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Strong and Electromagnetic Decay of Mesons*

LAI-HIM CHAN†
Harvard University, Cambridge, Massachusetts
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Partial widths of strong and electromagnetic decays of mesons are calculated within the framework of Schwinger's field theory of matter. Results are consistent with present experimental data. The model requires a two-photon–pseudoscalar-meson interaction and a direct four-pseudoscalar-meson interaction. The former is shown to be consistent with the $\pi\to\gamma e^+e^-$ form factor. The latter explains the $\delta\to\eta\pi\pi$ and $\eta\to\pi^+\pi^\circ\pi^-$ decays, and the coupling constant also agrees with the $\pi\pi$ S-wave coupling constant. Some justification in terms of the heavy scalar-meson nonet is given for the four-meson interaction.

SINCE decay rates and branching ratios comprise most of the accessible experimental data, they have been subjected to intensive study. In particular, they serve as a check for various unitary-symmetry models. In this paper, we report some calculations using Schwinger's field theory of matter.[1] With the introduction of some new interactions, we obtain strong and electromagnetic decay rates for all mesons, which are consistent with experiment. In particular, we are able to describe the $\eta\to 3\pi$ decay modes, their branching ratios relative to other modes, and the π^0 energy spectrum. There seems to be no need for the assumption of the existence of a low-mass scalar meson[2] to enhance, or a new quantum number[3] to suppress, certain decay modes.

In Schwinger's field theory of matter, mesons are constructed from the fundamental field ψ. In matrix form, the vector mesons and the pseudoscalar mesons are, respectively,

$$U^\mu{}_{ab}=C_1\bar{\psi}_a\gamma^\mu\psi_b=\begin{bmatrix}-2^{-1/2}\rho^0+2^{-1/2}\omega & \rho^+ & K^{*+}\\ -\rho^- & 2^{-1/2}\rho^0+2^{-1/2}\omega & K^{*0}\\ K^{*-} & \bar{K}^{*0} & \phi\end{bmatrix}, \quad (1)$$

$$\Phi_{ab}=C_2\bar{\psi}_a\gamma_5\psi_b=\begin{bmatrix}-2^{-1/2}\pi^0+\frac{1}{2}\eta+\frac{1}{2}\delta & \pi^+ & K^+\\ -\pi^- & 2^{-1/2}\pi^0+\frac{1}{2}\eta+\frac{1}{2}\delta & K^0\\ K^- & \bar{K}^0 & -2^{1/2}\eta+2^{-1/2}\delta\end{bmatrix}, \quad (2)$$

in which δ is identified as the 960-MeV meson and C_1 and C_2 are constants with dimension of inverse mass squared. A more precise version is obtained by replacing $2^{-1/2}\omega$ by $0.705\omega+0.058\phi$ wherever ω appears.

The couplings between these two fields are given by

$$\mathcal{L}_{U\Phi^2}=g_{U\Phi^2}\,\mathrm{Tr}U^\lambda(1/2i)(\Phi\Phi_\lambda-\Phi_\lambda\Phi) \quad (3)$$

and

$$\mathcal{L}_{\Phi U^2}=g_{\Phi U^2}\,\mathrm{Tr}[\Phi\tfrac{1}{4}\epsilon_{\mu\nu\lambda K}U^{\mu\nu}U^{\lambda K}]. \quad (4)$$

The coupling constants are estimated to be $(g_{U\Phi^2})^2/4\pi=4.6\pm0.5$ and $(g_{\Phi U^2}m_\omega)^2/4\pi=5.9\pm1.7$.[1] The relevant terms in the following calculation are

$$\mathcal{L}_{U\Phi^2}=g_{U\Phi^2}[\phi\bar{K}K+2^{-1/2}\rho(\bar{\pi}t\pi)+2^{-1/2}\pi\bar{K}tK^*+2^{1/2}\bar{K}^*tK\pi] \quad (5)$$

and

$$\mathcal{L}_{\Phi U^2}=g_{\Phi U^2}[\tfrac{1}{2}\delta(\omega\omega+\bar{\rho}\rho)+\tfrac{1}{2}\eta(\omega\omega+\bar{\rho}\rho)+2^{1/2}\bar{\pi}\rho\omega+2^{-1/2}\omega\bar{K}^*K+2^{-1/2}\bar{K}K^*\omega-2^{1/2}\rho\bar{K}^*tK-2^{1/2}\bar{K}tK^*\rho]. \quad (6)$$

All strong decay properties can be calculated from (5) and (6).

The electromagnetic field interacts with the charge current of the fundamental field in the form $A_\mu[\bar{\psi}_1\gamma^\mu\psi_1+i(\bar{V}_1{}^{\mu\nu}V_{\nu1}-\bar{V}_{\nu1}V_1{}^{\mu\nu})]$, ψ_1 and V_1 being the charged components of ψ and V. If $\bar{\psi}_1\gamma^\mu\psi_1$ or $i(\bar{V}_1{}^{\mu\nu}V_{\nu1}-\bar{V}_{\nu1}\bar{V}_1{}^{\mu\nu})$ (or the combination) is identified as $U_{11}{}^\mu$, then the interaction becomes

$$\mathcal{L}_A=\lambda A_\mu 2^{-1/2}(\omega^\mu-\rho^{0\mu}). \quad (7)$$

The electromagnetic interaction of the strongly interacting particles can be viewed as a two-step process: a strong interaction with ρ^0 or ω following by ρ^0 or ω going into a photon. The ρ^0 and ω intermediate states serve as a crude model for form factors. The value of λ is fixed by the requirement that a real photon

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[3] J. B. Bronzan and F. E. Low, Phys. Rev. Letters 12, 522 (1964).

coupled to a conserved current through (7) should reduce to a simple electromagnetic interaction. Hence

$$(1/4\pi)[(g_{U\Phi^2}\lambda)/(2m_\rho^2)]^2=e^2/4\pi=\alpha. \quad (8)$$

With (6), (7), and (8), a number of radiative decays can be estimated; for example, $\omega\to\pi+\gamma$ with $\omega\to\rho^0+\pi$ and $\rho^0\to\gamma$, or $\delta\to\pi^++\pi^-+\gamma$ with $\delta\to\rho^0+\rho^0$ and $\rho^0\to\gamma$, $\rho^0\to\pi^++\pi^-$.

While the identification of the particles is unique in the first-order electromagnetic interaction, the identification may not be unique in second order. This suggests the additional interaction

$$\begin{aligned}\mathcal{L}_A'&=(k/m_\pi)\Phi_{11}\epsilon_{\mu\nu\lambda K}F^{\mu\nu}F^{\lambda K}\\&=(k/m_\pi)(-2^{-1/2}\pi^0+\tfrac{1}{2}\eta+\tfrac{1}{2}\delta)\tfrac{1}{4}\epsilon_{\mu\nu\lambda K}F^{\mu\nu}F^{\lambda K} \quad (9)\end{aligned}$$

which contributes only to the two-phonon process. Using the recently measured mean lifetime of π^0, 1.05×10^{-16} sec,[4] we obtain $k=-1.5\alpha$ with α being the fine-structure constant. k is negative in order to give the correct cancellation to the contribution from the process $\pi\to\omega+\rho$ and $\rho\to\gamma$, $\omega\to\gamma$ which alone would predict too small a lifetime for π^0. The same cancellation occurs for $\eta\to2\gamma$ and $\delta\to2\gamma$. The interaction (9) can be considered as a high-mass contribution to the electromagnetic form factor for the two-photon process in addition to that given by ρ^0 and ω.[5] Some experimental data for the form factor is available through the process $\pi^0\to\gamma+e^++e^-$. For small q^2, the form factor can be approximated by

$$G(q^2)=G(0)(1+aq^2/m_\pi^2), \quad (10)$$

where $q^2=(p_++p_-)^2$ is the invariant energy of the Dalitz pair (e^+,e^-). Since the range of q^2 is small and the spectrum is highly peaked at low q^2, the determination of a is very crude but indicates[6,7]

$$a=-0.24\pm0.16, \quad a=-0.30\pm0.2.$$

Our model gives $a=-0.14$ in agreement with these measurements.

Recently, a 960-MeV $TJ^{GP}=00^{-+}$ meson has been observed[8,9] and identified as a δ meson by Schwinger.[1] Experiments show that δ has two major decay modes $\delta\to\eta+\pi+\pi$ and $\delta\to\pi^++\pi^-+\gamma$ with branching ratio[8]

$$\frac{\Gamma(\delta\to\pi^++\pi^-+\gamma)}{\Gamma(\delta\to\pi^++\pi^-+\eta)}=0.25\pm0.14.$$

There is some evidence that $\delta\to\eta+\pi^++\pi^-$ is accompanied by a comparable amount of $\delta\to\pi^0+\pi^0+\eta$ decay. The interaction is strong. The Dalitz-Fabbri plot is consistent with that of the phase space alone. Such characteristic properties strongly suggest a possible structureless four-pseudoscalar-meson interaction. Within unitary symmetry, such an interaction can take on the form

$$\begin{aligned}\mathcal{L}_{\Phi^4}&=f\,\mathrm{Tr}(\Phi\Phi\Phi\Phi)\\&=f[\tfrac{1}{2}(\bar\pi\pi)^2+3\eta\delta(\bar\pi\pi)+\tfrac{3}{2}\delta^2(\bar\pi\pi)+(\bar KK)(\bar\pi\pi)\\&\quad+\tfrac{1}{2}(\delta+\eta)\pi\bar KtK+(\bar KK)^2+\tfrac{1}{2}(\eta^2+\delta^2)\bar KK\\&\quad+\tfrac{1}{8}(\eta+\delta)^4+\tfrac{1}{4}(\eta-\delta)^4]. \quad (11)\end{aligned}$$

For the time being, we take this as an assumption. The coupling constant f is a dimensionless real number and is taken to be positive. f may be estimated by the known branching ratio

$$\frac{\Gamma(\delta\to\pi^++\pi^-+\gamma)}{\Gamma(\delta\to\pi^++\pi^-+\eta)}.$$

We obtain $f^2/4\pi=0.62\pm0.36$ and $f=2.9\pm0.9$. The $\pi\pi$ S-wave effective coupling constant λ is defined by

$$\mathcal{H}=-4\pi\lambda(\bar\pi\pi)^2=-\mathcal{L}.$$

In terms of f, $\lambda=-(1/4\pi)(\frac{1}{2}f)=-0.13\pm0.05$ which is in agreement with the best known value[10] -0.18 ± 0.05 with the sign correctly chosen.

Another possible interaction involving four mesons is $\eta\to3\pi$. This decay has been of particular theoretical interest. Several models have been proposed to explain the branching ratios and the energy spectrum.[11] All these features can be accommodated in our model with no additional free parameters. The electromagnetic nature of the decay is realized by introducing an electromagnetic mixing term into the Lagrangian

$$\begin{aligned}\mathcal{L}_\Phi=h(\Phi\Phi)_{11}=h[\tfrac{1}{2}(\bar\pi\pi)-2^{-1/2}(\delta+\eta)\pi^0\\+\tfrac{1}{4}(\delta+\eta)^2+K^+K^-]. \quad (12)\end{aligned}$$

We can consider this mixing term to come from the nonvanishing vacuum expectation value of the scalar-meson field, $\langle S_{11}\rangle$, in the interaction $\mathrm{Tr}(\Phi\Phi S)$. This so-called tadpole-dominance picture has been used by Coleman and Glashow to calculate electromagnetic mass differences.[12] According to these authors, tadpole and nontadpole contributions are distinct; thus, the total mass difference should be the sum of both. h can be estimated by the mass difference of the K meson. $h=m_{K^0}{}^2-m_{K^\pm}{}^2+\Delta m_{KN}{}^2$, where $\Delta m_{KN}{}^2$ is the nontadpole contribution which we calculate to be 2200 (MeV)^2,

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assuming a one-photon and one-K-meson intermediate state with ρ^0 and ω mesons as form factor according to the picture prescribed above. We get $h/m_K{}^2=0.026$.

The similarity of the structure of vector meson and pseudoscalar meson as seen from the fundamental field and the empirical relation $m_{K^*}{}^2-m_\rho{}^2=m_K{}^2-m_\pi{}^2$ enables us to infer that the symmetry-breaking coefficients must be equal for both cases.[1] Hence we obtain the electromagnetic mixing for vector mesons

$$\mathcal{L}_U=h(U^\mu U_\mu)_{11}=h[\tfrac{1}{2}\bar{\rho}\rho-\rho^0\omega+K^{*+}K^{*-}]. \quad (13)$$

The processes $\omega\to\rho^0\to 2\pi$ and $\omega\to\gamma\to\rho^0\to 2\pi$ give a partial width of 0.08 MeV to $\omega\to 2\pi$ decay.

The 3π decay of η is then considered to be a combination of electromagnetic mixing and a four-meson interaction. The possible processes are

$$\eta\to\pi+\pi+\delta\,,\quad \delta\to\pi^0\,,$$
$$\eta\to\pi+\pi+\eta\,,\quad \eta\to\pi^0\,,$$
$$\eta\to\pi^0\,,\qquad \pi^0\to\pi^0+\pi+\pi \quad \text{in } S \text{ wave},$$

and

$$\eta\to\pi^0\to\pi^\pm+\rho^\mp\,,\quad \rho^\mp\to\pi^0+\pi^\mp \quad \text{in } P \text{ wave}.$$

There is no P-wave contribution in the neutral modes.

The decay rate of $\eta\to\pi^++\pi^-+\pi^0$ is given by

$$w=\left(\frac{f^2}{4\pi}\right)\frac{h^2}{8\pi^2 m_\eta{}^3(m_\eta{}^2-m_\pi{}^2)}\int_{4m_\pi{}^2}^{(m_\eta-m_\pi)^2} ds\, b$$
$$\times\left\{\left(1+\frac{3}{2}\frac{m_\eta{}^2-m_\pi{}^2}{m_\delta{}^2-m_\pi{}^2}\right)^2+\frac{g_{U\Phi^2}{}^2}{2f}\left(1+\frac{3}{2}\frac{m_\eta{}^2-m_\pi{}^2}{m_\delta{}^2-m_\pi{}^2}\right)\right.$$
$$\times\left[\frac{(c+a)}{2b}\ln\left(\frac{c+b}{c-b}\right)-1\right]+\left(\frac{g_{U\Phi^2}{}^2}{4f}\right)^2$$
$$\left.\times\left[1+\frac{(c+a)^2}{2(c^2-b^2)}-\frac{(3c-a)}{4bc}\ln\left(\frac{c+b}{c-b}\right)\right]\right\}, \quad (14)$$

where $s=(p_++p_-)^2=(m_\eta-m_\pi)^2-2m_\eta T_0$, T_0 being the kinetic energy of the neutral pion, and

$$a(s)=3s-m_\eta{}^2-3m_\pi{}^2\,,$$
$$c(s)=s+2m_\rho{}^2-m_\eta{}^2-3m_\pi{}^2\,,$$

and

$$b(s)=\{[(m_\eta+m_\pi)^2-s][(m_\eta-m_\pi)^2-s][S-4m_\pi{}^2]/s\}^{1/2}$$

in the phase-space density.

The first, second, and third terms in expression (14) correspond to the S-wave, S-P-wave-interference, and P-wave contributions, respectively. While the P-wave contribution is negligible, the S-P-wave interference gives the correct asymmetry with f being positive. The calculated spectrum is plotted against three sets of experimental data[13] in Fig. 1; each is normalized to unit area.

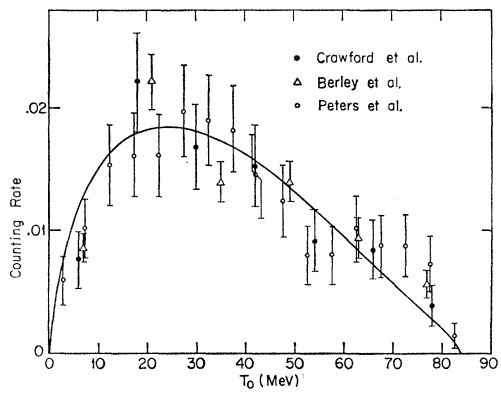

FIG. 1. Kinetic-energy spectrum of π^0 in $\eta\to\pi^+\pi^-\pi^0$ decay.

The results of the calculation are summarized in Table I. Among these partial widths, four have been chosen from experiment to determine the coupling constants $g_{U\Phi^2}$, $g_{\Phi U^2}$, f, and k. We have not included errors in these numbers since the errors are large, strongly correlated, and thus very misleading in the sense that on taking a ratio of any two numbers, cancellation of errors may occur, and in practice, branching ratios are the basic quantities in many experiments. Experimental data mostly compiled by Rosenfeld *et al.*[14] are shown next to the calculated values as reference. Since inconsistencies have been noted among experiments, for fair comparison one should refer to original measurements. A few decays are still subject to experimental verification.

Since we base our estimate of the coupling constants on particular processes, some of which are not very accurately known, we should not overemphasize the quantitative agreement with experiments; however, we are encouraged by the qualitative consistency. In addition to Schwinger's field theory of matter, we have made a crucial assumption concerning the existence of a four-pseudoscalar-meson direct interaction. Such an interaction may indeed occur directly in nature or be mediated by heavy particles. Phenomenologically a unitary nonuplet of scalar mesons may be used to obtain symmetry-breaking interactions through the Lagrangian

$$\mathcal{L}_{S\Phi^2}=G\,\mathrm{Tr}(\Phi\Phi S) \quad (15)$$

with nonvanishing vacuum expectation values of $\langle S_{33}\rangle$, $\langle S_{11}\rangle$, $\langle S_{23}\rangle$, and $\langle S_{32}\rangle$. One reason that such mesons have not been observed may be that the masses are relatively heavy compared with the available machine

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TABLE I. Widths of meson decays.

	Partial modes	Calculation Γ (total)	Calculation Γ	Calculation Fraction (%)	Experiment Γ (total)	Experiment Γ	Experiment Fraction (%)
π^0	$\gamma\gamma$	6.62 eV	6.5 eV[a]	98.8	6.5 eV[4]		98.8
	γe^+e^-		0.12 eV	1.2			1.19±0.05
η	$\gamma\gamma$	1.4 keV	0.475 keV	33.8	<10 MeV		38.6 ±3.8
	$\pi^0\gamma\gamma$		0.034 keV	2.4			
	$3\pi^0$		0.445 keV	31.7			30.8 ±3.2
	$\pi^+\pi^-\pi^0$		0.312 keV	22.2			25.0 ±2.2
	$\pi^+\pi^-\gamma$		0.140 keV	9.9			5.5 ±1.7
δ	$\eta\pi^0\pi^0$	~1.5 MeV	0.350 MeV		<4 MeV	$\frac{\Gamma(\eta\pi^+\pi^-)}{\Gamma(\gamma\pi^+\pi^-)}=0.25\pm0.14$[a]	
	$\eta\pi^+\pi^-$		0.660 MeV				
	$\gamma\pi^+\pi^-+\gamma\rho$		0.165 MeV				
	$\gamma\pi^+\pi^-\pi^0+\gamma\omega$		0.155 MeV				
ρ	$\pi\pi$	111 MeV	110 MeV[a]	99.1	112 ±4 MeV		
	$\pi\gamma$		1.0 MeV	0.9		1.65 MeV[b]	
ω	$\pi^+\pi^-\pi^0$	9.4 MeV	8.1 MeV[a]	86.2	9.3±1.7 MeV		89
	$\pi^0\gamma$		1.06 MeV	11.3			10 ±1
	$\pi^+\pi^-\gamma$		0.15 MeV	1.6			<3.2±1
	$\pi\pi$		0.08 MeV	0.8			1
ϕ	K_1K_2	3.47 MeV	0.96 MeV	28	3.1±0.6 MeV	1.08 MeV	35 ±6
	K^+K^-		1.51 MeV	43.5		1.55 MeV	50 ±6
	$\pi\rho+3\pi$		1.00 MeV	29.5		0.46 MeV	15 ±7
K^*	K^{*+} $K^+\pi^0$	33.3 MeV	11.65 MeV		50 ±3 MeV		
	K^{*+} $K^0\pi^+$		20.5 MeV				
	K^{*+} $K^+\gamma$		1.14 MeV				
	K^{*0} $K^+\pi^-$	34.45 MeV	23.05 MeV				
	K^{*0} $K^0\pi^0$		11.4				

[a] Input of the calculation.
[b] H. R. Crouch *et al.*, Phys. Rev. Letters 13, 640 (1964).

energy. If such is indeed the case, then an effective four-meson interaction may result from the elimination of the scalar-meson intermediate state. Assuming that the splitting of the scalar-meson masses is small compared with their common mass if they are not degenerate, then the Green's function is approximately given by

$$\langle S_{ab}S_{ca}\rangle\simeq\delta_{ad}\delta_{bc}/m_s^2$$

and the effective Lagrangian becomes

$$\mathfrak{L}_{\mathrm{eff}}=(G^2/2m_s^2)\mathrm{Tr}(\Phi\Phi\Phi\Phi) \qquad (16)$$

with $G^2/2m_s^2$ identified as f which must indeed be positive.

If we take this argument seriously, we realize we should treat separately the singlet interaction as well. The actual Lagrangian is not (15) but

$$\mathfrak{L}S\Phi^2=G[\mathrm{Tr}(\Phi\Phi S)+\gamma\,\mathrm{Tr}\Phi\,\mathrm{Tr}\Phi\,\mathrm{Tr}S+\beta\,\mathrm{Tr}\Phi\,\mathrm{Tr}(\Phi S)]. \qquad (17)$$

From this Lagrangian follows the same mass formula and mixing as that obtained by Schwinger.[1] In terms of λ, λ', and α defined by him,

$$G=\lambda/\langle S_{33}\rangle,\quad \gamma=\tfrac{1}{6}(\lambda'/\lambda)-(2/9)(\alpha-1),\quad \beta=\tfrac{2}{3}(\alpha-1),$$

where α is the overlapping factor $0<\alpha<1$. To produce the correct mass formula, $|\alpha|$ is chosen to be 0.53 and then $\gamma=0.64$, $\beta=-0.313$.

The effective Lagrangian in this case is

$$\mathfrak{L}_{\mathrm{eff}}'=(G^2/2m_s^2)[\mathrm{Tr}(\Phi\Phi\Phi\Phi)+2\beta\,\mathrm{Tr}(\Phi\Phi\Phi)\,\mathrm{Tr}\Phi+(2\gamma+\beta^2)\mathrm{Tr}(\Phi\Phi)\mathrm{Tr}\Phi\,\mathrm{Tr}\Phi+(3\gamma^2+2\gamma\beta)\mathrm{Tr}\Phi\,\mathrm{Tr}\Phi\,\mathrm{Tr}\Phi\,\mathrm{Tr}\Phi]. \qquad (18)$$

The relevant term used in this paper is given by

$$\mathfrak{L}_{\mathrm{eff}}'=f[\tfrac{1}{2}(\bar{\pi}\pi)^2+2.5\eta\delta(\bar{\pi}\pi)+1.34\eta^2(\bar{\pi}\pi)+\cdots]. \qquad (19)$$

This Lagrangian differs from that of (11) only slightly. With (19), we obtain $f^2/4\pi=0.89\pm0.60$ and $\lambda=-0.155\pm0.05$. The same calculation has been repeated for the $\eta\rightarrow3\pi$ decay. Since the differences are small within the crudeness of the calculation, we shall not present them as separate results. It suffices to mention that they show small improvement but are by no means decisive.

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