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Baryon-Meson Vertex in a Relativistic $SU(6)$ Theory*

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A calculation is presented of the vector and pesudoscalar currents for the baryon-meson vertex within the context of a relativistic $SU(6)$-invariant S-matrix theory. This symmetry is *not* intrinsically broken and is exact in the limit of exact $SU(6)$ symmetry.

I. INTRODUCTION

With the recent introduction[1] of a new technique which provides a relativistic method of treating spin independence, it has become possible to construct theories in which spin and internal symmetries are combined to form one larger symmetry in a completely covariant manner. By combining spin independence with isotopic-spin symmetry to obtain relations between the nucleon electromagnetic form factors, this technique has already been tested[2] in a relativistic situation, and remarkable agreement with the experimental data was achieved. The next logical step, that of combining this spin independence with unitary-spin[3] symmetry to produce a relativistic version of the fashionable $SU(6)$ theory,[4] is presented in the present paper for the case of the baryon-meson vertex. This treatment has the unique advantage that in the limit of exact $SU(6)$ symmetry (when the masses of all the baryons are equal, etc.) the symmetry is exact. The intrinsic breaking of the symmetry by the kinetic-energy terms, which occurs in many recent attempts[5–8] to make the symmetry relativistic, is completely avoided in the present approach.

A treatment of the spin invariance of the couplings which is more general than the original one given in Refs. 1 and 2 is given in Sec. II, and this allows the physical masses of the baryons to be introduced into the theory in a simple way. Thus, it is hoped, the breaking of the symmetry indicated by the large mass differences between particles within $SU(6)$ multiplets will be compensated to some extent. This approach is somewhat speculative and may prove controversial, but the reduction to the equal-mass limit $(w=m)$ is trivial and will enable the more conservative reader to extract the relativistic results of the exact $SU(6)$ calculations with a minimum of labor.

The combined symmetry is treated in Sec. III, while in Sec. IV a general discussion is given of the method employed and its possible generalization to other processes.

II. THE GENERALIZED COUPLING

The present approach to spin independence has been treated in detail in Refs. (1) and (2) and the results given in those papers will be used freely in the present work.

If $U(p)$ and $\bar{U}(p')$ represent incoming and outgoing fermions at some strong-interaction three-particle vertex, then invariance is assumed under the "spin transformations"

$$U(\Gamma)\rightarrow[+i(a_I 1+ia_{\mu 5}\gamma^\mu\gamma_5+\tfrac{1}{2}a_{\mu\nu}\sigma^{\mu\nu})]U(p)\,, \quad (1)$$

where

$$p^\mu a_{\mu 5}=p'^\mu a_{\mu 5}=p^\nu a_{\mu\nu}=p'^\nu a_{\mu\nu}=0\,. \quad (2)$$

These transformations are the intersection of the two little groups of the full $\tilde{U}(4)$ group which are defined by the momenta p and p'. It should be emphasized that the transform of a spinor of momentum p will *not* depend on p'; the momenta merely serve to specify *which* of the $\tilde{U}(4)$ generators form the generators of the allowed transformation group. The interaction vertex is moreover to be formed only from the generators of the $P\tilde{U}(4)$ transformations, which are effectively generalized spin transformations applied only to the fermions.

It is however proposed that the $P\tilde{U}(4)$ transformations discussed in those references be generalized so that they may contain the scalars p'^2, p^2 but they do *not* explicitly contain the mass of either fermion. Thus the generalized $P\tilde{U}(4)$ transformations from which the couplings are to be constructed are the full set of transformations

$$U(p)\rightarrow S(p,p')U(p)\,, \quad (3)$$

where S has the properties:

(A) $(p\!\!\!/-m)SU(p,m)=0=\bar{U}(p',w)S(p\!\!\!/'-w)\,,$

(B) $\bar{U}(p)U(p)$ and $\bar{U}(p')U(p)$ are invariants,

(C) the product of any two generators of these transformations is again a generator.

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[3] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[4] See, for example: M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965) and Refs. 1 and 2 contained therein; F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1964); M. A. B. Bég, B. W. Lee, and A. Pais, *ibid.* **13**, 514 (1964); B. Sakita, *ibid.* **13**, 643 (1964); K. Johnson and S. B. Treiman, *ibid.* **14**, 189 (1965).
[5] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965).
[6] B. Sakita and W. C. Wali, Phys. Rev. Letters **14**, 404 (1965).
[7] K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **13**, 698 (1964).
[8] S. Okubo and R. E. Marshak, Phys. Rev. Letters **13**, 818 (1964).

The transformations thus defined have all the properties of the $P\tilde{U}(4)$ transformations but moreover may be applied to incoming and outgoing fermions of different masses.

It is straightforward but somewhat tedious to establish that the most general form for S subject to the above conditions is

$$S(p,p')=1+i\alpha_\mu\tilde{\Gamma}^\mu, \tag{4}$$

where

$$\begin{aligned}\tilde{\Gamma}^\mu=a[&pp'\Gamma^\mu pp'-p\cdot p'(pp'\Gamma^\mu+\Gamma^\mu pp')-p'^2p\Gamma^\mu p\\&-p^2p'\Gamma^\mu p'+p\cdot p'(p\Gamma^\mu p'+p'\Gamma^\mu p)\\&+(2(p\cdot p')^2-p^2p'^2)\Gamma^\mu]\\&+b[pp'\Gamma^\mu p'-p'\Gamma^\mu pp'-p'^2p\Gamma^\mu\\&+2p\cdot p'p'\Gamma^\mu-p'^2\Gamma^\mu p]\\&+b'[p\Gamma^\mu pp'-pp'\Gamma^\mu p-p^2\Gamma^\mu p'\\&-p^2p'\Gamma^\mu+2p\cdot p'\Gamma^\mu p].\end{aligned} \tag{5}$$

Here the α_μ are real parameters, $\Gamma^\mu=\{1,\gamma^\mu,i\gamma^\mu\gamma_5,\sigma^{\mu\nu},\gamma_5\}$ and a, b, and b' are functions of p'^2, p^2, and $p\cdot p'$, and are subject to the conditions

$$a(p,p')=a(p',p), \tag{6}$$

$$b(p,p')=b'(p',p), \tag{7}$$

$$b^2p'^2+b'^2p^2-2p\cdot p'bb'+\tfrac{1}{4}a=0. \tag{8}$$

Finally, the normalization requirement that the generator with $\Gamma^\mu=1$ shall be the identity operator yields

$$4a=[(p\cdot p')^2-p^2p'^2]^{-1}. \tag{9}$$

It therefore follows that the three-particle interaction [defined in Eq. (25) of Ref. 2] now takes the form

$$g\bar{\psi}(p',w)\tilde{\Phi}(q)\psi(p,m), \tag{10}$$

where the tilde indicates the same construction as in Eq. (5) above. Fortunately, this expression reduces with a little effort to the much simpler form

$$\begin{aligned}g\bar{\psi}(p',w)\Bigg\{&[2b(p\cdot p'-w^2)+2b'(p\cdot p'-m^2)]\\&\times\left[\phi_\mu\left(P^\mu+\frac{r^\mu}{\mu}\right)+\frac{\phi}{\mu}\gamma_5[(w+m)^2-q^2]\right]\\&+q_\mu\phi^\mu[2b(w^2+p\cdot p')-2b'(m^2+p\cdot p')]\Bigg\}\psi(p,m),\end{aligned} \tag{11}$$

where μ is the meson mass, $r_\mu=\epsilon_{\mu\nu\rho\lambda}P^\nu q^\rho\gamma^\lambda\gamma_5$, $P=p'+p$, and $q=p'-p$. Notice that in the equal-mass limit ($w=m$, $b=b'$) this form reduces to

$$\begin{aligned}&g[4m^2-q^2]^{-1/2}\bar{\psi}(p')\\&\quad\times[\phi\gamma_5(4m^2-q^2)+\phi_\mu(P^\mu+(r^\mu/\mu))]\psi(p),\end{aligned} \tag{12}$$

in agreement with Eq. (27) of Ref. 2.

III. THE COMBINED SYMMETRY

The basic assumption now to be made is that of invariance under the transformations which have as their generators all possible products of the generators of the spin transformations and those of unitary-spin symmetry[3]; specifically for the lowest dimensional representation

$$U(p)\rightarrow[1+i(a_I{}^j1+ia_{\mu5}{}^j\gamma^\mu\gamma_5+\tfrac{1}{2}a_{\mu\nu}{}^j\sigma^{\mu\nu})T^j]U(p), \tag{13}$$

where the parameters are real and subject to the conditions

$$p^\mu a_{\mu5}{}^j=p'^\mu a_{\mu5}{}^j=p^\nu a_{\mu\nu}{}^j=p'^\nu a_{\mu\nu}{}^j=0 \tag{14}$$

and $T^j=\frac{1}{2}\lambda^j$, where λ^j is as defined by Gell-Mann.[3] Thus the basic spinor has 12 components which may be conveniently described by two indices; one Greek index in the range 0–3 representing spin variables, and one Latin index in the range 1–3 representing unitary-spin variables. This convention as to Latin and Greek indices will be extended to higher dimensional spinors with no futher explanation.

The physical baryons are now assigned to the fully symmetric multispinor which transforms as the product of three basic spinors of the same momentum. Under the direct product of the spin transformations and those of unitary spin, this symmetric third-rank spinor decomposes in the following manner[5]

$$\begin{aligned}\psi_{\alpha p\beta q\gamma r}=&\tfrac{1}{2}(\sqrt{\tfrac{3}{2}})D_{\alpha\beta\gamma,pqr}\\&+\frac{1}{2\sqrt{6}}[\epsilon_{pqs}M_{\alpha\beta\gamma,r}{}^s+\epsilon_{qrs}M_{\beta\gamma\alpha,p}{}^s+\epsilon_{rps}M_{\gamma\alpha\beta,q}{}^s],\end{aligned} \tag{15}$$

where D is totally symmetric in both spin and unitary indices, and M is of the mixed symmetry (in the spin indices) which is specified by

$$M_{\alpha\beta\gamma}+M_{\beta\alpha\gamma}=0, \tag{16}$$

$$M_{\alpha\beta\gamma}+M_{\beta\gamma\alpha}+M_{\gamma\alpha\beta}+0. \tag{17}$$

Thus the interaction defined in Eq. (10) takes the form

$$g\bar{\psi}^{\alpha p\beta q\gamma r}\tilde{\Phi}_{\alpha p}{}^{\alpha' p'}\psi_{\alpha' p'\beta q\gamma r}=gJ^{Ri}\Phi_{Ri}, \tag{18}$$

where (taking only the terms in Φ which are traceless in the unitary indices) J^{Ri} is given by[5]

$$\begin{aligned}J^{Ri}=&\tfrac{3}{8}\bar{D}^{\alpha\beta\gamma pqr}(\tilde{\gamma}^R)_\alpha{}^{\alpha'}(T^i)_p{}^{p'}D_{\alpha'\beta\gamma p'qr}\\&+\tfrac{1}{4}[\bar{D}^{\alpha\beta\gamma pqr}(\tilde{\gamma}^R)_\alpha{}^{\alpha'}(T^i)_p{}^{p'}\epsilon_{p'qs}N_{\alpha'\beta\gamma r}{}^s\\&+\bar{N}^{\alpha\beta\gamma r}{}_s\epsilon^{pqs}(\tilde{\gamma}^R)_\alpha{}^{\alpha'}(T^i)_p{}^{p'}D_{\alpha'\beta\gamma p'qr}]\\&-(1/24)[\bar{N}^{\beta\alpha\gamma}(\tilde{\gamma}^R)_\alpha{}^{\alpha'}N_{\alpha'\beta\gamma}]_{3D+2F}\\&+\tfrac{1}{12}[\bar{N}^{\beta\alpha\gamma}(\tilde{\gamma}^R)_\alpha{}^{\alpha'}N_{\alpha'\gamma\beta}]_{3D+5F},\end{aligned} \tag{19}$$

where

$$(\bar{N}N)_F=\bar{N}_r{}^pT_p{}^{p'}N_q{}^r-\bar{N}_r{}^pN_p{}^qT_q{}^r \tag{20}$$

and

$$(\bar{N}N)_D=\bar{N}_r{}^pT_p{}^{p'}N_q{}^r+\bar{N}_r{}^pN_p{}^qT_q{}^r. \tag{21}$$

Notice that Eq. (11) indicates how this reduces in a very simple way to the sum of a vector and pseudoscalar current. All that remains is to substitute the expressions[2,9]

$$D_{\alpha\beta\gamma}(p,m)=m^{-1}[(p\!\!\!/+m)\gamma_\mu\gamma_5 B]_{\alpha\beta}D_\gamma{}^\mu(p,m)\,, \tag{22}$$

where

$$(p\!\!\!/-m)=\gamma_\mu D^\mu=p_\mu D^\mu=0 \tag{23}$$

and

$$M_{\alpha\beta\gamma}(p,m)=m^{-1}[(p\!\!\!/+m)B]_{\alpha\beta}N_\gamma(p,m)\,, \tag{24}$$

where

$$(p\!\!\!/-m)N=0 \tag{25}$$

for the spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ states, and thus finally obtain the results for the pseudoscalar and vector currents,

$$J^5=\frac{[(w+m)^2-q^2]}{4mw\mu}[2b(p\cdot p'-w^2)+2b'(p\cdot p'-m^2)]\{(m+w)^2-q^2]\bar{N}\gamma_5 N_{D+2F/3}$$
$$+3[(m+w)^2-q^2]\bar{D}^\mu\gamma_5 D_\mu+2q^\mu q_\nu\bar{D}^\nu\gamma_5 D_\mu+(1/m)q_\nu\bar{D}^\nu N+(1/w)q^\mu\bar{N}D_\mu\}\,, \tag{26}$$

$$J^\mu=(1/4mw)[(w+m)^2-q^2][2b(p\cdot p'-w^2)+2b'(p\cdot p'-m^2)]$$
$$\times\{\bar{N}P^\mu N_F+(1/\mu)\bar{N}r^\mu N_{D+2F/3}+3[\bar{D}^\lambda V^\mu D_\lambda+((w+m)^2-q^2)^{-1}2q^\lambda q_\nu\bar{D}^\nu V^\mu D_\lambda]$$
$$+[(2/\mu)\epsilon_{\mu\alpha\beta\nu}\bar{D}^\nu NP^\alpha q^\beta+\text{H.c.}]\}+(1/4mw)[(w+m)^2-q^2][2b(w^2+p\cdot p')-2b'(m^2+p\cdot p')]$$
$$\times\{\bar{N}q^\mu N_F+3[\bar{D}^\lambda q^\mu D_\lambda+((w+m)^2-q^2)^{-1}2q^\lambda q_\nu\bar{D}^\nu q^\mu D_\lambda]\}\,, \tag{27}$$

where

$$V^\mu=P^\mu+r^\mu/\mu \tag{28}$$

and the unitary-spin dependence has been suppressed.

IV. DISCUSSION OF THE RESULTS

It is now clear that the baryon-meson vertex contains just one form factor. Assuming that the coupling of the photon to the baryons is dominated by the vector-meson contribution this leads immediately to the conclusion[2,10] that the electric and magnetic form factors[11] of the baryons are all proportional. In particular the conclusions that for the proton

$$G_M{}^P(q^2)=(2m/\mu)G_E{}^P(q^2)\,, \tag{29}$$

while for the neutron

$$G_E{}^N(q^2)=0\,, \tag{30}$$

$$G_M{}^N(q^2)=-\tfrac{2}{3}G_M{}^P(q^2) \tag{31}$$

are found thus confirming the calculation in Ref. 2, where the fine agreement between these relations and the experimental data[12] was discussed. It should be emphasized that the device employed in this paper to insert the physical masses of the particles and thus obtain the kinematical factors does not also correct the couplings for the changes which may occur in them when the symmetry breaking splits the masses. Thus the most reliable predictions might be expected to be those relating physical quantities for particles which have nearly equal physical masses. It is extremely satisfying in this respect to note the very well verified validity of Eq. (29), which relates the form factors of one particle but depends crucially on the present treatment of the coupling. This form equality is *not* merely a result of demanding vector-meson dominance of the G form factors as has been suggested by Freund *et al.*[13]; it *is* the direct consequence of the difference between the basic strong-interaction couplings of mesons to baryons suggested here and those proposed in Ref. 5. Notice that since the correct value for the mean meson mass μ is not known, the only crucial difference between the predictions of the present theory and that of Salam *et al.*[5] for the form factors is the form equality in Eq. (29). Although the experimental result favors the present approach, it is clearly very desirable to obtain further nonstatic predictions of the two approaches which may be accurately compared with experiment.

It cannot be emphasized too strongly that in the present approach invariance is demanded only under the transformations (13) and *not* under the $P\tilde{U}(4)$ transformations which depend explicitly on the momenta and serve only to specify the couplings. The transformations (13) are a subgroup of the $\tilde{U}(12)$ group proposed in Refs. 5 and 6, specified but *not* depending explicitly on the momenta. As a consequence of this less restrictive nature of the subgroup as compared to the entire $\tilde{U}(12)$ group, it follows that $p\!\!\!/$ and $p\!\!\!/'$ are invariants, and that the imposition of the Bargmann-Wigner[9] equations is compatible with the symmetry proposed. Furthermore, the matrix B used to construct the baryon wave functions[2] *is* well defined and invariant under the spin transformations in contrast to the charge-conjugation matrix C used in Refs. 5 and 6. The realization of this fact and

[9] See also Refs. 5 and 6 and V. Bargmann and E. Wigner, Proc. Natl. Acad. Sci. **34**, 211 (1948); F. J. Belinfante, Physics **6**, 870 (1939).

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that the charge conjugate $\psi_C = C\bar{\psi}^T$ of ψ does *not* belong to the same representation[14] as ψ removes at once the crossing symmetry difficulties pointed out by Riazuddin *et al.*[15] It is perhaps worth emphasizing that if the coupling suggested in Eq. (18) is replaced by the much simpler (and equally invariant) form

$$g\bar{\psi}\Phi\psi\,, \tag{32}$$

then the resulting theory gives exactly the same *results* as those of Salam *et al.*[5] and Sakita *et al.*,[6] except that there is now no conflict with the Bargmann-Wigner[9] equations or crossing.[16] The differing *predictions* of those theories and the present one for the baryon-meson vertex lie entirely in the introduction of the momentum-dependent $P\tilde{U}(4)$ transformations which define the coupling in Eq. (18).

In view of the success of the present scheme it is most desirable to investigate its possible extension to other processes. The first and most crucial point to be made is that in general there is no immediate extension of the spin transformations to four point (or higher) functions[17]; this is essentially a theory of three-point functions. Any restrictions which the theory imposes on four-point functions must be through the implicit effects of restrictions on the three-point functions (e.g., through unitarity, or the decomposition of the amplitude into pole contributions with only three-point vertices). The three-meson interaction however should be amenable to treatment along the present lines, although the author has not yet succeeded in the endeavor to define this interaction in a way fully consistent with the above work.

[14] That ψ_C transforms with the opposite sign to ψ for infinitesimal transformations of the type $i\gamma_\mu\gamma_5$ was noted by H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965), who independently discovered the spin transformations of Ref. 10 as a subgroup of $\tilde{U}(12)$, and suggested the name W spin.

[15] Riazuddin, L. K. Pandit, and S. Okubo, University of Rochester Report U.R.-875-79 1965 (unpublished).

[16] No difficulties with unitarity arise in this case, as there is no direct restriction on four-piont functions. In fact unitarity offers possible information on the four-point functions through the restrictions imposed by the symmetry on three-point functions.

[17] The exceptions to this statement arise perhaps when particles of degenerate mass have collinear momenta, i.e., forward and backward scattering.

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Σ Radiative Decay and the Angular Momentum of Σ Pionic Decay

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We have studied the pion spectrum in the $\Sigma^\pm \to n+\pi^\pm+\gamma$ decay in order to determine the angular-momentum channel of the Σ pionic decay. We discuss the results from measurements of a sample of 14 800 $\Sigma^+ \to \pi^+ + n$ decays and 25 000 $\Sigma^- \to \pi^- + n$ decays. After subtraction of the background, we find 26 Σ^+ radiative decays and 28 Σ^- radiative decays with $P_{\text{c.m.}} < 166$ MeV/c. The combination $\Sigma^+ \to \pi^+ + n$ decays via P wave and $\Sigma^- \to \pi^- + n$ decays via S wave is 45 times more likely than the combination $\Sigma^+ \to \pi^+ + n$ decays via S wave and $\Sigma^- \to \pi^- + n$ decays via P wave. This means that our result is 2.7 standard deviations in favor of the first combination.

RECENTLY some interest has been centered on a determination of the pion spectrum in the decay $\Sigma^\pm \to n+\pi^\pm+\gamma$. This comes about because the pion momentum spectrum in this decay is sensitive to the angular-momentum channel of the decay $\Sigma^\pm \to n+\pi^\pm$.[1] It is well known that the $\Delta I = \frac{1}{2}$ rule for nonleptonic decays[2] combined with the experimental measurements on the α parameter and rates of the Σ decays[3] predict that the $\Sigma^+ \to \pi^+ + n$ and the $\Sigma^- \to \pi^- + n$ decays must occur, respectively, through the S- and P-wave channels or

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[3] Bruce Cork, L. T. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. **120**, 1000 (1960); R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters **9**, 66 (1962).