Validity of the ξ Approximation for the $\frac{7}{2}^-(\beta)\frac{7}{2}^+$ Decay in ${}_{58}Ce^{141}$

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The energy dependence of the beta-gamma directional correlation was measured for the $\frac{7}{2}^-(\beta)\frac{7}{2}^+(\gamma)\frac{5}{2}^+$ beta-gamma cascade in the decay of ${}_{58}Ce^{141}$ with a slow-fast scintillation assembly in the beta energy region 165.5–422.5 keV. Results from the integral correlation showed that the correlation coefficient ϵ_4 was zero within experimental errors, while those from the differential correlation experiment showed that the coefficient ϵ_2 was small throughout the energy range and the results were in accordance with the use of the ξ approximation to explain the inner beta transition of Ce^{141}.

INTRODUCTION

THE decay scheme of Ce^{141} is well known[1] and is shown in Fig. 1. 70% of Ce^{141} de-excites to Pr^{141} at 145-keV excitation, which subsequently returns to the ground state through the emission of the 145-keV gamma component, while 30% of beta emission proceeds directly to the ground state of Pr^{141}. The beta-gamma cascade, which is of present interest, follows the spin sequence $\frac{7}{2}^-(\beta)\frac{7}{2}^+(\gamma)\frac{5}{2}^+$. This inner beta group has an end-point energy of 447 keV or $1.87mc^2$ and has an allowed spectral shape, with a log ft value of 6.9. The ξ value[2] $\alpha Z/2\rho$ for Ce^{141} is 12.8, which is relatively larger than the end-point energy $1.87mc^2$. Thus here is a case with characteristics which fit into the ξ approximation.[2–4] The ξ approximation is explained by expanding the beta transition probability in powers of $\mathbf{p}\cdot\mathbf{r}$ and $\mathbf{q}\cdot\mathbf{r}$, where $\mathbf{p}$ and $\mathbf{q}$ are the electron and the neutrino momenta, respectively, and $\mathbf{r}$ is the radius vector. The various terms of this expansion can be grouped so that the orders of magnitude form a series in descending powers of the parameter ξ. If Z, the atomic number, is greater than 20, and if W_0, the end-point energy of the beta transition, is small enough that $\xi \gg W_0-1$, then the terms of highest power in ξ predominate over all other terms. Retaining only the highest term leads to the so-called ξ approximation. In the present case the 447-keV beta transition is involved in a change in parity with zero spin difference, is contributed by matrix elements of ranks $\lambda=0$, $\lambda=1$, and $\lambda=2$, and is of nonunique first-forbidden type. Validity of the ξ approximation requires that all the first-forbidden experimentally measurable quantities must have the same energy and angle dependence as the allowed quantities. Hence, a measurement of the energy dependence of the directional correlation of this cascade forms a critical test for the validity of the ξ approximation. If the ξ approximation holds good, the anisotropy is expected to be of small magnitude of the order of $1/\xi$. Previous data on beta-gamma directional-correlation measurements in Ce^{141} seem to have been available only from the work of Deutsch *et al.*[5] In the present work, a systematic measurement of the energy dependence of the beta-gamma directional correlation is made with a slow-fast scintillation assembly possessing a good coincidence resolution (about 20 nsec), and the results are discussed on the basis of the ξ approximation formalism.

EXPERIMENTAL DETAILS

The experimental setup used in the present work is a conventional slow-fast scintillation assembly, the block diagram of which is shown in Fig. 2. The fast coincidence resolution is determined to be 20 nsec. The gamma channel comprises a $1\frac{1}{2}$-in.×1-in. size NaI(Tl) crystal, optically coupled to an RCA 6810-A photomultiplier. A conical lead shield as shown in Fig. 3 houses the gamma crystal for the collimation of the gamma radiation. The gamma-energy calibration is accomplished with the standard sources Ba^{133} (81 and 360 keV), Co^{57} (137 keV), Hg^{203} (280 keV), Na^{22} (511 keV), and Cs^{137} (661 keV). The beta detection is accomplished with a plastic scintillator with a conical well cut in it to reduce the low-energy back-scattering

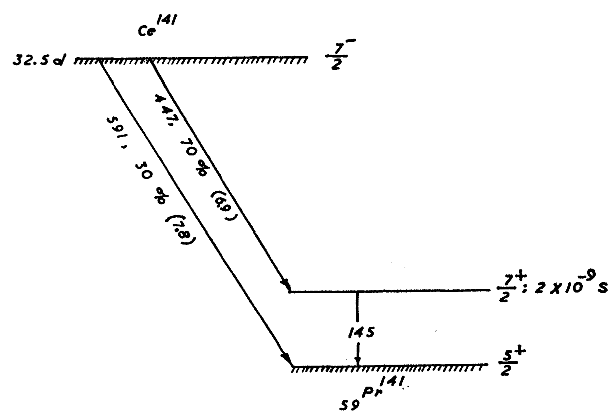

FIG. 1. Decay scheme of Ce^{141}.

[1] B. S. Dzhelepov and L. K. Perker, *Decay Schemes of Radioactive Nuclei* (Pergamon Press, Inc., New York, 1961).
[2] E. J. Konopinski and G. F. Uhlenbeck, Phys. Rev. **60**, 308 (1941).
[3] T. Kotani and M. Ross, Phys. Rev. Letters **1**, 140 (1958); *ibid.* **113**, 622 (1959).
[4] Hans A. Weidenmüller, Rev. Mod. Phys. **33**, 574 (1961).
[5] J. P. Deutsch, L. Grenacs, and P. Lipknik, J. Phys. Radium **22**, 662 (1961).

effects and eliminate the low-energy back-scattered tail which will generally accompany the beta spectra. The beta head assembly and the source are enclosed in an aluminum vacuum chamber of thickness $\frac{1}{32}$-in. A constant pressure of 5 mm Hg was maintained throughout the course of the experiment. The geometry of the two detectors is shown in Fig. 3. The source is situated at the apex of the beta crystal to focus the electrons. The effective thickness of the beta crystal is $\frac{1}{8}$-in., while the effective solid angle is 2% of 4π. The beta spectrometer is calibrated with the 195- and 624-keV conversion lines in the decay of Hg^{203} and Cs^{137} in coincidence with 85- and 37-keV x rays, respectively. The angular-correlation table is adjusted to be perfectly horizontal and the two detectors are mounted on it.

SOURCE

The source is obtained from the Atomic Energy Establishment, Government of India in the form of liquid cerium chloride. The source is prepared by slowly evaporating to dryness a small drop of Ce^{141} solution on a Mylar film of thickness 0.6 mg/cm^2, over an area of about 3 mm diameter. A drop of insulin aided uniform spreading of the source. The source film is glued to a very thin aluminum ring of 1-in. diameter and 0.3-mm thickness. The source is centered properly so as to ensure that the variation of the counting rate in the movable gamma detector lies below 1%.

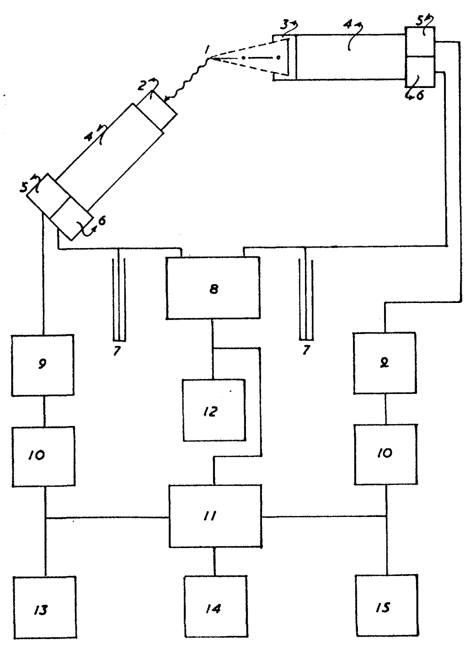

FIG. 2. Block diagram of slow-fast system: 1–source; 2–NaI(Tl) crystal; 3–plastic scintillator; 4–6810-A photomultipliers; 5–cathode followers; 6–limiters; 7–clippers; 8–fast-coincidence circuit; 9–linear amplifiers; 10–single-channel analyzers; 11–triple-coincidence unit; 12, 13, 14, and 15–scalers.

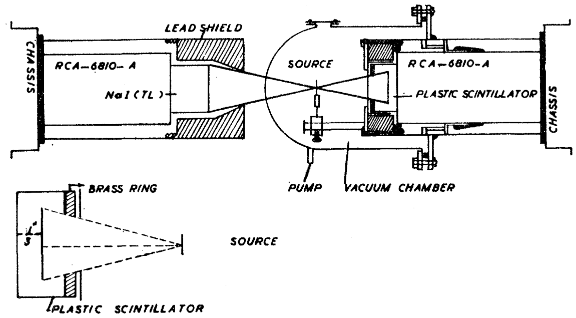

FIG. 3. Experimental arrangement and the vacuum chamber.

EXPERIMENTAL PROCEDURE AND COLLECTION OF DATA

The correlation function $W_{\beta\gamma}(\theta)$ can be expressed in the form[6]

$$W_{\beta\gamma}(\theta)=1+\epsilon_2P_2(\cos\theta)+\epsilon_4P_4(\cos\theta)\,,$$

where ϵ_2 and ϵ_4 are correlation coefficients. In the case of first-forbidden beta transitions ϵ_4 vanishes, as can be established by the integral correlation experiment. Then $W_{\beta\gamma}(\theta)$ becomes equal to $1+\epsilon P_2(\cos\theta)$, where we write ϵ for ϵ_2. Hence, from the differential correlation experiment, ϵ as a function of energy can be measured by recording the beta-gamma coincidence rates at two angles 180° and 90° (including 270°).

The anisotropy A is given by $A=(W_{180}-W_{90})/W_{90}$, where W_{180} and W_{90} are the true beta-gamma coincidence rates at 180° and 90°, respectively. To evaluate the directional correlation coefficients, the data have been treated along the following lines.

(1) Rose's method of least squares[7] has been adopted to obtain the correlation coefficients from the measured coincidence rates at various angles in the integral correlation experiment. However, the differential correlation coefficient ϵ could be calculated directly from the measured coincidence rates, inasmuch as the coincidence rates are required to be recorded only at two angles, namely 180° and 90° (including 270°). In this case ϵ is given by $\epsilon=2A/(A+3)$.

(2) In the case of integral correlation, the errors are obtained directly from the Rose formula. In the case of differential correlation experiment, the errors are estimated from the statistical errors involved in the coincidence rates at 180° and 90°.

(3) The experimentally obtained correlation coefficients are corrected for the finite-solid-angle effects[7-9] of both the detectors.

The intrinsic asymmetry of the system is measured

[6] H. Frauenfelder, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. 19, p. 531.
[7] M. E. Rose, Phys. Rev. **91**, 610 (1953).
[8] S. Frankel, Phys. Rev. **83**, 673 (1951).
[9] E. Breitenberger, Proc. Phys. Soc. (London) **A66**, 846 (1953).

by undertaking gamma-gamma and beta-gamma correlation experiments with Co^{60} and found to be insignificant and within experimental errors.

In the 447-keV-beta–145-keV-gamma cascade in the decay of Ce^{141}, the 145-keV gamma photopeak is accepted in a window of 14.5 keV in integral and differential correlation experiments. For the integral beta-gamma correlation measurements, the beta particles of energy 100 keV and above are accepted in the single-channel analyzer. The experiment is conducted in the angular range 90° to 270° in steps of $22\frac{1}{2}$°. In the differential correlation measurements, beta particles in the energy region 150 to 430 keV are selected in steps of 65 keV (channel width). Coincidences are recorded in this experiment only at angles 90°, 180°, and 270°. About 40 000 coincidences are collected in several runs at each energy and at each angle. The ratio of true to chance counts lies in the range 8–12 for the energy region investigated (the true-to-chance ratio is less at the ends of the energy spectrum). The true coincidences are obtained after subtracting the chance and gamma-gamma background coincidences. The gamma singles rate is used to normalize the coincidence rates, which accounts for the slight differences in the singles count rates of the movable counter at different positions. The anisotropy is calculated from the formula

$$A=\frac{N_c(180)N_\gamma(90)}{N_c(90)N_\gamma(180)}-1,$$

N_c and N_γ representing the coincidence and singles gamma rates. $N_c(90)$ stands for the average of the beta-gamma coincidence rates at 90° and 270°; $N_c(180)$ stands for the coincidence rate at 180°. The expression

$$[N_c(90)N_\gamma(270)]/[N_\gamma(90)N_c(270)]$$

is found to be unity within statistics.

The outer beta group of Ce^{141} representing the ground-state transition to Pr^{141} with an end-point energy of 591 keV has no influence in coincidence experiments of present kind.

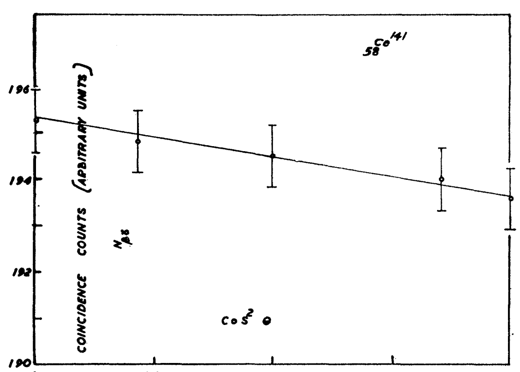

FIG. 4. Integral correlation: $N_{\beta\gamma}$, the true beta-gamma integral coincidences, are plotted against $\cos^2\theta$.

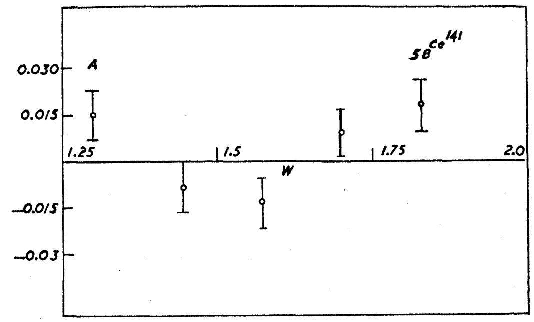

FIG. 5. Anisotropy A is plotted against energy W in mc^2 units.

RESULTS

Integral Correlation

In Fig. 4 the correlation function $W_{\beta\gamma}(\theta)$ is plotted as $N_{\beta\gamma}(\theta)$ against $\cos^2\theta$. The straight line represents the least-squares fit to the data[7] in the form

$$W_{\beta\gamma}(\theta)=1+\epsilon_2P_2(\cos\theta)+\epsilon_4P_4(\cos\theta).$$

The values obtained are

$$\epsilon_2=-0.006\pm0.004,$$
$$\epsilon_4=-0.001\pm0.006.$$

The coefficient of $P_4(\cos\theta)$ is zero within experimental errors, while that of $P_2(\cos\theta)$ is of small magnitude.

Differential Correlation

The final results of the differential-correlation experiment are summarized in Table I. A plot of A versus energy is shown in Fig. 5, with vertical flags representing the statistical errors. The errors involved are of the order of 50% or more. Because of the large uncertainties no definite conclusion could be drawn about the existence of any anisotropy.

However, a small anisotropy may exist in accordance with the ξ approximation. Deutsch *et al.*[5] concluded that the value of A was zero, but no numerical values were given. The values of the correlation coefficient ϵ and reduced correlation coefficient ϵ' defined as $\epsilon'=\epsilon W/p^2$ are also given in the table (the correlation coefficients are corrected for the finite-solid-angle effects). Here W is the energy of the beta particles

TABLE I. Directional correlation data of 447–145-keV beta-gamma cascade in the decay of Ce^{141}.

Energy (keV)	W (mc^2)	Anisotropy A	Correlation coefficient ϵ	Reduced correlation coefficient $\epsilon'=\epsilon W/p^2$
162.5	1.318	0.015±0.008	0.01 ±0.005	0.018±0.009
227.5	1.445	−0.009±0.008	−0.006±0.005	−0.008±0.007
292.5	1.572	−0.014±0.008	−0.009±0.005	−0.009±0.006
357.5	1.699	0.009±0.008	0.006±0.005	0.009±0.005
422.5	1.827	0.018±0.009	0.011±0.005	0.009±0.004

expressed in mc^2 units and p is the momentum of the electron, given by $p^2=W^2-1$. The correlation coefficient and the reduced correlation coefficient are shown in Figs. 6 and 7, respectively, as functions of energy. It is to be noted that no resolution correction has been applied for the finite resolution of the beta spectrometer, as the resolution correction is expected to fall within the experimental errors.

It may also further be noted that the attenuation of the angular correlation due to the finite lifetime ($\sim 10^{-9}$ sec) of the 145-keV level in Pr^{141} is not expected to be of such a magnitude as to enhance the anisotropy by a sizable amount, so that the ξ approximation fails.

The magnitudes of the anisotropy values for the $\frac{7}{2}^-(\beta)\frac{7}{2}^+(\gamma)\frac{5}{2}^+$ cascade from the present work are small, and this fact is consistent with the ξ approximation. However, the energy dependence of the angular correlation, resulting in the occurrence of a dip in the low-energy side of the angular-correlation function, shows

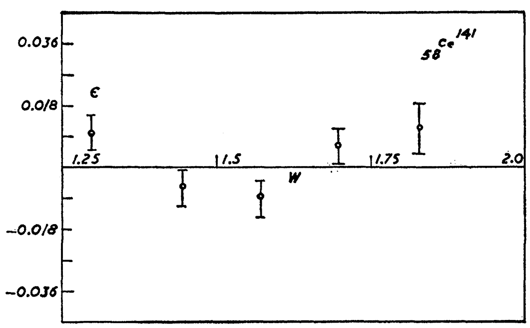

FIG. 6. Correlation coefficient ϵ is plotted against energy W.

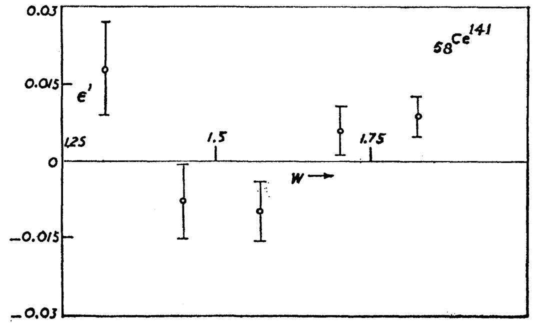

FIG. 7. A plot between reduced correlation coefficient $\epsilon'=\epsilon W/p^2$ and energy W. It is seen that the graph is almost parallel to the energy axis.

a deviation from the ξ approximation if the errors in the measurement of $\epsilon(W)$, though large, are taken seriously. But other factors like the beta spectral shape of the $\frac{7}{2}^-(\beta)\frac{7}{2}^+$ transition, the $\log ft$ value and the ξ value as well as the anisotropy are in accordance with the ξ approximation. It may be noted that the errors involved in the measurement of $\epsilon(W)$ range from $\sim 50\%$ to $\sim 100\%$, which leaves some doubt as to whether we can draw any definite conclusion about the apparent deviation of the energy dependence of angular correlation from the ξ approximation.

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