Differential Cross Section* for $e^+ + e^- \to W^+ + W^- \to e^- + \bar{\nu}_e + \mu^+ + \nu_\mu$

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The cross section $e^+ + e^- \to W^+ + W^- \to \mu^+ + \nu_\mu + e^- + \bar{\nu}_e$ in which e^- and μ^+ are detected in coincidence in the colliding-beam experiment is computed with the mass, magnetic moment, and leptonic mode branching ratio of the W boson as parameters. The kinematical correlations necessary for the identification and mass determination of the W meson are discussed. Numerical examples show that the energy-angle correlations of the final e and μ are very sensitive to the W mass. The analytical expression for the cross section was obtained by an electronic computer. The characteristics of dynamical correlations were investigated by numerical examples of angular distributions of e^- and μ^+ for different values of magnetic moment of W. It was found that the rate of increase of cross section with respect to the relative angle between the final electron and muon is the most sensitive dynamical correlation needed for the determination of the W magnetic moment. We ignore the possibility that W may have form factors and an anomalous quadrupole moment. Symmetries in the differential cross section are discussed. Because of one-photon exchange, the differential cross section e^- and μ^+ must be symmetric with respect to the plane perpendicular to the incident beam. Because of time-reversal invariance, the differential cross section for μ^+ must be symmetric with respect to the plane formed by the incident beam and the final electron. Similarly the differential cross section for e^- must be symmetric with respect to the plane formed by the incident beam and the μ^+. It is also shown that the charge-conjugate decay mode $e^+ + e^- \to W^+ + W^- \to \mu^- + \bar{\nu}_\mu + e^+ + \nu_e$ can be obtained from our result by simply putting $\mu^+ \to \mu^-$ and $e^- \to e^+$ in the final state if one considers only the lowest order process. It is pointed out that the techniques used in this paper can be employed to calculate many other processes in which two unstable particles are produced.

I. INTRODUCTION

With the success of the Stanford electron-electron colliding-beam project[1] and the building of electron-positron colliding-beam machines[2] at various places in the world, it may be useful to consider again the production of weak vector bosons which have so far escaped detection.[3] The cross section $e^+ + e^- \to W^+ + W^-$ via the one-photon intermediate state has been calculated by Cabibbo and Gatto.[4] In this paper we would like to consider the particular decay modes

$$e^+ + e^- \to W^+ + W^- \begin{array}{l} \nearrow e^- + \bar{\nu} \\ \\ \searrow \mu^+ + \nu , \end{array} \qquad (1.1)$$

in which e^- and μ^+ are detected in coincidence. The particular W decay modes given above have the minimum background problem. Other decay modes of W, such as $\pi\pi$, $\rho\pi$, $\omega\pi$, etc., are extremely interesting from general weak interaction theory[5] and can be incorporated into our calculation easily. However, there are so many ways W can decay into pions that even if π's are detected, it would be much harder to interpret the result, aside from the fact that many more pions are produced directly via $e^+ + e^- \to \gamma \to$ multiple π's.

Since e^- and μ^+ are to be detected in coincidence, they are correlated both kinematically and dynamically. The kinematical correlations are given by Eqs. (2.20)–(2.28) which give the constraints among the final electron energy, the muon energy and their relative angle. These kinematical constraints are sensitive functions of mass of W, and hence they must be used to determine the mass of W. There are two other unknown parameters[6]

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[3] See, for example, G. Bernardini, in *Proceedings of the 12th Annual International Conference on High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[4] N. Cabibbo and R. Gatto, Phys. Rev. **124**, 1577 (1961). See also H. Überall, Nucl. Phys. **58**, 625 (1964). These two papers also treated the effects caused by the polarization of one of the W's produced. Our paper treats the effects caused by the correlation of the polarization of two W bosons.

[5] H. S. Mani and J. C. Nearing, Phys. Rev. **135**, B1009 (1964).

[6] In this paper we adopt the convention of T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962) in which the W has no anomalous quadrupole moment and no electromagnetic form factors. We could have included these effects into our formulation easily by a computer. It would just make the expression for C more complicated. According to the usual arguments, W cannot have form factors because it does not interact strongly. But it is evident from discussion in Sec. IV that there must be some mechanism of damping at high energies in order to preserve unitarity. Probably every particle has some finite intrinsic extension such that its mass and charge renormalization constants are finite and the cross section, such as discussed in Sec. IV, preserves unitarity at high energies. After all, it is very hard to believe that any particle can be truly a geometrical point in which all its mass, electric charge, magnetic moment, weak charge, etc., are located. In this sense, the measurement of the electromagnetic form factors of the W boson is as fundamental as the measurement of electromagnetic form factors of electrons and muons.

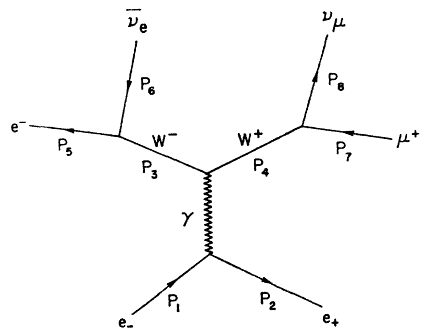

FIG. 1. Feynman diagram for the process $e^+ + e^- \rightarrow W^+ + W^- \rightarrow \mu^+ + \nu_\mu + e^- + \bar{\nu}_e$.

besides mass in our calculation, namely the branching ratio $R = \Gamma(W \rightarrow e + \nu)/\Gamma_{tot}$ and the magnetic moment $(1+k)e\hbar/2Wc$. The expression for our differential cross section is proportional to R^2 and hence the relative angular distribution depends only upon k, after the W mass is determined from the kinematics. Once the magnetic moment is determined from the angular distribution, the branching ratio R can be determined by the magnitude of the cross section, without even measuring other decay modes of W directly. The angular distribution depends upon the dynamical correlation. This correlation arises from the fact that the two W's produced are polarized and the polarization of each is correlated with the other, and that the angular distribution of leptons from the polarized W is different from that of an unpolarized W. The polarization state of two correlated vector particles can be described in general by a 9×9 Hermitian density matrix. In a covariant description this density matrix is represented by a rank-4 tensor, each vector index satisfying the usual subsidiary condition for the relativistic polarization vector of a particle. The possibility of such a representation comes from the requirement that the fourth component of the polarization vector vanishes in the rest frame of the particle. This covariant density matrix is obtained in Sec. 2 and its properties are given there. The analytical expression for the matrix element squared (C) was obtained by a computer.[7]

[7] The computer can take traces of the γ matrices, contract tensor indices, use kinematics to reduce the expression in terms of a minimum number of invariants, re-express invariants in terms of quantities like (E, P, $\cos\theta$), rearrange the whole expression in a dictionary form such as descending powers of each variable, and set the mass of the electron m equal to zero, etc. Actually the expression for C was obtained directly from the Feynman diagram without using any of the intermediate expressions given in Eqs. (2.30), (2.32), (2.33), and (2.35). However, the computer was used to check all of these intermediate expressions. It was found in addition that computation time could be saved if these intermediate expressions were actually used, because symmetry properties such as gauge invariance, subsidiary conditions for polarization vectors, symmetries under exchange of certain tensor indices, etc., were employed to simplify these expressions, whereas the computer did not use these properties in the intermediate stages. The computer program used to obtain C was constructed by the second named author, A.C.H. The analytical expression for C is available from the authors upon request.

In Sec. III we discuss symmetries in the cross sections. In Sec. IV the differential cross section $e^+ + e^- \rightarrow W^+ + W^-$ is discussed. In Sec. V the energies of the electron and muon are integrated and the characteristics of their angular distributions are investigated for an arbitrary set of parameters with the mass of the boson $W = 2$ BeV, incident electron energy $E = 3$ BeV, magnetic moment $k = -2$, 0, 2, branching ratio $R = 0.25$. We found that the cross section increases rapidly as we increase the relative angle θ_{57} between the final electron and muon. The rate of increase from 30° to 150° is approximately 1 to 10 for $k = -2$, 1 to 30 for $k = 0$, and 1 to 15 for $k = 2$. Thus the different rates of increase in the differential cross section with respect to the relative angle between the final e^- and μ^+ are the most sensitive dynamical correlation for determining k. Of course the over-all rate is also a very sensitive function of k, but we think it should be reserved to determine the branching ratio R unless R can be found by some other means. In Sec. VI we discuss some general aspects of our calculation and make some additional remarks relevant to the planning of the experiment.

We have tried to write this paper in such a way that all the results can be used readily by the experimenters. Thus many trivial details are also included whenever we think they are useful.

II. CALCULATIONS

All the desired information including kinematical and dynamical correlations of the problem under consideration can be obtained by computing the Feynman diagram shown in Fig. 1, provided one replaces the square of each denominator of the W boson propagator which occurs in the square of the matrix element by a δ function

$$|p_W{}^2 - W^2|^{-2} \rightarrow \pi\delta(p_W{}^2 - W^2)/\Gamma W, \qquad (2.1)$$

where W, Γ and p_W are the mass, the total width, and the four-momentum of the vector boson. This replacement is allowed if $W \gg \Gamma$. Denoting the branching ratio of the mode $W^- \rightarrow e^- + \bar{\nu}$ as R and the Fermi constant as G, we have[8]

$$\Gamma = \Gamma(W^- \rightarrow e^- + \bar{\nu})/R = g^2 W/6\pi R = GW^3/6\sqrt{2}\pi R$$
$$= 1.02 \times 10^{-5} W^3/6\sqrt{2}\pi R M_p{}^2,$$

where M_p is the mass of proton and g is the coupling constant between W and the leptonic current. From the last relation one can obtain criteria under which the replacement (2.1) is allowed. For example, for $W = 2M_p$ and $R = 0.25$ we have $\Gamma = 1.14 \times 10^{-2}$ MeV (corresponding to mean life 5×10^{-20} sec) which is much less than W and thus (2.1) is justified. On the other hand, if $W = 100 M_p$ and $R = 0.001$, we can no longer use (2.1), but under such circumstances the experiment is unfeasible, at least for the foreseeable future.

[8] T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).

We shall try to formulate our presentation in such a way that those who intend to design the experiment can make maximum use of it. The kinematical correlations which are important for the mass determination are presented in detail. We shall see that for each choice of final electron and muon momenta, there correspond two production angles of W's.

The notations used in this paper are as follows. The four-momenta of particles are denoted by p_1=initial electron, p_2=initial positron, $p_3=W^-$ boson, $p_4=W^+$ boson, p_5=final electron, $p_6=\bar{\nu}_e$ neutrino, p_7=muon, and $p_8=\nu_\mu$ neutrino. The masses of the electron, muon, and W boson are denoted by m, μ, and W, respectively. E_i and P_i represent the energy and momentum of the ith particle; the exception is $E_1=E_2=E_3=E_4=E$. θ_{ij} is the angle between P_i and P_j. θ_6, φ_6 and φ_{17} are defined in Figs. 2(a) and 2(b). The coupling constants are defined as $e^2/4\pi=\alpha$ and $g^2/W^2=G/\sqrt{2}$, where $G=1.02\times10^{-5}/M_p^2$. The metric used is such that $(p_3\cdot p_7)=EE_7-P_3P_7\cos\theta_{37}$.

We adopt the quantum electrodynamics of vector bosons[8] by Lee and Yang in which W has an arbitrary magnetic moment $\mathfrak{M}=(1+k)(e/2W)S$, the quadrupole moment is not arbitrary but is given by $Q=-ek/W^2$.

For convenience of discussion and computation we write the differential cross section in the following way[9]:

$$d\sigma=(4\pi)^4\frac{1}{4[(p_1\cdot p_2)^2-m^4]^{1/2}}\int\frac{d^3P_5}{2E_5}\frac{d^3P_6}{2E_6}\frac{d^3P_7}{2E_7}\frac{d^3P_8}{2E_8}$$
$$\times\frac{1}{(2\pi)^{12}}\frac{1}{4}\frac{\pi^2}{\Gamma^2W^2}\frac{e^4g^4}{(2E)^4}\delta^4(p_1+p_2-p_5-p_6-p_7-p_8)$$
$$\times\delta((p_5+p_6)^2-W^2)\delta((p_7+p_8)^2-W^2)128C\equiv ABC\,; \tag{2.2}$$

A is a numerical factor and is given by

$$A=\frac{\alpha^2g^4 16}{(2\pi)^4\Gamma^2W^2(2E)^6}=\frac{9\alpha^2R^2}{4(2\pi)^2W^4E^6}, \tag{2.3}$$

R is the branching ratio

$$R\equiv\Gamma(W^-\to e^-+\bar{\nu})/\Gamma\equiv g^2W/\Gamma6\pi\,, \tag{2.4}$$

and C is essentially the matrix-element squared with propagators and coupling constants taken out and will be defined in Eq. (2.29).

Kinematical Correlations

B represents the phase space and contains all the information about kinematical correlations which are important in the verification of the existence of W and the determination of its mass.

$$B\equiv\int\frac{d^3P_5}{2E_5}\int\frac{d^3P_7}{2E_7}\int\frac{d^3P_6}{2E_6}\int\frac{d^3P_8}{2E_8}$$
$$\times\delta^4(p_1+p_2-p_5-p_6-p_7-p_8)$$
$$\times\delta((p_5+p_6)^2-W^2)\delta((p_7+p_8)^2-W^2)$$
$$=\tfrac{1}{8}P_5dE_5P_7dE_7d\Omega_5d\Omega_7\int_0^{2\pi}d\varphi_6\delta((p_1+p_2-p_5-p_6-p_7)^2)$$
$$\times\int_0^\infty dE_6\delta((p_1+p_2-p_5-p_6)^2-W^2)$$
$$\times\int_{-1}^1 d(P_6\cos\theta_6)\delta((p_5+p_6)^2-W^2)\,. \tag{2.5}$$

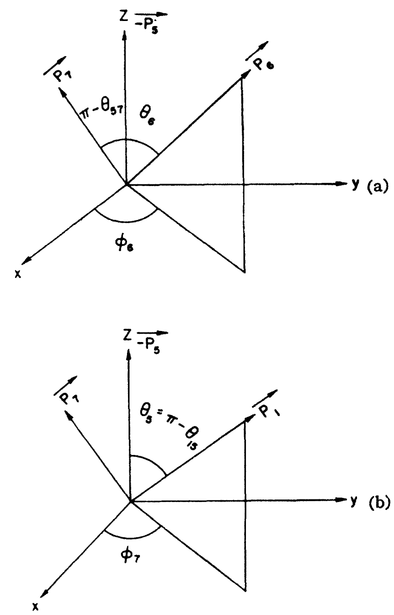

FIG. 2. The coordinate system chosen to define θ_{57}, θ_6, φ_6, θ_{15} and φ_7.

Using the coordinate system shown in Fig. 2, the integrations can be performed by using the δ functions.

$$\int_{-1}^1 d(P_6\cos\theta_6)\delta((p_5+p_6)^2-W^2)=\frac{1}{2p_5}\ \text{if}\ \cos\theta_6\leq1\,,$$
$$=0\quad\text{otherwise}\,, \tag{2.6}$$

where

$$\cos\theta_6=\frac{W^2-2E_5(E-E_5)}{2E_5(E-E_5)}=\cos(\pi-\theta_{56})\,, \tag{2.7}$$

$$\int_0^\infty dE_6\delta((p_1+p_2-p_5-p_6)^2-W^2)=\frac{1}{4E}\ \text{if}\ E>E_5$$
$$=0\quad\text{otherwise}. \tag{2.8}$$

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[9] The factor $128=8\times4\times4$ [in Eq. (2.2)] comes from the numerical factors in the definitions of C, Y, X in Eqs. (2.29), (2.32), (2.33).

The integration with respect to φ_6 is slightly more complicated because the argument of the δ function vanishes at two points in the range of integration. The matrix element squared C depends upon φ_6 as well as other variables. For the moment we will write $C=C(\varphi_6)$ and evaluate

$$\int_0^{2\pi} d\varphi_6 C(\varphi_6)\delta((p_1+p_2-p_5-p_6-p_7)^2)$$
$$\equiv\int_0^{2\pi} C(\varphi_6)\delta(a-b\cos\varphi_6)d\varphi_6$$
$$=\frac{C(\varphi_6)+C(-\varphi_6)}{(b^2-a^2)^{1/2}} \quad \text{if} \quad |\cos\varphi_6|=|a/b|\leq 1,$$
$$=0 \quad \text{otherwise}, \tag{2.9}$$

where

$$a=W^2+\mu^2-2EE_7+p_7E_5^{-1}(W^2-2EE_5)\cos\theta_{57}, \tag{2.10}$$

and

$$b=Wp_7E_5^{-1}[4E_5(E-E_5)-W^2]^{1/2}\sin\theta_{57}. \tag{2.11}$$

We choose

$$\pi\geq\varphi_6\geq 0. \tag{2.12}$$

For convenience of discussion let us write

$$\cos\theta_{35}=(EE_5-\tfrac{1}{2}W^2)/E_5P_4 \tag{2.13}$$

and

$$\cos\theta_{47}=[EE_7-\tfrac{1}{2}(W^2+\mu^2)]/P_7P_4. \tag{2.14}$$

These two equations can be obtained trivially from

$$(p_3-p_5)^2=p_6^2=0 \quad \text{and} \quad (p_4-p_7)^2=p_8^2=0.$$

In terms of θ_{35} and θ_{47} we may write a and b in Eqs. (2.10) and (2.11) as

$$a=-2P_4P_7(\cos\theta_{47}+\cos\theta_{35}\cos\theta_{57}) \tag{2.15}$$

$$b=2P_4P_7\sin\theta_{35}\sin\theta_{57}. \tag{2.16}$$

The two values of φ_6 allowed for each choice of P_5 and P_7 correspond to two production angles for the W pair. To see this we write

$$x_\pm\equiv-\mathbf{P}_1\cdot\mathbf{P}_3{}^\pm/E=-\mathbf{P}_1\cdot(\mathbf{P}_5+\mathbf{P}_6{}^\pm)/E$$
$$=-E_5\cos\theta_{15}-(E-E_5)\cos\theta_{16}{}^\pm, \tag{2.17}$$

where

$$\cos\theta_{16}{}^\pm=-\cos\theta_{15}\cos\theta_6+\sin\theta_{15}\sin\theta_6\cos\varphi_6\cos\varphi_7$$
$$\pm\sin\theta_{15}\sin\theta_6\sin\varphi_6\sin\varphi_7. \tag{2.18}$$

In summary the desired cross section can be written in the form

$$\frac{d\sigma}{dE_5dE_7d\Omega_5d\Omega_7}=\frac{9r_0{}^2m^2R^2[C(x_+)+C(x_-)]}{512(2\pi)^2W^4E^7P_4[\cos(\theta_{47}+\theta_{35})+2\cos\theta_{35}\cos\theta_{47}\cos\theta_{57}+\cos\theta_{57}]^{1/2}}, \tag{2.19}$$

where $C(x_+)$ and $C(x_-)$ correspond to $C(\varphi_6)$ and $C(-\varphi_6)$, respectively, in Eq. (2.9).

The allowed range of E_5, E_7, $d\Omega_5$, and $d\Omega_7$ of the cross section can be obtained from the inequalities in Eqs. (2.6), (2.8), and (2.9). From Eqs. (2.6) and (2.8) we obtain

$$\tfrac{1}{2}(E+P_4)>E_5>\tfrac{1}{2}(E-P_4) \tag{2.20}$$

and

$$\tfrac{1}{2}(E+P_4)+\mu^2(E-P_4)/2W^2>E_7>\tfrac{1}{2}(E-P_4)$$
$$+\mu^2(E+P_4)/2W^2. \tag{2.21}$$

These two inequalities give the energy ranges of the electrons and muons if they are not detected in coincidence. The kinematical constraints due to coincidence are imposed by Eq. (2.9) which can be written as

$$\cos(\theta_{47}+\theta_{35})+2\cos\theta_{35}\cos\theta_{47}\cos\theta_{57}+\cos^2\theta_{57}>0. \tag{2.22}$$

From Eqs. (2.13) and (2.14), we see that θ_{35} and θ_{47} are related to energy of the electron E_5 and of the muon E_7, respectively. Thus Eq. (2.22) gives the range of one of the variables (E_5,E_7,θ_{57}) when the other two are fixed. The three situations are described below.

1. For a given E_5 and E_7, which necessarily must satisfy Eqs. (2.20) and (2.21), the range of θ_{57} is given by

$$(\cos\theta_{57})_{\mathbf{max,min}}=-\cos\theta_{47}\cos\theta_{35}\pm\sin\theta_{47}\sin\theta_{35} \tag{2.23}$$

(where $+$ goes with max and $-$ with min), or

$$|\pi-(\theta_{35}+\theta_{47})|<\theta_{57}<\pi-|\theta_{47}-\theta_{35}|. \tag{2.24}$$

2. For given E_5 and θ_{57}, the range of θ_{47} is given by

$$(\cos\theta_{47})_{\mathbf{max,min}}=-\cos\theta_{35}\cos\theta_{57}\pm\sin\theta_{35}\sin\theta_{57}. \tag{2.25}$$

$E_{7\,\mathbf{max,min}}$ can be obtained by letting $(\cos\theta_{47})_{\mathbf{max,min}}=\cos\theta_{47}$ in the following expression:

$$E_7=\frac{E(W^2+\mu^2)+P_4\cos\theta_{47}((W^2-\mu^2)^2-4\mu^2P_4{}^2\sin^2\theta_{47})^{1/2}}{2(E^2-P_4{}^2\cos^2\theta_{47})} \tag{2.26}$$

3. Similarly, for a given E_7 and θ_{57}, the range of θ_{35} is given by

$$(\cos\theta_{35})_{\mathbf{max,min}}=-\cos\theta_{47}\cos\theta_{57}\pm\sin\theta_{47}\sin\theta_{57}. \tag{2.27}$$

$(E_5)_{\mathbf{max,min}}$ can be obtained by letting $(\cos\theta_{35})_{\mathbf{max,min}}=\cos\theta_{35}$ in the following expression:

$$E_5=W^2/2(E-P_4\cos\theta_{35}). \tag{2.28}$$

The relations (2.23)–(2.28) can also be obtained by drawing pictures. Suppose the electron with energy E_5 is moving along the $-\hat{z}$ direction. From Eq. (2.13), the W^- meson (P_3) must be on a cone around P_5 with angle θ_{35} given by (2.13). Let us invert this cone and call it cone C_{-3} as shown in Fig. 3. Let the muon momentum P_7 be on the xz plane and draw a similar cone for W^+ meson from Eq. (2.14) and call it C_4 as shown in Fig. 3. In order that P_5 and P_7 be detected in coincidence, P_3 and P_4 must come back to back, which means that the two cones C_{-3} and C_4 must intersect. In general there are two lines of intersection between the two cones C_{-3} and C_4, which correspond to two angles of production for W^+ for each set of P_5 and P_7, as mentioned previously. From the picture it is obvious that the condition for the intersection of the two cones is given by Eq. (2.24) and two other relations obtained by permutations $\theta_{57}\leftrightarrow\theta_{35}$ and $\theta_{57}\leftrightarrow\theta_{47}$, respectively.

To illustrate how sensitive these kinematical correlations are to the W mass, we give the following example.

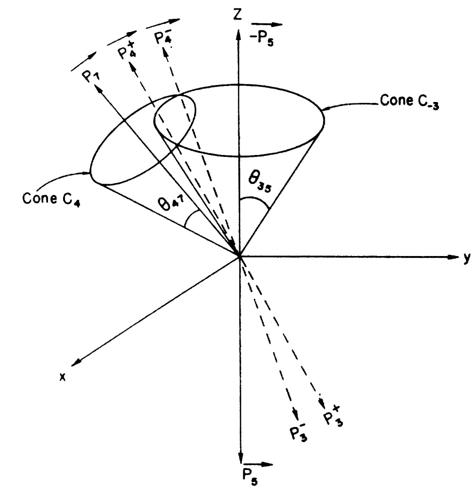

FIG. 3. Kinematical correlations. Two lines of intersection between cone C_4 and cone C_{-3} give the two possible directions of the W^+ boson produced for each choice of final electron and muon momenta.

Numerical Example (Determination of W Mass)

Suppose $E=3$ BeV, $W=1.5$ BeV or 2.0 BeV, $E_5=1$ BeV, and $\theta_{57}=\pi-\frac{1}{6}\pi$. From Eq. (2.13) we obtain

$$\theta_{35}=43.8^\circ \quad \text{for} \quad W=1.5 \text{ BeV}$$
$$=63.6^\circ \quad \text{for} \quad W=2.0 \text{ BeV}.$$

From Eq. (2.25),

$$(\cos\theta_{47})_{\max,\min}=(0.96,0.24) \quad \text{for} \quad W=1.5 \text{ BeV},$$

and

$$(\cos\theta_{47})_{\max,\min}=(0.834,0.060) \quad \text{for} \quad W=2.0 \text{ BeV}.$$

Therefore,

$$(E_7)_{\max,\min}=(2.96,0.49) \text{ BeV} \quad \text{for} \quad W=1.5 \text{ BeV},$$

and

$$(E_7)_{\max,\min}=(1.76,0.552) \text{ BeV} \quad \text{for} \quad W=2.0 \text{ BeV}.$$

From this example we can see that the mass of W can be determined easily from kinematics alone.

Dynamical Correlations

The function C represents the matrix elements squared and can be conveniently written as

$$C=\tfrac{1}{8}t_{\mu\nu}V_{\mu\alpha\beta}V_{\nu\alpha'\beta'}Y_{\beta\beta'}X_{\alpha\alpha'}. \tag{2.29}$$

$t_{\mu\nu}$ is the tensor obtained by taking the trace of the initial electron-positron system,

$$\begin{aligned} t_{\mu\nu}&=-\mathrm{Tr}(-\not{p}_2+m)\gamma_\mu(\not{p}_1+m)\gamma_\nu \\ &=4(p_{1\mu}p_{2\nu}+p_{1\nu}p_{2\mu}-2E^2g_{\mu\nu}) \qquad (2.30)\\ &=-8[E^2g_{\mu\nu}+Q_\mu Q_\nu], \end{aligned}$$

where $\not{p}_i\equiv p_i\cdot\gamma$ and

$$Q=\tfrac{1}{2}(p_1-p_2).$$

$V_{\mu\alpha\beta}$ is the γW^-W^+ vertex,

$$V_{\mu\alpha\beta}=g_{\alpha\beta}(p_4-p_3)_\mu+(1+k)p_{3\beta}g_{\mu\alpha} -(1+k)p_{4\alpha}g_{\mu\beta}. \tag{2.31}$$

$Y_{\beta\beta'}$ is $\frac{1}{4}$ the trace of the $\mu^++\nu$ system and the square of the numerator of the W^+ boson propagator:

$$\begin{aligned} Y_{\beta\beta'}&=-\tfrac{1}{4}\,\mathrm{Tr}[(-\not{p}_7+\mu)(1+\gamma_5)\gamma_\delta\not{p}_8\gamma_{\delta'}(1-\gamma_5)] \\ &\quad\times(p_{4\beta}p_{4\delta}W^{-2}-g_{\beta\delta})(p_{4\beta'}p_{4\delta'}W^{-2}-g_{\beta'\delta'}) \\ &=(W^2-\mu^2)(p_{4\beta}p_{4\beta'}W^{-2}-g_{\beta\beta'}) \\ &\quad-4[p_{4\beta}(p_4\cdot p_7)W^{-2}-p_{7\beta}] \\ &\quad\times[p_{4\beta'}(p_4\cdot p_7)W^{-2}-p_{7\beta'}]-2i\epsilon_{a\beta b\beta'}p_{7a}p_{4b}. \end{aligned} \tag{2.32}$$

$X_{\alpha\alpha'}$ is the corresponding expression for the $e^-+\bar{\nu}$ system,

$$\begin{aligned} X_{\alpha\alpha'}&=-\tfrac{1}{4}\,\mathrm{Tr}[-\not{p}_6(1+\gamma_5)\gamma_\gamma(\not{p}_5+m)\gamma_{\gamma'}(1-\gamma_5)] \\ &\quad\times(p_{3\alpha}p_{3\gamma}W^{-2}-g_{\alpha\delta})(p_{3\alpha'}p_{3\gamma'}W^{-2}-g_{\alpha'\gamma'}) \\ &=(W^2-m^2)(p_{3\alpha}p_{3\alpha'}W^{-2}-g_{\alpha\alpha'}) \\ &\quad-4[p_{3\alpha}(p_3\cdot p_5)W^{-2}-p_{5\alpha}] \\ &\quad\times[p_{3\alpha'}(p_3\cdot p_5)W^{-2}-p_{5\alpha'}] \\ &\quad+2i\epsilon_{c\alpha d\alpha'}p_{5c}p_{3d}. \end{aligned} \tag{2.33}$$

The analytical expression for C was obtained by a computer. We set the mass of the electron $m=0$ for simplicity. C is first written as a function of invariants μ^2, W^2, $(p_1+p_2)^2$, $p_1\cdot p_5$, $p_1\cdot p_7$, $p_2\cdot p_5$, $p_2\cdot p_7$, $p_5\cdot p_7$, and $p_1\cdot p_3$. It was found that the expression simplifies greatly and also exhibits the symmetries of the problem more clearly if one uses the variables E, E_5, E_7, x, y, z,

TABLE I. Differential cross section for $e^+ + e^- \to W^+ + W^-$ at $E=3$ BeV, $W=2$ BeV.

k	θ (degrees)	$d\sigma/d\Omega$ (10^{-33} cm^2/sr)
2	0	2.33
	30	2.77
	60	3.65
	90	4.10
−1	0	0
	30	0.1
	60	0.307
	90	0.401

and u defined by

$$\begin{aligned}
&(p_1+p_2)^2=S^2=4E^2,\\
&p_1\cdot p_5=E(E_5-P_5\cos\theta_{15})\equiv E(E_5-y),\\
&p_1\cdot p_7=E(E_7-P_7\cos\theta_{17})\equiv E(E_7-z),\\
&p_2\cdot p_5\equiv E(E_5+y),\\
&p_2\cdot p_7\equiv E(E_7+z),\\
&p_1\cdot p_3\equiv E(E_3+x),\\
&p_5\cdot p_7=E_5(E_7-p_7\cos\theta_{57})\equiv E_5(E_7-u).
\end{aligned}\tag{2.34}$$

All the quantities except x in the above are directly measurable experimentally. As shown in (2.17), x is not an independent variable but takes two values $x^\pm$ which are expressible in terms of observable quantities.

It should be noted that if other decay modes of W's are to be considered we need to change only the expressions for $X_{\alpha\alpha'}$ and $Y_{\beta\beta'}$. The expression for $t_{\mu\nu}V_{\mu\alpha\beta}V_{\nu\alpha'\beta'}$ remains unaltered. By explicit calculation we obtain

$$\begin{aligned}
\rho_{\alpha'\beta\beta'}\equiv{}&\tfrac{1}{8}t_{\mu\nu}V_{\mu\alpha\beta}V_{\nu\alpha'\beta'}\\
={}&-g_{\alpha\beta}g_{\alpha'\beta'}[E^2(p_4-p_3)^2+4(Q\cdot p_4)^2]\\
&-2(1+k)[E^2(g_{\alpha\beta}p_{4\alpha'}p_{3\beta'}+g_{\alpha'\beta'}p_{4\alpha}p_{3\beta})\\
&+g_{\alpha\beta}(Q\cdot p_4)(p_{3\beta'}Q_{\alpha'}-p_{4\alpha'}Q_{\beta'})\\
&+g_{\alpha'\beta'}(Q\cdot p_4)(p_{3\beta}Q_\alpha-p_{4\alpha}Q_\beta)]\\
&+(1+k)^2[E^2(p_{3\beta}p_{4\alpha'}g_{\alpha\beta'}+p_{4\alpha}p_{3\beta'}g_{\alpha'\beta}\\
&-p_{3\beta}p_{3\beta'}g_{\alpha\alpha'}-p_{4\alpha}p_{4\alpha'}g_{\beta\beta'})\\
&-(p_{3\beta}Q_\alpha-p_{4\alpha}Q_\beta)(p_{3\beta'}Q_{\alpha'}-p_{4\alpha'}Q_{\beta'})].
\end{aligned}\tag{2.35}$$

The density matrix of the W pair produced is actually defined as

$$\begin{aligned}
D_{\gamma\gamma'\delta\delta'}={}&\rho_{\alpha\alpha'\beta\beta'}(p_{3\alpha}p_{3\gamma}W^{-2}-g_{\alpha\gamma})(p_{3\alpha'}p_{3\gamma'}W^{-2}-g_{\alpha'\gamma'})\\
&\times(p_{4\beta}p_{4\delta}W^{-2}-g_{\beta\delta})(p_{4\beta'}p_{4\delta'}W^{-2}-g_{\beta'\delta'}).
\end{aligned}\tag{2.36}$$

We have merely incorporated the last four factors into the definitions of X and Y to make the writing more compact. The rank-4 tensor D has the following properties:

1. It is symmetric under simultaneous exchange of two indices $\gamma\leftrightarrow\gamma'$ and $\delta\leftrightarrow\delta'$.
2. It is invariant under exchange $p_1\leftrightarrow p_2$.
3. It is symmetric under simultaneous exchange $p_3\leftrightarrow p_4$, $\delta\leftrightarrow\gamma$ and $\delta'\leftrightarrow\gamma'$.
4. It satisfies the subsidiary condition $p_{3\gamma}D_{\gamma\gamma'\delta\delta'}=0$.

III. SYMMETRIES IN THE CROSS SECTION

(a) The parity-violating effect of the weak interaction does not show up in the differential cross section. Since only P_1, P_2, P_5, and P_7 are measured experimentally, the only pseudoscalar quantity one can construct is

$$\epsilon_{\mu\nu\alpha\beta}p_{1\mu}p_{2\nu}p_{5\alpha}p_{7\beta}=2E\mathbf{P}_1\cdot(\mathbf{P}_5\times\mathbf{P}_7).\tag{3.1}$$

But this quantity is not time-reversal invariant, hence will not appear in the cross section. The absence of such a term in the cross section implies that the differential cross section for P_7 must be symmetric with respect to the P_1-P_5 plane, and the differential cross section for P_5 must be symmetric with respect to the P_1-P_7 plane.

(b) The cross section must be symmetric with respect to the plane perpendicular to the incident beam. This is the consequence of the one-photon exchange model. This must be so because of the fact that $t_{\mu\nu}$ is symmetric with respect to the interchange $p_1\leftrightarrow p_2$, and hence C must also be invariant under this exchange. The only other places where p_1 and p_2 occur are in the flux factor and the δ function, both of which are invariant under the exchange $p_1\leftrightarrow p_2$. Thus the differential cross section should not be able to tell the sense of the current of the incident beam.

(c) The differential cross section for the process

$$e^++e^-\to W^++W^-,\quad W^+\to e^++\nu_e,\quad W^-\to\mu^-+\bar\nu_\mu\tag{3.2}$$

is identical to the one we are considering [Eq. (1.1)]. This can be proved by the following steps.

1. The mass of μ inside the trace of (2.32) does not contribute.

2. The expression of matrix-element squared C for (1.1) can be written as

$$\begin{aligned}
C=\tfrac{1}{4}D_{\gamma\gamma'\delta\delta'}(p_5+p_6,\,p_7+p_8)\,&\mathrm{Tr}[\not p_6(1+\gamma_5)\gamma_\gamma\not p_5\gamma_{\gamma'}]\\
&\times\mathrm{Tr}[\not p_8(1-\gamma_5)\gamma_\delta\not p_7\gamma_{\delta'}],
\end{aligned}\tag{3.3}$$

where D (p_3,p_4) is the density matrix defined by Eq. (2.36). Since D is symmetric under $\gamma\leftrightarrow\gamma'$ and $\delta\leftrightarrow\delta'$, C is symmetric under $\gamma_5\leftrightarrow-\gamma_5$.

3. Let us denote e^+ by p_5, μ^- by p_7, ν_e by p_6, and $\bar\nu_\mu$ by p_8 for the process in (3.2).

Then the matrix element squared can be written as

$$\begin{aligned}
C'=\tfrac{1}{4}D_{\gamma\gamma'\delta\delta'}(p_7+p_8,\,p_5+p_6)\,&\mathrm{Tr}[\not p_8(1+\gamma_5)\gamma_\gamma\not p_7\gamma_{\gamma'}]\\
&\times\mathrm{Tr}[\not p_6(1-\gamma_5)\gamma_\delta\not p_5\gamma_{\delta'}].
\end{aligned}\tag{3.4}$$

Now

$$D_{\gamma\gamma'\delta\delta'}(p_7+p_8,\,p_5+p_6)=D_{\delta\delta'\gamma\gamma'}(p_5+p_6,\,p_7+p_8)$$

from the symmetry property No. 3 of D. Rearranging the dummy tensor indices and remembering the symmetry under $\gamma_5\leftrightarrow-\gamma_5$, we arrive at the desired result

$$C=C'.\tag{3.5}$$

The processes (3.2) and (1.1) are related by the charge conjugation. The theorem we have just proved combined with the invariance under $p_1 \leftrightarrow p_2$ of C shows that the charge conjugation violating effect of the weak interaction does not show up in the differential cross section. Experimentally this theorem implies that if the detectors can distinguish between e and μ but cannot distinguish the sign of their charges, one will get exactly twice the coincident counting rate we have given in this paper.[10]

(d) If $E-W \gg \mu$ then the mass of the muon can be ignored from our consideration. Under these conditions the four leptonic decay modes of W pair will all have the same differential cross sections.

IV. CROSS SECTION FOR $e^+ + e^- \to W^+ + W^-$

For completeness we give the differential cross section for this process summed over the polarization of the W's.

$$d\sigma = \frac{e^4}{(2\pi)^2} \frac{1}{32E^2} \frac{1}{(2E)^4} \int \frac{d^3P_3}{2E_3} \int \frac{d^3P_4}{2E_4} \times \delta^4(p_1+p_2-p_3-p_4) 8\rho_{\alpha\alpha'\beta\beta'}(p_{3\alpha}p_{3\alpha'}W^{-2}-g_{\alpha\alpha'}) \times (p_{4\beta}p_{4\beta'}W^{-2}-g_{\beta\beta'}). \quad (4.1)$$

From the above we obtain the differential cross section

$$\frac{d\sigma}{d\Omega_4} = \frac{\alpha^2\beta^3}{32\gamma^2W^2}\{4\gamma^4k^2\sin^2\theta + [4(1+k)^2 - 2(1+k^2)\sin^2\theta]\gamma^2 + 3\sin^2\theta\}, \quad (4.2)$$

where $\gamma = E/W$ and $\beta = (1-\gamma^{-2})^{1/2}$. Notice that this cross section has a maximum at $\theta = 90°$ and is symmetric with respect to 90°.

The total cross section is

$$\sigma = (\pi\alpha^2\beta^3/3\gamma^2W^2)[\gamma^4k^2 + (k^2+3k+1)\gamma^2 + \tfrac{3}{4}]. \quad (4.3)$$

Equation (4.2) agrees with the result obtained by Cabibbo and Gatto[4] if one lets their form factors be equal to unity, identifies their μ with our k and puts their anomalous quadrupole moment $\epsilon = 0$. The numerical examples of (4.2) and (4.3) are given in Tables I and II, respectively.

As pointed out by Cabibbo and Gatto, the expression for the total cross section (4.3) cannot possibly be right at high energies because it violates unitarity. The unitarity relation says that the sum of total cross sections of all channels from electron-positron annihilation via a single time-like photon intermediate state cannot exceed $3\pi/4E^2$, because the initial total angular momentum of the electron-positron system must be unity.

[10] Y. S. Tsai, Stanford Linear Accelerator Center Report No. SLAC-PUB-117, 1965 (unpublished), to appear in Proceedings of International Symposium on Electron and Photon Interactions at High Energies, Hamburg, Germany, 1965 (to be published). If radiative corrections are included in the reaction $e^+ + e^- \to \gamma \to A^+ + B^-$ in the center-of-mass system, there will be more B^- coming out along the direction e^- than A^+. This phenomenon is very similar to the difference between e^+p and e^-p scatterings where e^+p in general has a larger cross section at a fixed angle than e^-p if higher order terms are included.

TABLE II. Total cross section for $e^+ + e^- \to W^+ + W^-$.

E (BeV)	W (BeV)	k	σ (10^{-32} cm^2)
3	2	2	4.41
4	2	2	9.24
10	2	2	54.7
100	2	2	5240.0
3	2	−1	0.343
4	2	−1	1.08
10	2	−1	11.8
100	2	−1	1310.0
3	2	0	0.289
3	2.2	0	0.191
3	2.4	0	0.116
3	2.6	0	0.060
3	2.8	0	0.020

The cross section (4.3) increases with energy as γ^2 at high energies if $k \neq 0$ and stays constant if $k=0$ in the asymptotic limit. The cross section reaches its unitarity limit at an energy equal to

$$E = (\tfrac{3}{2})^{1/2}(137/k)^{1/2}W \quad \text{if} \quad k \neq 0,$$

and

$$E = \tfrac{3}{2}W \times 137 \quad \text{if} \quad k = 0.$$

The energies at which these limits are reached are considerably higher than those of the various colliding beam machines proposed. Nevertheless, it is still a serious defect of the theory. It is not immediately obvious that by considering the higher order electromagnetic effects this difficulty can be circumvented.[6]

V. NUMERICAL EXAMPLES OF THE DIFFERENTIAL CROSS SECTION

$$e^+ + e^- \to e^- + \bar{\nu}_e + \mu^+ + \nu_\mu$$

In order to facilitate the design of the experiment, it is useful to know approximately how the electrons and muons are distributed and what their energy and angular correlations are. We were told by David Ritson that a spark chamber with nearly 4π solid angle can be used, and that the muon energy can be measured with a high accuracy from its range and the electron energy can be measured from its shower production. We have integrated the expression (2.19) with respect to the energies of the muon and electron, and have obtained $d\sigma/d\Omega_5 d\Omega_7$ numerically by a computer.

$$\frac{d\sigma}{d\Omega_5 d\Omega_7} = \frac{9r_0^2m^2R^2}{(2\pi)^2 512E^7W^4P_4} \int_{(E_7)_{\min}}^{(E_7)_{\max}} dE_7 \int_{(E_5)_{\min}}^{(E_5)_{\max}} dE_5 \times \frac{C_+ + C_-}{[\cos(\theta_{47}+\theta_{35}) + 2\cos\theta_{35}\cos\theta_{47}\cos\theta_{57} + \cos^2\theta_{57}]^{1/2}}.$$

The limits of integrations are:

$$(E_7)_{\max,\min} = (E \pm P_4)/2 + \mu^2(E \mp P_4)/2W^2,$$

$$(E_5)_{\max,\min} = W^2/2[E - P_4(\cos\theta_{35})_{\max,\min}],$$

where $(\cos\theta_{35})_{\max,\min} = -\cos\theta_{47}\cos\theta_{57} \pm \sin\theta_{47}\sin\theta_{57}$; the upper sign goes with "max" and the lower with "min."

TABLE III. Differential cross section for $e^+ + e^- \rightarrow \mu^+ + e^- + \nu_\mu + \bar{\nu}_e$ at $E=3$ BeV with $W=2$ BeV, $R=0.25$, and $k=-2$, 0, 2.

θ_{15}	φ_7	θ_{57}	$d^2\sigma/d\Omega_5 d\Omega_7$ in 10^{-34} cm^2/sr^2 $k=-2$	$k=0$	$k=2$
30	0	30	0.01289	0.0009347	0.02537
		60	0.03293	0.001503	0.05245
		90	0.07312	0.002514	0.1166
		120	0.1180	0.01035	0.2412
		150	0.1286	0.03207	0.4519
30	30	30	0.01289	0.001017	0.02524
		60	0.03244	0.001658	0.05153
		90	0.07099	0.002940	0.1152
		120	0.1138	0.01107	0.2394
		150	0.1262	0.03294	0.4606
30	60	30	0.01277	0.001194	0.02440
		60	0.03074	0.002013	0.04632
		90	0.06501	0.004616	0.1084
		120	0.1028	0.01295	0.2338
		150	0.1185	0.03576	0.4632
30	90	30	0.01251	0.001380	0.02331
		60	0.02755	0.002537	0.04206
		90	0.05438	0.005730	0.1000
		120	0.08622	0.01503	0.2293
		150	0.1119	0.03796	0.4722
30	120	30	0.01215	0.001542	0.02227
		60	0.02335	0.002919	0.03757
		90	0.04188	0.006471	0.08857
		120	0.06914	0.01626	0.2083
		150	0.1049	0.03947	0.4715
30	150	30	0.01187	0.001644	0.02125
		60	0.01967	0.002867	0.03492
		90	0.03194	0.006852	0.07688
		120	0.05485	0.01682	0.1968
		150	0.1003	0.04014	0.4688
30	180	30	0.01164	0.001678	0.02099
		60	0.01813	0.002931	0.03260
		90	0.02790	0.006914	0.07121
		120	0.04807	0.01694	0.1907
		150	0.09963	0.04018	0.4681
60	0	30	0.01165	0.001510	0.02018
		60	0.02765	0.002040	0.04634
		90	0.07355	0.002229	0.1183
		120	0.1401	0.008009	0.2642
		150	0.1545	0.02390	0.4502
60	30	30	0.01035	0.001583	0.02051
		60	0.02819	0.002292	0.04713
		90	0.07326	0.002870	0.1169
		120	0.1365	0.009516	0.2645
		150	0.1505	0.02596	0.4529
60	60	30	0.01039	0.001773	0.02039
		60	0.02886	0.002866	0.04514
		90	0.07039	0.005507	0.1177
		120	0.1258	0.01276	0.2630
		150	0.1442	0.03009	0.4517
60	90	30	0.01033	0.002008	0.01937
		60	0.02669	0.003421	0.04165
		90	0.06162	0.006828	0.1096
		120	0.1095	0.01535	0.2549
		150	0.1337	0.03290	0.4607
60	120	30	0.01008	0.002141	0.01778
		60	0.02133	0.003256	0.03783
		90	0.04725	0.007305	0.09436
		120	0.09086	0.01590	0.2378
		150	0.1295	0.03371	0.4574
60	150	30	0.009372	0.002184	0.01654
		60	0.01565	0.003705	0.03101
		90	0.03325	0.007076	0.08047
		120	0.07827	0.01524	0.2141
		150	0.1255	0.03274	0.4506
60	180	30	0.009078	0.002188	0.01567
		60	0.01027	0.003630	0.02756
		90	0.02720	0.006905	0.07099
		120	0.07256	0.01474	0.2100
		150	0.1265	0.03219	0.4528
90	0	30	0.008996	0.002078	0.01535
		60	0.01791	0.003087	0.03372
		90	0.05041	0.005153	0.09572
		120	0.1166	0.01024	0.2477
		150	0.1570	0.02387	0.4409
90	30	30	0.009320	0.002137	0.01575
		60	0.01993	0.003274	0.03593
		90	0.05411	0.005709	0.1014
		120	0.1173	0.01162	0.2507
		150	0.1535	0.02571	0.4442
90	60	30	0.009833	0.002259	0.01681
		60	0.02390	0.003643	0.03949
		90	0.06151	0.006789	0.1113
		120	0.1207	0.01421	0.2617
		150	0.1476	0.02904	0.4513
90	90	30	0.009237	0.002318	0.01748
		60	0.02625	0.003311	0.04094
		90	0.06544	0.007211	0.1154
		120	0.1220	0.01533	0.2663
		150	0.1435	0.03125	0.4549

The result of the computation is shown in Table III. The unit of the cross section is 10^{-34} cm^2/sr^2.

We make the following comments and observations on Table III.

(a) Because of the symmetry with respect to $\varphi_7 \leftrightarrow -\varphi_7$, we computed the cross section only from $\varphi_7=0$ to π. This symmetry is due to the time-reversal invariance as discussed in Sec. III(a).

(b) The cross section is symmetric with respect to a simultaneous exchange:

$$\theta_{15} \leftrightarrow \pi - \theta_{15},$$

$$\varphi_7 \leftrightarrow \pi - \varphi_7.$$

This is due to the symmetry with respect to the interchange $P_1 \leftrightarrow P_2$ as discussed in Sec. III(b). Because of this symmetry we took θ_{15} from 0 to $\frac{1}{2}\pi$.

(c) The values of the differential cross section at $\theta_{57}=0°$ and 180° were not given in Table III, because of the limits of the E_5 integration pinch [i.e., $(E_5)_{\max} = (E_5)_{\min}$] and at the same time the denominator of the integral vanishes at these two points. However, by taking the limit, the integrals at these two points give finite numbers as shown in Table IV. In general the cross section increases rapidly with θ_{57} from 0° to 180°. The rate of increase depends critically upon k. For $k=-2$, the ratio of the cross section at $\theta_{57}=30°$ to $\theta_{57}=150°$ is approximately 1/10 or 1/15 depending upon whether $\theta_{15}=30°$ or $\theta_{15}=90°$; for $k=0$ the correspond-

TABLE IV. An example of the behavior of the differential cross section near $\theta_{57}=0$ and 180°, for $E=3$ BeV, $W=2$ BeV, $R=0.25$, $k=-2$, $\theta_{15}=30°$, $\varphi_7=30°$.

θ_{57} (degrees)	$d\sigma/d\Omega_5 d\Omega_7$ (10^{-34} cm²/sr²)
1	0.009584
5	0.009852
30	0.01289
90	0.07312
150	0.1286
170	0.1392
179	0.1422

ing ratio is 1/33 or 1/13; and for $k=2$ the corresponding ratio is 1/18 or 1/28. In the absence of dynamical correlations all these ratios should be identical for all k. Thus we conclude that the effect of dynamical correlations is strong and should be utilized advantageously to determine k (and the anomalous quadrupole moment if it is there).

VI. DISCUSSION

(a) All of our considerations will be only of academic interest if there is no W meson, or if its mass is so large that it cannot be produced in the foreseeable future. However we believe various considerations made in this paper can be applied to many other similar problems which involve creation of unstable particles by $e^+ + e^-$ collisions. For example

$$e^- + e^+ \to \Lambda + \bar{\Lambda}, \quad \Lambda \to \pi^- + p, \quad \bar{\Lambda} \to \pi^+ + \bar{p}.$$

This reaction gives the electric and magnetic form factors of Λ for a time like momentum transfer.

(b) We have completely ignored the fact that some extra photons are always emitted either from initial or final charged particles (the so-called radiative corrections). If a photon is emitted from the initial system, the virtual photon in our problem will no longer be a pure time-like vector $(2E,0)$, but will acquire a certain energy and momentum distribution. As a result the kinematical correlations we have discussed will no longer have a sharp edge at the boundary, but will be smeared by some radiative tail. In general the radiative tail smears the particle energy on the low energy side. Thus it will change, for example, $(E_7)_{\rm min}$ to a lower value but will not affect $(E_7)_{\rm max}$ in the numerical example given in Sec. 2. Since $(E_7)_{\rm max}$ depends very critically upon W for fixed E_5 and θ_{57}, we conclude that the mass determination via kinematical correlation will not be affected by the radiative corrections. If the radiative corrections are included then the symmetry under $P_1 \leftrightarrow P_2$ will also be violated by a few percent.[10]

(c) The major background to the process considered is expected to be due to the accidental coincidence from two reactions

$$e^+ + e^- \to e^+ + e^-$$

and

$$e^+ + e^- \to \mu^+ + \mu^- .$$

Neglecting the radiative corrections and possibilities of form factors, their cross section can be written, respectively,[11] as

$$\frac{d\sigma}{d\Omega(e-)} = \frac{r_0^2}{8}\frac{m^2}{E^2}\left[\frac{1+\cos^4(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)} - \frac{2\cos^4(\frac{1}{2}\theta)}{\sin^2(\frac{1}{2}\theta)} + \frac{1+\cos^2\theta}{2}\right], \quad (6.1)$$

$$\frac{d\sigma}{d\Omega(\mu+)} = \frac{r_0^2}{8}\frac{m^2}{E^2}\left(1-\frac{m^2}{E^2}\right)^{1/2}\left[\frac{1+\cos^2\theta}{2} + \frac{m^2}{2E^2}\sin^2\theta\right]. \quad (6.2)$$

At $\theta=90°$ and $E=3$ BeV, we have

$$d\sigma/d\Omega(e-) = 12.5\times 10^{-34}\ \mathrm{cm^2/sr}$$

and

$$d\sigma/d\Omega(\mu+) = 1.4\times 10^{-34}\ \mathrm{cm^2/sr}.$$

Compare these with the result of our Table III at $\theta_{15}=90°$, $\theta_{57}=150°$, $\varphi_7=90°$, with $k=-2$ and $W=2$:

$$d\sigma/d\Omega_5 d\Omega_7 = 0.1435\times 10^{-34}\ \mathrm{cm^2/sr^2}.$$

The accidental coincidence is proportional to the product of (6.1) and (6.2) if one detects e^- and μ^+ or e^+ and μ^- and therefore it is completely negligible. However, if W really exists, then one would expect the (e^+e^-), $(\mu^+\mu^-)$, $(e^-\mu^+)$ and $(e^+\mu^-)$ decay modes of the W pair to have almost identical probability. Turning the argument around, the near identity of all these four decay modes will serve as an additional proof that W's were actually produced. The radiative corrections to processess (6.1) and (6.2) will then be the major background for the (e^+e^-) and $(\mu^+\mu^-)$ decay modes, respectively, of the W pair. The main effects of radiative corrections to processes (6.1) and (6.2) are: (1) the final particles will no longer all come out exactly back-to-back, and (2) their energies will be smeared. These effects are all rather easy to calculate[10] and in general the cross sections drop down very quickly as one deviates from the elastic kinematics. Thus in principle there is no major difficulty in distinguishing the processes (6.1) and (6.2) from the (e^+e^-) and $(\mu^+\mu^-)$ decay modes of the W pairs.

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[11] Y. S. Tsai, Phys. Rev. **120**, 269 (1960), Eqs. (57) and (58).