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Measurement of the K^0 Mass and the K^0-K^- Mass Difference*

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From the decay of the K_1^0 in a hydrogen bubble chamber, we have measured the mass of the K^0 to be 497.44±0.33 MeV. From the reaction

$$K^-p \to K^0 n,\quad K^0 \to \pi^+\pi^-$$

we have measured the K^0-K^- mass difference to be 3.71±0.35 MeV.

I. INTRODUCTION

The K^0 mass was measured by two independent methods in the Columbia-BNL 30-in. hydrogen bubble chamber exposed to $\bar{p}$ and K^- from the low-energy separated beam at the Brookhaven alternating gradient synchrotron (AGS). The first procedure was based on the study of a large sample of two-body decays of K^0 mesons produced by stopping antiprotons. The second procedure was to determine Δ, the K^0-K^- mass difference, from the charge exchange reaction

$$K^-p \to K^0 n,\quad K^0 \to \pi^+\pi^-$$

followed by elastic scattering of the neutron.

A discussion is included of the tests used to determine an absolute scale factor for the conversion of curvature measurements to momenta, and to check the validity of the error assignments.

II. K^0 MASS FROM DECAY

A sample of V's was fitted to the three-constraint hypothesis $K^0 \to \pi^+\pi^-$ using the measured directions of the neutral K^0 meson and the measured momenta and directions of the pions. The fits were performed using the GRIND kinematics program.[1] Since the program is unable to treat the mass as a variable, we fit each event using five values of the K^0 mass, ranging from 496.5 to 498.5 MeV in steps of 0.5 MeV. Near the minimum, the curve of χ^2 versus K^0 mass is a parabola with the minimum located at the best value of the mass. For each event, the lowest three values of χ^2 are fitted to a parabola to determine the value of χ^2 at the minimum, and if this value exceeds 5.0, the event is rejected. The accepted sample contains 2223 events. The best value of the K^0 mass from these events is determined by fitting the curve $\sum_{i=1}^{2223}\chi_i^2$ versus mass to a parabola as shown in Fig. 1. The result is $m_{K^0}=497.44\pm0.23$ MeV, where the error is obtained from the values of the mass for which $\sum_i\chi_i^2=(\sum_i\chi_i^2)_{\min}+1.0$. This error reflects the measurement and multiple-scattering errors assigned to each track.

We have two independent checks on the error assignments. The experimental width of the momentum distribution for a set of events produced at a fixed momentum should equal the width of the resolution function for the same sample. The resolution function is obtained by multiplying the assigned momentum error for each event by $\sqrt{2}$ and then plotting a Gaussian ideogram about the fixed momentum. The pion momenta from the reactions $K^-p\to\Sigma\pi$ and $\bar{p}p\to\pi^+\pi^-$ are used to perform this test. Good agreement is achieved in both cases, indicating not only a correct magnitude, but also a correct momentum dependence for the errors. The curves for the reaction $\bar{p}p\to\pi^+\pi^-$ are plotted in Fig. 2.

The second check is whether or not the errors which are determined by the fitting program based on the assigned errors are correct for the three-constraint fit.

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This is checked by examining the K_1^0 momentum ideograms after fitting the K_1^0 decay from the reaction $\bar{p}p \to K_1^0K_2^0$. Again the widths of the experimental distribution and the resolution function are comparable.

From a measurement of the momenta of pions produced in the reaction $\bar{p}p \to \pi^+\pi^-$ we can experimentally determine the scale factor for the conversion of curvature measurements to momenta. The scale factor takes into account systematic effects such as optical distortions, uncertainties in the magnetic field, and approximations in the geometrical reconstruction program. Since this is a two-body decay occurring from an initial state of fixed energy and zero momentum, the outgoing prongs must be collinear and have unique momentum. The momentum distribution of 1632 pion tracks which dip less than 60°, and which are collinear within 1.15° were used to determine $p_{\rm exp}$, the mean experimental value of the pion momentum from $\bar{p}p \to \pi^+\pi^-$. The expected value, $p_{\rm theor}$, is based only upon the pion and proton mass. The scale factor, $p_{\rm theor}/p_{\rm exp}$, is applied to the momentum of all tracks used in a determination of the K^0 mass. There is an uncertainty in this ratio due to the uncertainty in $p_{\rm exp}$; $\delta p_{\rm exp}/p_{\rm exp}=0.1\%$. Each event, therefore, contributes an additional error,

$$\delta m_{K^0}=(\delta p_{\rm exp}/p_{\rm exp})(m_{K^0}/\sqrt{2})\times\{1-(E_K/E_1E_2)(2m_\pi^2/m_K^4)(m_K^2+2m_\pi^2)+2(m_\pi/m_K)^4(E_K/E_1E_2)^2\}^{1/2},$$

where E_K, E_1, and E_2 are the total lab energies of the K^0 and decay pions, respectively. The average contribution to the error in the K^0 mass is 0.24 MeV, so that the result of this measurement is

$$m_{K^0}=497.44\pm0.33 \text{ MeV}.$$

III. K^0 MASS FROM K^- CHARGE EXCHANGE

For a systematic study of low-energy K^-p cross sections, 60 000 pictures were scanned for all K^- interactions. Those events which were kinematically compatible with the hypothesis $K^-p \to K^0n$ were rescanned. The scanners were given the predicted direction of the neutron as projected into each camera view, and the maximum length of the recoil proton track resulting from an n-p elastic scattering. They were instructed to accept any event within ±5° of the predicted direction and within 15 cm of the production vertex. Twenty-five events met this criteria. Since the incident beam is inadequately shielded, many of these events are due to a background of unassociated recoil protons. The events were fitted to the hypothesis

$$K^-p \to K^0n \searrow n+p \to n+p; \quad K^0 \searrow \pi^+\pi^-$$

where the momentum of the K^0, the incident neutron, and the outgoing neutron, and the direction of the outgoing neutron, are not observable. The program simultaneously fits three vertices so that the problem has seven constraints.

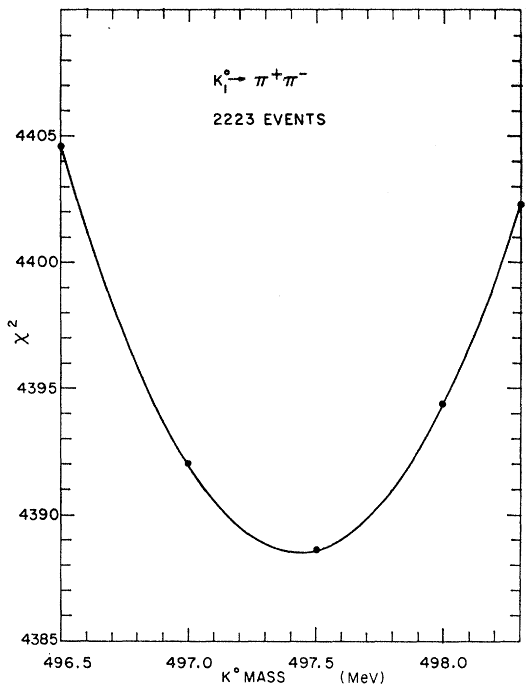

FIG. 1. Distribution of χ^2 versus K^0 mass for 2223 $K^0 \to \pi^+\pi^-$ events.

Seven of the events fit with $\chi^2<12$, whereas among the remainder, none has $\chi^2<55$. Each of the seven acceptable events is fit to five values of the K^0 mass ranging from 496.5 to 498.5 in steps of 0.5 MeV, using a K^- mass of 493.8 MeV. The three lowest values of χ^2 are used to fit a parabola to determine the best value of the K^0 mass. Table I gives the value of the mass difference and its error for each event. The weighted

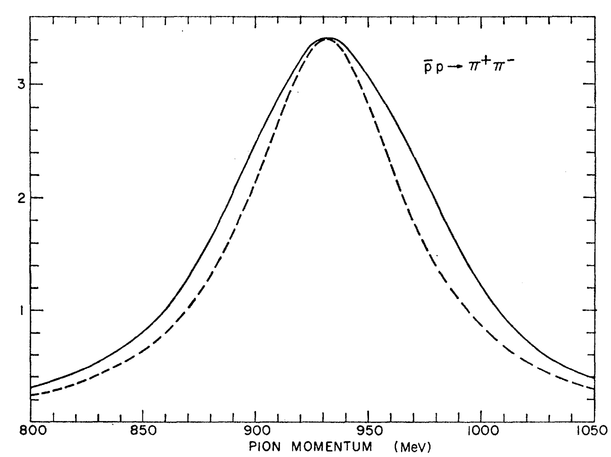

FIG. 2. The solid curve is a Gaussian ideogram of the momentum of 1632 pion tracks from the reaction $\bar{p}p \to \pi^+\pi^-$. The dashed curve is the resolution function for the same events. The vertical scale is in arbitrary units.

TABLE I. Values of K^0-K^- mass difference for individual events.

Event number	K^0-K^- mass difference (MeV)
309 941	5.37±1.12
309 691	3.60±0.69
359 964	2.44±3.13
368 562	3.92±0.64
372 533	4.08±1.12
372 823	3.95±1.24
487 277	2.25±0.87
Weighted average	Δ=3.71±0.35 (MeV)

average is $\Delta=3.71\pm0.35$ MeV. Combining this result with the assumed value[2] of the K^- mass

$$m_{K^-}=493.8\pm0.15 \text{ MeV}$$

gives $m_{K^0}=497.51\pm0.41$ MeV.

The determination of the mass from these events depends upon the K^- and pion momenta. Since both the $\bar{p}$ and K^- exposures were made in the same chamber, the momenta were corrected in the same manner as discussed in Sec. II. The 0.1% momentum uncertainty results in less than 0.01-MeV change in the K^0 mass.

The proton momentum is determined from a range-momentum table which has a scale factor known to 0.5%. The mass change due to this uncertainty is 0.03 MeV.

Both of our results are in agreement with the values of the mass obtained by Burnstein and Rubin,[3] $m_{K^0}=497.70\pm0.32$ MeV, and the value given in the compilation by Rosenfeld *et al.*,[4] $m_{K^0}=498.0\pm0.5$ MeV. The weighted average of these four values is $\bar{m}_{K^0}=497.62\pm0.18$ MeV.

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Faddeev Formalism for Four-Particle Systems*

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From the four-particle Lippmann-Schwinger equation a set of equations of the Faddeev type with properly connected kernels has been obtained. The kernels contain only the two- and three-particle scattering amplitudes, and do not involve the potentials.

I. INTRODUCTION

RECENTLY, there has been considerable interest in the three-particle problem[1–5] and in particular the Faddeev equation[1] for the three-body scattering amplitudes. The main advantages of the Faddeev approach are (1) only the off energy shell two-body scattering amplitudes rather than the potentials occur in the kernel, and (2) the square of the kernel is completely connected, consequently the kernel is likely to be square integrable since the troublesome delta functions arising from disconnected graphs are absent. Of course, the Faddeev method is only one of an infinite number of ways of obtaining L^2 kernels.[4] Weinberg[3] has given a

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