Cross-Section Relations and Polarization in a Static $SU(6)$ Model*

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Within the framework of a static (p-wave) $SU(6)$ model, cross-section relations for the scattering of pseudoscalar mesons from baryons are found and the polarization of the final baryon is investigated. The predicted value of 4/19 for the ratio $|M(K^-p\rightarrow K^+\Xi^-)|^2/|M(\pi^-p\rightarrow K^+\Sigma^-)|^2$ is found to be in agreement with experiment. In this model, it is predicted that for several reactions, including $K^-p\rightarrow K^+\Xi^-$ in which the Ξ^- is observed to be polarized near threshold, the p-wave part of the nonspin-flip and spin-flip amplitudes cannot cause polarization. We suggest that breaking of the symmetry may be able to account for the experimental polarization.

I. INTRODUCTION

An $SU(6)$ model has been recently proposed by Capps,[1] and by Belinfante and Cutkosky[2] (hereafter abbreviated CBC) in which the mesons are thought of as being in a p-wave relative to the (static) baryons. The mesons are assigned to the 35-dimensional representation of $SU(6)$ in a way which incorporates the features of the baryon-meson (pseudoscalar or vector) vertices for low energies. That is, the spinless pseudoscalar mesons, being in a p wave, would form a total angular momentum $J=1$ representation of the rotation subgroup, $SU(2)$, contained in $SU(6)$, and are thus assigned to the $(SU(2),SU(3))$ submodule (3,8) of the **35**. On the other hand, the vertex for the vector mesons is essentially $\chi_2^+\chi_1V_0$, where χ is a Pauli spinor describing the static baryon, and V_0 is the time component of the vector-meson field V_μ. The subsidiary condition, usually imposed on a massive vector-meson field, implies that $V_0=(\mathbf{k}\cdot\mathbf{V})/k_0$. Therefore, the intrinsic and orbital angular momenta of the vector meson are coupled together to give a total angular momentum of zero. They are accordingly assigned to the (1,8) submodule of the **35**. The remaining submodule, the (3,1), is a pseudoscalar meson, possibly the X_0 (960 MeV).

The baryons and the $P^{3/2}$ decuplet are assigned respectively to the submodules (2,8) and (4,10) of the 56-dimensional representation.

It is our desire to study some of the aspects of the CBC model (those concerned with cross sections and polarizations) and compare the consequences with experiment, and with the predictions of the $SU(6)$ symmetry scheme proposed by Gürsey and Radicati,[3] Pais,[4] and Sakita[5] (abbreviated GR).

In Sec. II we will present the general formalism needed to study cross sections and the polarization of the final baryon. Section III contains predictions for cross sections, along with a method for removing certain kinematical factors from the experimental data. A discussion of the limitations of the CBC model and a comparison of its results with the GR scheme are presented. In this connection, we note that owing to the inability of the GR scheme to give nonzero values for the baryon-resonance–pseudoscalar-meson production amplitudes we cannot compare the two schemes for such reactions. Section IV contains a brief discussion of the unsuccessful polarization predictions of the CBC model and the suggestion that symmetry breaking, to which the polarization is expected to be very sensitive, may remedy this situation.

II. GENERAL FORMALISM

We will consider the following kinds of reactions:

$$\text{(a)}\ P_1+B_1\rightarrow P_2+B_2;\quad \text{(b)}\ P+B_1\rightarrow B_2+V;$$
$$\text{(c)}\ P_1+B\rightarrow B^*+P_2;\quad \text{(d)}\ P+B\rightarrow B^*+V;$$
$$\text{(e)}\ K^-+p\rightarrow \Lambda^0+X_0;$$

(where P denotes a pseudoscalar meson; B, a baryon; B^*, a member of the $P^{3/2}$ decuplet of the baryon resonances; and V, a vector meson).

As an illustration of the formalism employed, we will consider scattering of type (a). The scattering will be described by a static-model amplitude, which can be written as a sum of two orthogonal terms, the nonspin-flip (nsf) and the spin-flip (sf) amplitudes:

$$\langle\mathbf{k}_f s_f|T|\mathbf{k}_i s_i\rangle =\chi^\dagger(s_f)[A(\omega,\cos\theta)-iB(\omega)\boldsymbol{\sigma}\cdot\mathbf{k}_f\times\mathbf{k}_i]\chi(s_i)\,,\quad (1)$$

where the momentum of the initial (final) meson is denoted by $\mathbf{k}_{i(f)}$; the spin orientation of the initial (final) baryon, by $s_{i(f)}$; and θ is the angle between $\mathbf{k}_f$ and $\mathbf{k}_i$. The amplitudes also depend on the lengths of the momenta (which are equal in this case because of the mass degeneracy assumed in the limit of complete symmetry) through ω, the meson energy. $A(\omega,\cos\theta)$ is the nsf amplitude, while $B(\omega)k^2\sin\theta$ is the sf amplitude.

The CBC $SU(6)$ describes the pseudoscalar meson as having a total angular momentum $J=1$. The scattering amplitude in this description therefore depends on the angular-momentum states of the initial and final particles: $T_{m_i\mu_i,m_f\mu_f}(\omega)$ (m denotes the magnetic quan-

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[1] R. H. Capps, Phys. Rev. Letters 14, 31 (1965).
[2] J. G. Belinfante and R. E. Cutkosky, Phys. Rev. Letters 14, 33 (1965).
[3] F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964).
[4] A. Pais, Phys. Rev. Letters 13, 175 (1964).
[5] B. Sakita, Phys. Rev. 136, B1756 (1964).

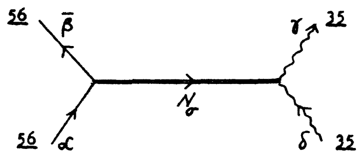

FIG. 1. Pictorial representation of the coupling scheme used in expressing the scattering amplitude in terms of the $SU(6)$ independent amplitudes. N_σ, the dimension of the "exchanged" representation, can be **1**, **35**, **405**.

tum number of the baryon and μ denotes the same quantity for the meson).

The connection between the two sets of amplitudes is established by means of the transformation coefficients relating these two descriptions of the mesons. The transformation coefficients are essentially the spherical harmonics, $Y_{l=1}{}^m$, $\hat{k}$ being the direction of **k**.

$$
\begin{aligned}
\langle \mathbf{k}_f s_f | T | \mathbf{k}_i s_i \rangle
&= \sum_{m_i \mu_i m_f \mu_f} \chi_{m_f}{}^*(s_f) Y_1{}^{\mu_f *}(\hat{k}_f) \\
&\qquad \times T_{m_i\mu_i, m_f\mu_f} Y_1{}^{\mu_i}(\hat{k}_i) \chi_{m_i}(s_i) . \qquad (2)
\end{aligned}
$$

In order to connect the sf and nsf amplitudes with $SU(6)$ amplitudes, we will have to extract the $SU(2)$ Clebsch-Gordan (CG) coefficients from the $SU(6)$ CG coefficients. To perform this extraction as simply as possible, we will adopt the following point of view. As far as the group-induced structure of the amplitude $T_{m_i\mu_i, m_f\mu_f}$ is concerned, we may think of viewing the scattering process as being described by $\mathbf{56}\otimes\mathbf{56}^* \rightarrow \mathbf{35}\otimes\mathbf{35}$, instead of by $\mathbf{56}\otimes\mathbf{35}\rightarrow\mathbf{56}\otimes\mathbf{35}$. The former description has nothing to do with dynamical calculations in the annihilation channel and, in fact, can be related to the direct $\mathbf{56}\otimes\mathbf{35}$ description by recoupling the representations. Pictorially, we are thinking of the scattering process as proceeding through an "exchanged representation" **N** (Fig. 1). One advantage of this description lies in the symmetry or antisymmetry of the $\mathbf{35}\otimes\mathbf{35}\rightarrow\mathbf{N}$ vertex; i.e., $\mathbf{35}\otimes\mathbf{35} = (\mathbf{35}\otimes\mathbf{35})_{\mathrm{sym}} \oplus (\mathbf{35}\otimes\mathbf{35})_{\mathrm{antisym}}$. The symmetric and antisymmetric terms are

$$
(\mathbf{35}\otimes\mathbf{35})_{\mathrm{sym}} = \mathbf{1}\oplus\mathbf{35}_s\oplus\mathbf{189}\oplus\mathbf{405},
$$

and

$$
(\mathbf{35}\otimes\mathbf{35})_{\mathrm{antisym}} = \mathbf{35}_a\oplus\mathbf{280}\oplus\mathbf{280}^* .
$$

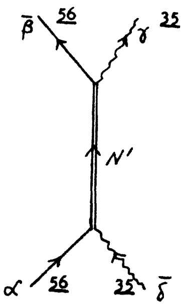

FIG. 2. Coupling scheme appropriate to a "direct"-channel description. $N' = \mathbf{56}$, **70**, **700**, **1134**.

Since $\mathbf{56}\otimes\mathbf{56}^* = \mathbf{1}\oplus\mathbf{35}\oplus\mathbf{405}\oplus\mathbf{2695}$, the representations in common are **1**, $\mathbf{35}_s$, $\mathbf{35}_a$, **405**. Of course, in the other description, shown in Fig. 2, with $\mathbf{N} = \mathbf{56}$, **70**, **700**, or **1134**, we again have four independent, invariant amplitudes. These two equivalent sets of amplitudes may be related through $SU(6)$ Racah coefficients.

The amplitude $T_{m_i\mu_i; m_f\mu_f}$ may be written as a sum of invariant amplitudes:

$$
T_{\alpha\beta,\gamma\delta} = \sum_{N_\sigma,\nu} \langle \gamma\delta | \nu, N_\sigma \rangle \langle \nu, N | \alpha\beta \rangle \langle N_\sigma \| T \| N \rangle , \qquad (3)
$$

where N is the dimension of the exchanged representation and can be **1**, **35**, or **405**; ν is the relevant component of N; and σ distinguishes different representations of the same dimension. The indices α, β, γ, δ include both the quantum numbers characterizing $SU(3)$ and the quantum numbers characterizing $SU(2)$; i.e., m_i is part of these indices. In general, we will use only the $SU(2)$ magnetic quantum number when we want to emphasize the $SU(2)$ aspects of the amplitude.

The matrix elements $\langle \alpha\beta | \nu, N \rangle$ and $\langle \gamma\delta | \nu, N \rangle$ in Eq. (3) are $SU(6)$ CG coefficients, for which we introduce the notation

$$
\langle \alpha\beta | \nu, N \rangle = \begin{pmatrix} 56 & 56^* & \big| & N \\ \alpha & \beta & \big| & \nu \end{pmatrix},
$$

$$
\langle \gamma\delta | \nu, N_\sigma \rangle = \begin{pmatrix} 35 & 35 & \big| & N_\sigma \\ \gamma & \delta & \big| & \nu \end{pmatrix}.
$$

Hence,

$$
T_{\alpha\beta,\gamma\delta} = \sum_{N_\sigma,\nu} \begin{pmatrix} 56 & 56^* & \big| & N \\ \alpha & \beta & \big| & \nu \end{pmatrix} \begin{pmatrix} 35 & 35 & \big| & N_\sigma \\ \gamma & \delta & \big| & \nu \end{pmatrix} T_{N_\sigma}(\omega) . \qquad (4)
$$

We decompose the $SU(6)$ CG coefficients as follows:

$$
\begin{aligned}
\begin{pmatrix} N_1 & N_2 & \big| & N \\ \nu_1 & \nu_2 & \big| & \nu \end{pmatrix}
&= \sum_k \begin{pmatrix} j_1 & j_2 & \big| & j \\ m_1 & m_2 & \big| & m \end{pmatrix} \begin{pmatrix} \mu_1 & \mu_2 & \big| & \mu_k \\ \tau_1 & \tau_2 & \big| & \tau \end{pmatrix} \\
&\quad \times \begin{pmatrix} N_1 & N_2 & \big\| & N \\ (2j_1+1, \mu_1) & (2j_2+1, \mu_2) & \big\| & (2j+1, \mu_k) \end{pmatrix}, \qquad (5)
\end{aligned}
$$

where the first term is the $SU(2)$ CG coefficient for coupling angular momenta j_1 and j_2 to give j; the second term is the $SU(3)$ CG coefficient; and the last term is an $SU(6)$ scalar factor, which has been computed by Cook and Murtaza.[6]

Since we are considering a pseudoscalar meson and a baryon in both the initial and final states, we are concerned with the (2,8) submodule of the **56** and the (3,8) submodule of the **35**:

$$
T_{\alpha\beta,\gamma\delta} = \sum_{j,m} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \big| & j \\ m_i & m_f & \big| & m \end{pmatrix} \begin{pmatrix} 1 & 1 & \big| & j \\ \mu_i & \mu_f & \big| & m \end{pmatrix} T_j(\omega) , \quad j = 0, 1 \qquad (6)
$$

[6] C. L. Cook and G. Murtaza, Imperial College, London (unpublished).

where

$$T_j(\omega)=\sum_{\mu,k,l}\begin{pmatrix}8 & 8 & \mu_k\\ \alpha & \beta & \tau\end{pmatrix}\begin{pmatrix}56 & 56^* & N\\ (2,8) & (2,8) & (2j+1,\mu_k)\end{pmatrix}$$

$$\times\begin{pmatrix}8 & 8 & \mu_l\\ \gamma & \delta & \tau\end{pmatrix}\begin{pmatrix}35 & 35 & N_\sigma\\ (3,8) & (3,8) & (2j+1,\mu_l)\end{pmatrix}T_{N_\sigma}(\omega). \quad (7)$$

We are thus able to identify the sf and nsf amplitudes. It is advantageous to write $T_{m_i\mu_i;m_f\mu_f}$ as

$$T_{m_i\mu_i;m_f\mu_f}=\sum_{j,m}\begin{pmatrix}\frac{1}{2} & j & \frac{1}{2}\\ m_i & m & m_f\end{pmatrix}\times\begin{pmatrix}1 & j & 1\\ \mu_i & -m & \mu_f\end{pmatrix}(-)^m T_j'(\omega) \quad (8)$$

with

$$T_j'(\omega)=(-)^j(2j+1)(6)^{-1/2}T_j(\omega). \quad (9)$$

We now use Eq. (2) to find

$$T(m_i\mathbf{k}_i,m_f\mathbf{k}_f)$$
$$=\sum_j\sum_{\substack{mm_im_f\\ \mu_i\mu_f}}\chi_{m_f}^*(s_f)\chi_{m_i}(s_i)Y_1^{\mu_f*}(\hat{k}_f)Y_{1\mu_i}(\hat{k}_i)$$
$$\times(-)^m\begin{pmatrix}\frac{1}{2} & j & \frac{1}{2}\\ m_i & m & m_f\end{pmatrix}\begin{pmatrix}1 & j & 1\\ \mu_i & -m & \mu_f\end{pmatrix}T_j'(\omega)$$
$$=\chi_f^\dagger(\hat{k}_f\cdot\hat{k}_iT_0(\omega)-i(3/2)^{1/2}T_1(\omega)\boldsymbol{\sigma}\cdot\mathbf{k}_f\times\mathbf{k}_i)\chi_i. \quad (10)$$

Here we have absorbed a common factor into the amplitudes T_0 and T_1. Comparing this last equation with Eq. (2) the sf and nsf amplitudes may be defined in terms of T_0 and T_1 by

$$A=\hat{k}_f\cdot\hat{k}_iT_0,\quad(\text{nsf});\quad B=(3/2)^{1/2}k^{-2}T_1,\quad(\text{sf}) \quad (11)$$

(to within quantities involving the *lengths* of the momenta).

We can write the total cross section (averaged over spins) for processes of type (a) in terms of these $J=0$ and $J=1$ amplitudes, T_0 and T_1, as

$$\langle\sigma_{\text{BP}}\rangle_{\text{spins}}\propto|T_0(\omega)|^2+3|T_1(\omega)|^2. \quad (12)$$

The polarization of the final baryon (in the direction $\mathbf{k}_f\times\mathbf{k}_i$) is then given by

$$P_f=-\frac{(6)^{1/2}\,\mathrm{Im}(T_0T_1^*)\cos\theta\sin\theta}{|T_0(\omega)|^2\cos^2\theta+(3/2)|T_1(\omega)|^2\sin^2\theta}. \quad (13)$$

Equation (12) may be obtained directly from the amplitudes $T_{\alpha\beta\gamma\delta}$, Eq. (6), if we perform the sum over initial and final spins, $\sum_{\text{spins}}|T_{\alpha\beta\gamma\delta}|^2$. This corresponds to treating the pseudoscalar mesons as spin-1 particles and performing a sum over initial spins of both baryons and mesons.

Reactions of types (b), (c), (d), and (e) may be treated in a similar manner. For reactions of types (b) and (c), which proceed through representations having $j=1$, 2, we have

$$\langle\sigma_{B^*}\rangle_{\text{spins}}\propto3|T_1(\omega)|^2+5|T_2(\omega)|^2, \quad (14)$$

for reactions of type (d), which proceed only via $j=1$, we have:

$$\langle\sigma_{\text{BV}}\rangle_{\text{spins}}\propto3|T_1(\omega)|^2, \quad (15)$$

and for the X_0 production

$$\langle\sigma_{X_0}\rangle_{\text{spins}}\propto|T_0(\omega)|^2+3|T_1(\omega)|^2. \quad (16)$$

III. CROSS-SECTIONS

Using Eqs. (7), (12), (14), (15), and (16) we may calculate the coefficients of the products of the various $SU(6)$ invariant amplitudes entering $|T_j|^2$, for the final states PB, VB, PB^*, VB^* and $X_0\Lambda^0$. The squared magnitudes of the T-matrix elements, averaged over spins, have the general form:

$$\text{(a)}\ P_1+B_1\to P_2+B_2:\ |M_a|^2=|T_0|^2+3|T_1|^2$$
$$=|A_1T_1+A_{35s}T_{35s}+A_{35a}T_{35a}+A_{405}T_{405}|^2$$
$$+3|B_{35s}T_{35s}+B_{35a}T_{35a}+B_{405}T_{405}|^2, \quad (17)$$

$$\text{(b)}\ P+B_1\to V+B_2:\ |M_b|^2=3|T_1|^2$$
$$=3|B_{35s}T_{35s}+B_{35a}T_{35a}+B_{405}T_{405}|^2, \quad (18)$$

$$\text{(c)}\ P_1+B\to P_2+B^*:\ |M_c|^2=3|T_1|^2+5|T_2|^2$$
$$=3|B_{35s}T_{35s}+B_{35a}T_{35a}+B_{405}T_{405}|^2$$
$$+5|C_{405}T_{405}|^2, \quad (19)$$

$$\text{(d)}\ P+B\to V+B^*:\ |M_d|^2=3|T_1|^2$$
$$=3|B_{35s}T_{35s}+B_{35a}T_{35a}+B_{405}T_{405}|^2, \quad (20)$$

$$\text{(e)}\ K^-+p\to X_0+\Lambda^0:\ |M_e|^2=|T_0|^2+3|T_1|^2$$
$$=|A_{35s}T_{35s}+A_{405}T_{405}|^2$$
$$+3|B_{35s}T_{35s}+B_{405}T_{405}|^2. \quad (21)$$

Here the A's, B's, and C's are products of $SU(3)$ CG coefficients[7] and $SU(6)$ scalar factors [see Eq. (7)].

We have made a systematic study of the processes (a)–(d), and have compiled some results in Tables I, II, and III, which list the coefficients of $\mathrm{Re}T_NT_{N'}^*$, occuring in $|M|^2$, for a number of reactions in each group.[8] We will discuss the results for each of the four groups before examining the experimental situation.

Group (a) (Table I)

Simple relations for processes in this group are few in number, in contrast to the GR scheme as discussed in Ref. 9, and by Barger and Rubin, and by Binford,

[7] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963); P. McNamee and Frank Chilton, Rev. Mod. Phys. 36, 1005 (1964).

[8] Cross-section relations for reactions in groups (a), (b), and (d) have been found in the GR scheme by Carter, Coyne, and Meshkov (Ref. 9, below).

TABLE I. Coefficients of $\mathrm{Re}T_N T_{N'}^*$ for reactions of types (a) and (b).

		$\lvert T_{405}\rvert^2$	$\lvert T_{35s}\rvert^2$	$\lvert T_{35a}\rvert^2$	$2\,\mathrm{Re}T_{405}T_{35s}^*$	$2\,\mathrm{Re}T_{405}T_{35a}^*$	$2\,\mathrm{Re}T_{35s}T_{35a}^*$
(a)	$P_1+B \to B+P_2$						
(1)	$K^-+p \to \Xi^-+K^+$	12/5	0	0	0	0	0
(2)	$K^-+p \to \Xi^0+K^0$	57/5	0	0	0	0	0
(3)	$K^-+p \to \Sigma^-+\pi^+$	57/5	0	0	0	0	0
(b)	$P+p \to B+V$						
(4)	$K^-+p \to \Xi^-+K^{*+}$	48/5	0	0	0	0	0
(5)	$K^-+p \to \Xi^0+K^{*0}$	3/5	0	0	0	0	0
(6)	$K^-+p \to \Sigma^-+\rho^+$	3/5	0	0	0	0	0
(7)	$\pi^-+p \to \Sigma^-+K^{*+}$	15	0	0	0	0	0
(8)(e)	$K^-+p \to \Lambda^0+X_0$	27/160	9/32	0	$(27/32)/(15)^{1/2}$	0	0

Cline, and Olsson.[9,10] This complication is due to the appearance, in the CBC model, of the sf amplitude, which introduces many additional terms. In this connection, it is perhaps worth remarking that, in the case of elastic cross sections involving $K^\pm p$, K^0p, $\bar{K}^0p$, and $\pi^\pm p$, simple relations can be obtained because only the nsf amplitude contributes [Eqs. (7) and (11)]. These relations involve the differences $\sigma(PB-PB)-\sigma(\bar{P}B-\bar{P}B)$ which are proportional to the imaginary parts of the corresponding nsf amplitudes, made antisymmetric in the meson $SU(3)$ indices. Owing to the properties of the CG coefficients $\begin{pmatrix}35 & 35 \,|\, N\\ & \,|\,\end{pmatrix}$ only that coefficient coupling $\mathbf{35}\otimes\mathbf{35}$ to $\mathbf{35}_a$ is nonzero. The relative values, in the various meson states, of the $SU(3)$ coefficients $\begin{pmatrix}8 & 8\,|\,8_a\\ & \,|\,\end{pmatrix}$ determine the ratios of the above differences. These ratios are identical to those found by Johnson and Treiman.[11]

The reaction $K^-+p\to\Xi^-+K^+$ proceeds through the exchanged $SU(3)$ representation **27**, while the reaction $K^-+p\to\Sigma^-+\pi^+$ (or, alternatively, $\pi^-+p\to\Sigma^-+K^+$) involves both the **27** and the **10** of $SU(3)$. This latter reaction, therefore, has a structure peculiar to $SU(6)$ and the value of the cross section will depend on which $SU(6)$ scheme is used to calculate it:

$$|M_1|^2/|M_3|^2=4/19 \quad \text{(CBC)},$$
$$=4 \quad \text{(GR)}. \qquad (22)$$

The difference between these two results is due to the presence in the CBC model of the sf amplitude [whose magnitude, in the reaction $\pi^-+p\to\Sigma^-+K^+$, is $(18)^{1/2}$ times the magnitude of the nsf amplitude].

The next simplest reactions to consider would be $K^-+p\to\pi^0+\Lambda^0(\Sigma^0)$, but already for these reactions the CBC model does not yield a simple relation between the quantities $|M(K^-+p\to\pi^0+\Lambda^0)|^2$ and $|M(K^-+p\to\pi^0+\Sigma^0)|^2$, while the GR scheme predicts a value of 3 for their ratio.

The equality $|M_2|^2=|M_3|^2$ has been found to hold in $SU(3)$.[12]

Group (b) (Table I)

The equalities

$$(\tfrac{1}{16})|M_4|^2=|M_5|^2=|M_6|^2=(1/25)|M_7|^2$$

are purely $SU(6)$ predictions.

Group (c) (Table II)

The first five reactions are mediated solely by the **405**. However, only reaction (10) has a structure dependent on $SU(6)$. The other four involve the $SU(3)$ **27** and are, as a result, pure $SU(3)$ predictions.[12] The next four reactions satisfy the relation

$$|M_{14}|^2=|M_{15}|^2+3|M_{16}|^2-3|M_{17}|^2 \qquad (23)$$

also valid in $SU(3)$.[13]

In addition we obtain a number of $SU(6)$ predictions, some of which relate amplitudes involving approximately the same masses and which should provide a reasonable test of the CBC model.

Group (d) (Table III)

Again, of the first seven reactions, only numbers (25), (29), and (30) have a specifically $SU(6)$ structure, the others being true in $SU(3)$. The next four satisfy the $SU(3)$ relation

$$|M_{31}|^2=|M_{32}|^2+3|M_{33}|^2-3|M_{34}|^2. \qquad (24)$$

In this group we also find a number of pure $SU(6)$ predictions.

Of the four types of reactions we have discussed, only those of groups (b) and (d) will always fail to yield

[9] J. C. Carter, J. J. Coyne, and S. Meshkov, Phys. Rev. Letters 14, 523 (1965) and erratum Phys. Rev. Letters 14, 850 (1965).

[10] V. Barger and M. L. Rubin, Phys. Rev. Letters 14, 713 (1965); T. Binford, D. Cline, and M. Olsson, Phys. Rev. Letters 14, 715 (1965).

[11] K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965); see also R. F. Sawyer, Phys. Rev. Letters 14, 471 (1965).

[12] C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Letters 1, 44 (1962).

[13] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters 13, 212 (1964).

TABLE II. Coefficients of $\mathrm{Re}T_N T_{N'}^*$ for reactions of type (c). Note that the relations between (14) and (18), and (15) and (22) result from isospin invariance.

	$\|T_{405}\|^2$	$\|T_{35s}\|^2$	$\|T_{35a}\|^2$	$2\,\mathrm{Re}T_{405}T_{35s}^*$	$2\,\mathrm{Re}T_{405}T_{35a}^*$	$2\,\mathrm{Re}T_{35s}T_{35a}^*$
(c) $P_1+B \quad P_2+B^*$						
(9) $K^-+p\to\Xi^{*-}+K^+$	48	0	0	0	0	0
(10) $K^-+p\to\Xi^{*0}+K^0$	168/5	0	0	0	0	0
(11) $K^-+p\to Y_1^{*-}+\pi^+$	48	0	0	0	0	0
(12) $\pi^-+p\to N^{*-}+\pi^+$	144	0	0	0	0	0
(13) $\pi^0+p\to N^{*+}+\pi^0$	48	0	0	0	0	0
(14) $K^++p\to N^{*++}K^0$	189/25	3	8/3	$9/(15)^{1/2}$	$12/(30)^{1/2}$	$2(2)^{1/2}$
(15) $\pi^++p\to N^{*++}\pi^0$	162/5	6	0	$-36/(15)^{1/2}$	0	0
(16) $\pi^+p\to N^{*++}\eta$	276/25	0	16/9	0	$24/(30)^{1/2}$	0
(17) $K^-p\to Y_1^{*+}\pi^-$	483/25	1	8/9	$-15/(15)^{1/2}$	$20/(30)^{1/2}$	$-\frac{2}{3}(2)^{1/2}$
(18) $K^+p\to N^{*+}K^+$	63/25	1	8/9	$3/(15)^{1/2}$	$4/(30)^{1/2}$	$(\frac{2}{3})(2)^{1/2}$
(19) $K^-p\to Y_1^{*0}\pi^0$	441/50	$\frac{1}{4}$	2/9	$-(15/4)/(15)^{1/2}$	$5/(30)^{1/2}$	$-\frac{1}{6}(2)^{1/2}$
(20) $\pi^-p\to Y_1^{*0}K^0$	963/50	$\frac{1}{2}$	4/9	$-(15/2)/(15)^{1/2}$	$10/(30)^{1/2}$	$-\frac{1}{3}(2)^{1/2}$
(21) $\pi^-p\to N^{*0}\eta$	92/25	0	16/27	0	$-8/(30)^{1/2}$	0
(22) $\pi^+p\to N^{*+}\pi^+$	108/5	4	0	$-24/(15)^{1/2}$	0	0
(23) $\pi^-p\to N^{*0}\pi^0$	294/5	2	0	$-12/(15)^{1/2}$	0	0

polarized baryons. The reason for this is readily understood. The CBC model describes the coupling of the spin of the vector mesons to their orbital motion through the combination $\mathbf{k}\cdot\mathbf{V}$. In the reactions of all the groups except groups (b) and (d), this combination contributes to both the nsf and the sf amplitudes. In these exceptional cases, however, $\mathbf{k}\cdot\mathbf{V}$ enters only the sf amplitude. Here the angular dependence of the nsf amplitude is given by $\boldsymbol{\varepsilon}\cdot\hat{k}_f\times\hat{k}_i$, where $\boldsymbol{\varepsilon}$ is the polarization vector of the field V_μ. This is the projection of the vector-meson polarization onto the normal to the scattering plane, and it is the neglect of this component which prohibits the polarization of the final baryon. In the s channel, an intermediate state of spin $\frac{3}{2}$ emitting a static baryon and a total angular momentum $J=1$ ($\boldsymbol{\varepsilon}\times\mathbf{k}_f$) vector meson will contribute to the term $\boldsymbol{\varepsilon}\cdot\hat{k}_f\times\hat{k}_i$. Hence, the absence of the nsf term in the reactions $B_1+P\to B_2+V$ is closely related to the prediction in this model that the rate at which a baryon resonance decays into a baryon and a vector meson is zero.

One might contemplate comparing reactions from the different groups ($|M(PB\to PB^*)|^2$ with $|M(PB\to VB^*)|^2$, for example), but it is not clear that the energy dependence of the $SU(6)$ invariant amplitudes of one group will be the same as the energy dependence of the corresponding invariant amplitudes of another group. The possibility of such a comparison will be discussed in the latter part of this section.

TABLE III. Coefficients of $\mathrm{Re}T_N T_{N'}^*$ for reactions of type (d). Note that the relations between (31) and (37), and (32) and (35) result from isospin invariance.

	$\|T_{405}\|^2$	$\|T_{35s}\|^2$	$\|T_{35a}\|^2$	$2\,\mathrm{Re}T_{405}T_{35s}^*$	$2\,\mathrm{Re}T_{405}T_{35a}^*$	$2\,\mathrm{Re}T_{35s}T_{35a}^*$
(d) $P+B\to B^*+V$						
(24) $K^-+p\to\Xi^{*-}K^{*+}$	24/5	0	0	0	0	0
(25) $K^-+p\to\Xi^{*0}K^{*0}$	96/5	0	0	0	0	0
(26) $K^-+p\to Y_1^{*-}\rho^+$	24/5	0	0	0	0	0
(27) $\pi^-p\to Y_1^{*-}K^{*+}$	24/5	0	0	0	0	0
(28) $\pi^-p\to N^{*-}\rho^+$	72/5	0	0	0	0	0
(29) $\pi^0p\to N^{*+}\rho^0$	24/5	0	0	0	0	0
(30) $\pi^-p\to N^{*+}\rho^-$	0	0	16/9	0	0	0
(31) $K^+p\to N^{*++}K^{*0}$	3249/250	$\frac{3}{2}$	4/3	$(-171/10)/(15)^{1/2}$	$(114/5)/(30)^{1/2}$	$-(2)^{1/2}$
(32) $\pi^+p\to N^{*++}\rho^0$	36/5	0	8/3	0	$24/(30)^{1/2}$	0
(33) $\pi^+p\to N^{*++}\varphi^0$	243/125	1	0	$(-27/5)/(15)^{1/2}$	0	0
(34) $\pi^+p\to Y_1^{*+}+K^{*+}$	3/250	$\frac{1}{2}$	4/9	$(3/10)/(15)^{1/2}$	$(2/5)/(30)^{1/2}$	$(2)^{1/2}/3$
(35) $\pi^+p\to N^{*+}+\rho^+$	24/5	0	16/9	0	$16/(30)^{1/2}$	0
(36) $\pi^+p\to N^{*0}\rho^0$	12/5	0	8/9	0	$-8/(30)^{1/2}$	0
(37) $K^+p\to N^{*+}K^{*+}$	1083/250	$\frac{1}{2}$	4/9	$(-57/10)/(15)^{1/2}$	$(38/5)/(30)^{1/2}$	$-(2)^{1/2}/3$
(38) $K^-p\to Y_1^{*+}\rho^-$	1323/250	$\frac{1}{2}$	4/9	$(63/10)/(15)^{1/2}$	$(-42/5)/(30)^{1/2}$	$-(2)^{1/2}/3$
(39) $K^-p\to Y_1^{*0}\rho^0$	5547/1000	$\frac{1}{8}$	1/9	$(129/40)/(15)^{1/2}$	$(-43/10)/(30)^{1/2}$	$-(2)^{1/2}/12$

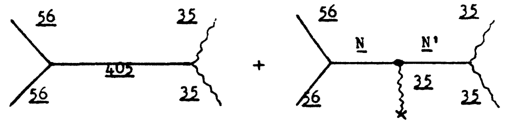

FIG. 3. Symmetry-breaking coupling scheme introduced in the discussion of the polarization of the final baryon.

Before attempting to compare the predictions of Tables I, II, and III with experiment, we must correct the experimental data by introducing appropriate kinematical factors. Moreover, we are faced with defining a "comparison" energy, E_C, such that the symmetry invariant amplitudes might be expected to have the same dependence on this energy. The introduction of the kinematical factors and the use of such a comparison energy are attempts at partially correcting for the fact that the masses of particles belonging to a certain multiplet are not equal, that the symmetry is broken. A comparison of some $SU(3)$ predictions with experiment, has been made,[13,14] in which each experimental cross section was multiplied by the factor (initial flux)/(final phase space) (evaluated in the center-of-mass system) appropriate for a given reaction. These quantities were compared at equal values of $Q=E^*-m_3-m_4$, E^* being the total energy in the center-of-mass system and m_3+m_4 being the sum of the final masses. This same correction has been introduced by Binford *et al.*[10] in comparing the predictions of the GR scheme with experiment.

Since the CBC $SU(6)$ scheme is essentially a static model and thus, inherently describes only the p wave, the kinematical factor which we have used is $(k_f/k_iW^2)(k_fk_i)^2$, where all of the quantities are barycentric variables; $k_{i(f)}$ is the magnitude of the initial (final) momentum and W is the total energy. Here, the first factor is essentially the ratio of phase space to initial flux, and k_fk_i is the product of the final centrifugal barrier factor for p waves times the same quantity for the initial state. The comparison energy which we will use is defined by

$$E_C=W-\langle m\rangle\,, \tag{25}$$

$\langle m\rangle$ being an average of the baryon masses possible in the s channel. In this channel, the hypercharge Y is zero for the initial state K^-p, while $Y=1$ for the initial system state π^-p. We have, for instance

$$\langle m\rangle=[2(m_{\Sigma^+}+m_{\Sigma^0}+m_{\Sigma^-}+m_{\Lambda^0}) +4(m_{Y_1^{*+}}+m_{Y_1^{*0}}+m_{Y_1^{*}})]/20\,. \tag{26}$$

This is the average mass (including spin multiplicities) in the **56** representation with $Y=0$.

In connection with the p-wave nature of the model we note that we should compare the predicted cross section, not with the *total* cross section, but instead with the *partial*, $\sigma_{l=1}$, experimental cross section. Without the additional information inherent in the differential cross section, we cannot say how much to subtract in order to rid ourselves of the s-wave contribution, which cannot be accounted for in this model.

At the present time, such detailed experimental evidence is not available to us. Therefore, we will try to compare the predictions of the CBC model for $|M(K^-p\to K^+\Xi^-)|^2/|M(\pi^-p\to K^+\Sigma^-)|^2$ with the experimental total cross sections,[15,16] keeping in mind that this cannot be an entirely convincing comparison.

We have chosen the reaction $\pi^-+p\to K^++\Sigma^-$ instead of $K^-+p\to\pi^-+\Sigma^-$ because the threshold for the former reaction in terms of the incident beam momentum in the laboratory system, is almost the same as for $K^-+p\to\Xi^-+K^+$ ($p_{\rm inc}=1.04$ BeV/c). These reactions are kinematically very similar and are, as a consequence, less ambiguously compared.

We must also try to avoid regions of energy where resonances, especially in the p wave, may occur. Fortunately, at beam momenta between threshold and, say, 1.2 BeV/c there are no resonances in the p wave.

Using the ratio of the total cross sections corrected by the kinematical factors and formed at the same comparison energy, we find

$$|M(\Xi^-K^+)|^2/|M(\Sigma^-K^+)|^2\cong 0.05-0.25 \tag{27}$$

in the range of incident momenta between 1.05 and 1.2 BeV/c. This is not in disagreement with the CBC prediction of 0.21 [see Eq. (22)].

IV. POLARIZATION OF THE FINAL BARYON

In the reactions $P_1+B_1\to P_2+B_2$, the final baryon is polarized, owing to the interference between the nsf and the sf amplitudes. It is found experimentally that in $K^-+p\to K^++\Xi^-$ the Ξ^- is almost maximally polarized at energies not far above threshold. It is also known that the average polarization is almost zero when the angle between the Ξ^- direction and the incident proton direction, in the center-of-mass system, is about 90°.[17] This last fact implies that, if the d wave were small, the polarization of the Ξ^- would be primarily due to the interference between the p-wave parts of the nsf and sf amplitudes. It is therefore important to examine the polarization predictions of a symmetry scheme.

From Eq. (13) it can be seen that nonzero polarization can only be obtained in CBC if there is an interference between different $SU(6)$ amplitudes. If, however, the sf and nsf amplitudes are both proportional to the same $SU(6)$ amplitude, no polarization can occur. For instance, for the reactions $K^-+p\to K^++\Xi^-$ and $\pi^-+p\to K^++\Sigma^-$ the sf and nsf amplitudes are proportional only to T_{405} (cf. Table I). Since the CG coefficients are real this implies that $\mathrm{Im}T_0T_1^*$ vanishes

[14] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters **10**, 361 (1963).

[15] J. A. Schwartz, Ph.D. thesis, University of California Radiation Laboratory Report UCRL-11360, 1964 (unpublished).

[16] J. Button-Shafer, Bull. Am. Phys. Soc. **9**, 482 (1964).

[17] M. L. Stevenson, University of California Radiation Laboratory Report UCRL-11493, 1964 (unpublished).

and therefore CBC predicts zero polarization for both reactions.[18] In order to overcome this difficulty, one might imagine that the polarization is generated by a symmetry-breaking term present in the "physical" T matrix (Fig. 3). In the case of $K^+\Xi^-$, the sf amplitude is zero in the symmetry limit (since the scalar factor

$$\left(\begin{array}{cc||c} 35 & 35 & 405 \\ (3,8) & (3,8) & (3,27) \end{array}\right)$$

is identically zero). Here, symmetry breaking must generate the entire sf amplitude. One way of achieving this is to include a term in the T matrix which transforms as the $I=Y=0$ member of the (1,8) contained in the **35**. This is essentially a breaking of the $SU(3)$ symmetry; because of the way in which (1,8) enters the **35**, however, there will be additional relations due to $SU(6)$.[19] The sf amplitude is generated solely by the contribution to T_1 induced by symmetry breaking as shown in Fig. 4. In the reaction $\pi^-+p\rightarrow K^++\Sigma^-$, the exchanged representation can contain an $SU(3)$ **10**, as well as the **27**. The nsf amplitude is due to the exchange of the (1,27) subrepresentation, while the sf amplitude

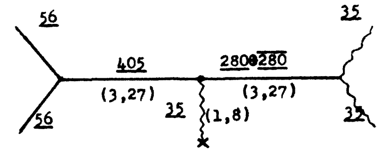

FIG. 4. The spin-flip amplitude of reaction 1 (Table I) which is generated by symmetry breaking.

arises from the exchange of the (3,10) of the **405**. Again the two amplitudes are proportional to T_{405}. A nonzero value of the polarization can be induced in this reaction only by symmetry breaking. In this regard, it is unfortunate that measurements of the polarization of the Σ^- are very difficult.

The polarization is expected to be quite sensitive to the symmetry breaking, especially if corresponding cross-section relations, in the symmetry limit, compare well with experiment. Then the symmetry-breaking-term amplitude may be small relative to the original term and yet still give the desired amount of polarization.

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[18] This situation is identical to that found in relativistic extensions of $SU(6)$ [J. M. Cornwall, P. G. O. Freund, and K. T. Mahanthappa, Phys. Rev. Letters **14**, 515 (1965); R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters **14**, 518 (1965)].

[19] That a breaking of the $SU(3)$ symmetry is sometimes needed has been pointed out by Meshkov, Snow, and Yodh (Ref. 14) in connection with their work on the $P^{3/2}$ baryon-resonance production cross sections.