one can obtain not only the ratio $-\frac{3}{2}$ of the magnetic moment of the proton to that of the neutron, but also the absolute value of the magnetic moment of the proton in terms of the quark magnetic moment. By using the value of the quark magnetic moments determined in this way, we calculate the rates of the $M1$ radiative transitions $V \to P+\gamma$, where V is a vector meson and P a pseudoscalar meson. The following results for the widths are obtained (in MeV): $\omega \to \pi^0\gamma$ (1.17); $\omega \to \eta\gamma$ (6.4×10⁻³); $\rho \to \pi\gamma$ (1.2×10⁻¹); $\rho_0 \to \eta\gamma$ (4.4×10⁻²); $K^{*+} \to K^+\gamma$ (7×10⁻²); $K^{*0} \to K^0\gamma$ (2.8×10⁻¹); $\varphi \to \eta\gamma$ (3.04×10⁻¹). The result for the ω agrees with the present experimental data, assuming that the $\pi^0\gamma$ decay dominates the neutral decay rate of the ω; no experimental data are available for the other decays and it is suggested that the $K^{*0} \to K^{*}\gamma$ and $\varphi \to \eta\gamma$ decay rates are sufficiently large to deserve a measurement. Two possible reasons for uncertainty are discussed in detail: (a) dependence of the vertex function upon the masses, and (b) choice of the ω and η unitary spin functions.

1. THE PROBLEM

IT has been remarked by one of us[1] that if quarks do exist as real massive particles, the internal dynamics of "elementary" particles might be nonrelativistic. In particular it could be possible, at least as a convenient

[1] G. Morpurgo, report presented at the Frascati meeting of the Istituto Nazionale di Fisica Nucleare (to be published).

approximation, to write an Hamiltonian for an "elementary" particle in terms of the quark coordinates only in the same way in which one writes an Hamiltonian for a nucleus in terms of the nucleon coordinates only.

For instance, the proton and the neutron might be conceived as being three-quark structures in much the same way as He³ and H³ are three-nucleon systems. If,

in this nonrelativistic model of the proton and of the neutron the space part of the three-quark wave function is taken to be completely antisymmetric, one obtains[1] the well-known $-\frac{3}{2}$ ratio between the magnetic moments of the proton and of the neutron; but this fact, as emphasized particularly by Bég, Lee, and Pais,[2] is not a test of the model because it can also be derived quite generally if one assumes that SU_6 is valid in the nonrelativistic limit and that the proton and the neutron belong to the representation **56** of SU_6. However, it is possible to have a definite test of the model, as we shall now show. Indeed, if one takes the three-quark model seriously, one can not only derive the ratio $-\frac{3}{2}$ between the magnetic moments, but also the individual values of the magnetic moments of the proton and the neutron in terms of the magnetic moments of the quarks. One obtains[1]

$$\mathbf{y}_i = \mu_p(e_i/e)\mathbf{\sigma}_i, \tag{1}$$

where the index $i = 1$, 2 specifies the kind of quark (respectively, the p quark and n quark), and $(e_1, e_2) = (\frac{2}{3}, -\frac{1}{3})$. In Eq. (1) μ_p is the magnetic moment of the real proton, $\mu_p = 2.79 eh/2M_p c$, where M_p is the proton mass.[3]

We can at this point state the test of the model which we propose: It consists in using the values (1) of the magnetic moments of the quarks obtained from fitting the n and p magnetic moments to predict the electromagnetic properties of other particles. One of the simplest possibilities appears to be that of calculating the rates of the $V \rightarrow P + \gamma$ processes, where V and P are, respectively, the mesons of the vector octet and of the pseudoscalar octet. These transitions, being pure $M1$ transitions, depend essentially on the quark magnetic moments. We list below the transitions which we shall consider:

(1) $\omega \rightarrow \pi^0 + \gamma$,   (2) $\omega \rightarrow \eta + \gamma$,

(3) $\rho \rightarrow \pi + \gamma$,   (4) $\rho^0 \rightarrow \eta + \gamma$,   (2)

(5) $K^{*+} \rightarrow K^+ + \gamma$,   (6) $K^{*0} \rightarrow K^0 + \gamma$.

There are also other matrix elements of interest, like those involving the φ and X_0 particles and that corresponding to the $\omega \rho_0 \gamma$ vertex, but we shall, for the moment, concentrate our attention on the matrix elements (1) to (6) above.

[2] M. A. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).

[3] Of course it must be strongly emphasized that underlying the values (1) of the magnetic moments of the quarks, is the assumption that such magnetic moments are proportional to the charges. As has already been often observed [M. A. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965)], this is, however, the only assumption which gives rise to the ratio $-\frac{3}{2}$ in the **56**, representation. Note also that the magnetic moments of the quarks (1) are very anomalous; for a quark mass of 5 GeV, the g factor of the quark is 18.6. This indicates that the quarks are, themselves, very far from being "elementary"; but they could hardly be expected to be, since they interact strongly. The only important point is that in the same way that nucleons (which are certainly not elementary) behave as the elementary constituents of nuclei, the quarks should behave as elementary in the internal dynamics of the lower states of particles.

2. GENERAL DISCUSSION OF THE MATRIX ELEMENTS FOR THE $V \rightarrow P + \gamma$ TRANSITIONS

To obtain a real test of the model, two conditions must be satisfied: (a) that in the calculation of the matrix element no uncertainties occur, and (b) that the process considered has a nonzero degree of likelihood of being experimentally accessible. For instance, the static magnetic moments of the ρ and K^* would be ideal as far as the condition (a) is concerned but do not easily satisfy condition (b). For this reason we omit giving the results for them and concentrate our attention on the transition moments (1) to (6). The uncertainties which occur in the matrix elements for these transitions can be divided into two classes:

(1) Relativistic uncertainties: Although the internal dynamics is nonrelativistic, the transitions (1) and (2) are relativistic transitions in the sense that the π^0 is clearly relativistic. As already pointed out in Ref. 1. this has consequences of two different kinds: (a) Dynamical effects: The relativistic form of the interaction between quarks and electromagnetic field must be used instead of the nonrelativistic form (7) of which we shall make use in this paper. (b) Kinematical effects: The wave function of a bound state of two particles in motion at a relativistic velocity must be appropriately transformed. In more conventional language, what we shall do is to calculate the transition rates without taking into account the dependence of the vertex function on the masses of the particles involved in the decay. In other words, the vertex function will be calculated assuming all the masses equal. The results presented in Table I, as we shall explain in more detail later, are based on this approximation. We hope to be able, in the future, to give an estimate of the reliability of this approximation.

(2) Uncertainties arising from violations of SU_3: The main point of the model is that the bosons are describable in terms of the coordinates of a quark and an antiquark, in the same way as positronium is describable in terms of the coordinates of an electron and a positron. Now if SU_3 were exactly true, this would fix the unitary spin structure of all the bosons. This is

TABLE I. The rates of the decays $V \rightarrow P + \gamma$ as calculated in the model. Column 1: the process; 2: the effective coupling constant; column 3: the momentum of the γ; column 4: the calculated width in MeV; column 5: the branching ratio.

Process	$|f_{ij}|^2$	k (MeV)	Γ_{ij} (MeV)	Γ_{ij}/Γ
$\omega \rightarrow \pi^0 + \gamma$	2/3	380	1.17	1.2×10^{-1}
$\omega \rightarrow \eta + \gamma$	2/81	200	6.4×10^{-3}	6.8×10^{-4}
$\rho \rightarrow \pi + \gamma$	2/27	370	1.2×10^{-1}	1.1×10^{-3}
$\rho_0 \rightarrow \eta + \gamma$	2/9	184	4.4×10^{-2}	4.15×10^{-4}
$K^{*+} \rightarrow K^+ + \gamma$	2/27	308	7×10^{-2}	1.4×10^{-5}
$K^{*0} \rightarrow K^0 + \gamma$	8/27	308	2.8×10^{-1}	5.6×10^{-3}
$\varphi \rightarrow \eta + \gamma$	16/81[a]	362	3.04×10^{-1} [a]	10^{-1}

[a] This calculation of the φ decay rate is performed with the conventional choice of the φ and η unitary-spin functions, namely, $\varphi = D_3{}^3$, $\eta = (1/\sqrt{6}) \times (D_1{}^1 + D_2{}^2 - 2D_3{}^3)$; note that if the η were, like the ω, a pure $(1/\sqrt{2}) \times (D_1{}^1 + D_2{}^2)$ state, the rate would vanish.

not the case, however, if violations of SU_3 of the kind $T_3{}^3$ are present; violations of this kind do not affect the unitary spin structure of the particles with isotopic spin different from zero, but may affect that of the particles with isotopic spin zero, since $T_3{}^3$ corresponds to isotopic spin zero. For instance, if the ω were a member of the vector octet, its unitary spin wave function would be

$$\omega_8 = (1/\sqrt{6})(D_1{}^1 + D_2{}^2 - 2D_3{}^3), \qquad (3)$$

while in the conventional solution of the so-called ω-φ problem the ω is mixed with the unitary singlet

$$\omega_1 = (1/\sqrt{3})(D_1{}^1 + D_2{}^2 + D_3{}^3)$$

in such a way that

$$\omega = (1/\sqrt{2})(D_1{}^1 + D_2{}^2). \qquad (4)$$

This conventional solution of the ω-φ problem appears very plausible, especially in view of the fact that presumably[1] the main part of the violating term $T_3{}^3$ is exhausted by the mass difference among quarks; and it is also the solution suggested by SU_6. However, some uncertainty remains. In other words, on writing

$$\omega = (2+\lambda_\omega{}^2)^{-1/2}(D_1{}^1 + D_2{}^2 - \lambda_\omega D_3{}^3) \qquad (5)$$

we prefer to think that the choice $\lambda_\omega = 0$ should be considered as extremely probable but not, as yet, absolutely certain. A situation in a sense more obscure holds for the η. Here it is usually accepted, mainly because of the success of the Gell-Mann–Okubo mass formula for the P octet, that the η is a pure (or practically pure) octet meson, and therefore has a unitary spin structure $(1/\sqrt{3})(D_1{}^1 + D_2{}^2 - 2D_3{}^3)$. Particularly in view of the fact that a clear justification of the *quadratic* mass formula for bosons is lacking (in spite of the success of such a formula), we do not consider this structure either as definitely established, and we prefer to write

$$\eta = (2+\lambda_\eta)^{-1/2}(D_1{}^1 + D_2{}^2 - \lambda_\eta D_3{}^3) \qquad (6)$$

where the choice $\lambda_\eta = 2$ appears to be not improbable, but not certain.

Finally, another reason for uncertainty, different from the ones described above, appears for the decays (5) and (6): Though the proton and the neutron determined the magnetic moments of quarks 1 and 2, they do not, of course, give any information on the magnetic moment of the third quark, whose knowledge is necessary for the calculations of the $K^* \to K + \gamma$ matrix element. In the following we shall calculate the rate of the $K^* \to K + \gamma$ transitions under the assumption that the third quark too has a magnetic moment proportional to its charge with the same constant of proportionality as for the proton and of the neutron; that is, we assume that (1) is valid not only for $i = 1, 2$ but also for $i = 3$ with of course $e_3/e = -\frac{1}{3}$. This assumption appears plausible. Indeed the SU_3-violating interaction $T_3{}^3$ has an order of magnitude such that it only changes the mass of the third quark with respect to that of the other two by ~ 200 MeV, a very small

difference compared with the quark mass itself (say 5 GeV). It is therefore not unreasonable to presume that a (negligible) correction of the same order of magnitude affects the magnetic moment.

We end this section with the following remark: The uncertainties due to the poor knowledge of λ_ω, λ_η, or the magnetic moment of the third quark can eventually be eliminated through appropriate experiments: For instance, the magnetic moment of the third quark might be determined through a measurement of the Λ magnetic moment and λ_ω might be determined by the methods discussed, for example, by Dalitz.[4] Therefore the only theoretical uncertainty is the mass dependence of the vertex function.[5]

3. A FEW DETAILS OF THE CALCULATIONS

In Table I we give the rates of the transitions (1) to (6) calculated in the way outlined in the previous section. To be sure, however, that the meaning of the numbers given in the table is clear, it appears appropriate to give some details of the calculations:

(a) The operator $\mathfrak{M}_i\theta$ which induces an $M1$ transition with polarization ε_a of the photon for the ith quark will be written in its nonrelativistic form:

$$\mathfrak{M}_i{}^a = \left[\frac{eh}{2Mc}\mathbf{L}_i \cdot (\mathbf{k} \times \boldsymbol{\varepsilon}_a) + \mu_p \frac{e_i}{e}\boldsymbol{\sigma}_i \cdot (\mathbf{k} \times \boldsymbol{\varepsilon}_a)\right]\exp i\mathbf{k}\cdot\mathbf{r}_i. \qquad (7)$$

The orbital magnetic-moment term is proportional to the quark magneton $eh/2Mc$ ($M =$ quark mass). For a value of $M = 5$ GeV its order of magnitude is ~ 15 times smaller than that of the spin term; since in addition the contribution of the orbital term vanishes in the long-wavelength approximation ($k = 0$) for the transitions in which we are interested here ($L = 0$), we are entitled to neglect the orbital term in the ensuing calculations.

(b) The states of the particles which enter into the calculation can be written as follows (for ρ, ω, and K^* the component with $S_z = +1$ will be given):

$$\pi^0 = \tfrac{1}{2}[(\alpha_1\beta_{\bar{1}} - \alpha_{\bar{1}}\beta_1) - (\alpha_2\beta_{\bar{2}} - \alpha_{\bar{2}}\beta_2)]f(r),$$

$$\eta^0 = [2(2+\lambda_\eta{}^2)]^{-1/2}[(\alpha_1\beta_{\bar{1}} - \alpha_{\bar{1}}\beta_1) + (\alpha_2\beta_{\bar{2}} - \alpha_{\bar{2}}\beta_2)$$
$$- \lambda_\eta(\alpha_3\beta_{\bar{3}} - \alpha_{\bar{3}}\beta_3)]f(r),$$

$$K^+ = (1/\sqrt{2})(\alpha_1\beta_{\bar{3}} - \alpha_{\bar{3}}\beta_1)f(r),$$

$$K^0 = (1/\sqrt{2})(\alpha_2\beta_{\bar{3}} - \alpha_{\bar{3}}\beta_2)f(r), \qquad (8)$$

$$\rho^0 = (1/\sqrt{2})(\alpha_1\alpha_{\bar{1}} - \alpha_2\alpha_{\bar{2}})f(r),$$

$$\omega^0 = (2+\lambda_\omega{}^2)^{-1/2}[\alpha_1\alpha_{\bar{1}} + \alpha_2\alpha_{\bar{2}} - \lambda_\omega\alpha_3\alpha_{\bar{3}}]f(r),$$

$$K^{*+} = \alpha_1\alpha_{\bar{3}}f(r),$$

$$K^{*0} = \alpha_2\alpha_{\bar{3}}f(r).$$

[4] R. H. Dalitz, *Proceedings of the Sienna International Conference on Elementary Particles, 1963*, edited by G. Bernardini and G. Puppi (Italian Physical Society, Bologna, 1963), Vol. II, p. 171.

[5] If the vertex function is assumed to depend on the masses only through the three-momentum of the photon (whether this is so questionable), and if the radius of the decaying $9\bar{9}$ system, that is, the radius of the vector boson, is taken of the order of $(5m_\pi)^{-1}$ or larger, the dependence of the vertex function on the masses is almost negligible.

In the above expressions we have indicated by α_i and β_i the spin functions (with spin up and spin down, respectively) for a quark ($i=1, 2, 3$) and by $\alpha_{\bar{i}}$, $\beta_{\bar{i}}$ the spin functions (with spin up and spin down, respectively) for an antiquark ($i=1, 2, 3$). The vector particles have been assumed in pure $^3S_1$ states (no mixing with $^3D_1$ states); the space part of the wave function $f(r)$ has been assumed to be the same for all P and V mesons. These last two assumptions may be regarded, if one prefers, as consequences of SU_6. Using (7) and (8), the squares $(|\langle V|P\rangle|^2)_{\mathrm{av}}$ of the matrix elements for the transitions of interest, summed over the polarization of the final photon and averaged over the spin direction of the initial vector meson, are immediately calculated as follows:

$$|f_{\omega\pi^0}|^2 \equiv \mu_p^{-2} k^{-2}(|\langle\omega|\pi^0\rangle|^2)_{\mathrm{av}} = 4/3(2+\lambda_\omega^2),$$

$$|f_{\omega\eta}|^2 \equiv \mu_p^{-2} k^{-2}(|\langle\omega|\eta\rangle|^2)_{\mathrm{av}}$$

$$= \frac{8}{27} \frac{1}{(2+\lambda_\omega^2)} \frac{1}{(2+\lambda_\eta^2)}(1-\lambda_\eta\lambda_\omega)^2,$$

$$|f_{\rho\pi}|^2 \equiv \mu_p^{-2} k^{-2}(|\langle\rho^0|\pi^0\rangle|^2)_{\mathrm{av}} = 2/27,$$

$$|f_{\rho\eta}|^2 \equiv \mu_p^{-2} k^{-2}(|\langle\rho^0|\eta\rangle|^2)_{\mathrm{av}} = 4/3(2+\lambda_\eta^2),$$

$$|f_{K^{*+}K^+}|^2 \equiv \mu_p^{-2} k^{-2}(|\langle K^{*+}|K^+\rangle|^2)_{\mathrm{av}} = 2/27,$$

$$|f_{K^{*0}K^0}|^2 \equiv \mu_p^{-2} k^{-2}(|\langle K^{*0}|K^0\rangle|^2)_{\mathrm{av}} = 8/27.$$

Here a long-wavelength approximation has been used ($\exp ikr = 1$). Note that owing to charge independence, the $\rho^{0,\pm} \to \pi^{0,\pm}+\gamma$ transition probabilities are equal. Note also that the following relations among matrix elements due to *nonviolated* SU_3 also hold quite generally, independently of the model[6]:

$$\langle\rho^0|\pi\rangle = \langle K^{*+}|K^+\rangle = -\tfrac{1}{2}\langle K^{*+}|K^0\rangle$$

$$= (1/\sqrt{3})\langle\rho^0|\eta\rangle = (1/\sqrt{3})\langle\omega_8|\pi^0\rangle.$$

These relations are obviously satisfied by our results (9) (the η appearing in these relations corresponds to the choice $\lambda_\eta = +2$).

The decay rates are obtained on multiplying the expressions (9) by $(2\pi\rho_f/2k)\times N$. Here $2\pi\rho_f$ is the usual phase-space factor, which we shall take in its *relativistic* form

$$2\pi\rho_f = 2\pi\frac{4\pi}{(2\pi)^3}k^2\frac{\omega_k}{M_V} = \frac{k^2}{\pi}\frac{\omega_k}{M_V} \quad (10)$$

where k is the energy of the photon, ω_k the energy of the pseudoscalar meson, and M_V the mass of the decaying vector meson; $1/2k$ is the standard factor arising from the normalization of the photon; and the factor N, given by

$$N = M_V/\omega_k \quad (11)$$

is a relativistic factor which requires a more careful discussion. This factor M_V/ω_k serves to take into account all the requirements of relativity except one, as we shall now explain. (We might omit this factor, but then, to be consistent, we should have to use the nonrelativistic phase space; the result is the same.)

To clarify this point imagine that we try to write the vertex for our process relativistically. This is uniquely determined as

$$f_{ij}\epsilon^{\alpha\beta\delta\gamma}\partial_\alpha A_\beta\partial_\gamma V_\delta^{(i)}P^{(J)} \quad (12)$$

where $A_\beta(X)$ is the electromagnetic field, $V_\delta(X)$ is the vector field and $P(X)$ is the pseudoscalar field; f_{iJ} is an effective coupling constant which depends on the kind i of vector meson and on the kind J of the pseudoscalar meson involved; f_{iJ} will depend on the masses M_{V_i} and M_{P_J}. But assume for a moment that this dependence is not strong and that we are allowed to use the value of $f_{iJ}(0)$ when the masses of the particles are taken to be equal. *We may consider our previous calculation of the matrix elements as a calculation of* $|f_{iJ}(0)|^2$. *What we have called the relativistic uncertainties is the fact that we really do not know how strongly f_{iJ} depends on the existing differences in masses.* But if we neglect this dependence our previous calculation is relativistically correct, provided that we insert the factor M_V/ω_k mentioned above, as results immediately on using (12).

4. RESULTS

The rates of the various decays given by the formula

$$\Gamma_{iJ} = |f_{iJ}|^2\frac{(2.79)^2}{2}\frac{1}{137}\left(\frac{k}{M_p}\right)^2 k \quad (13)$$

obtained by putting together the expressions (9) (which we have called $|f_{iJ}|^2$), (10), and (11), are given in Table I. For the ω and the η we have assumed, in the table, the standard solution $\lambda_\omega = 0$. $\lambda_\eta = +2$, but the general dependence on λ_ω and λ_η is of course contained in the previous formulas (9).

We only add that the present experimental value of the ratio $(\omega \to \text{neutrals})/(\omega \to \pi^++\pi^-+\pi^0)$ is given as 11×10^{-2}.[7] Since the total width of the ω is given as 9 MeV and since the $\omega \to \pi^0+\pi^0+\pi^0$ decay is forbidden by isotopic spin, it is possible, though not certain, that practically all the neutral decays are $\omega \to \pi^0+\gamma$. In such a case the rate would be in striking agreement with the value given in Table I. It is clearly very important to have a determination of the nature of the neutral decay mode of the ω. It appears also important to try to obtain some information on the $K^{*0} \to K^0+\gamma$ and $\varphi \to \eta+\gamma$ decays, which might not be entirely beyond experimental possibility.

[6] S. Okubo, Phys. Letters 4, 14 (1963); S. L. Glashow, Phys. Rev. Letters 11, 48 (1963); K. Tanaka, Phys. Rev. 133, B1509 (1964).

[7] A. H. Rosenfeld, A. Barbaro-Gualtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 36, 977 (1964).