New Sum Rule for Meson-Baryon Total Cross Sections at High Energy*

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Assuming that the dominant underlying mechanism of high-energy meson-nucleon elastic scattering is the exchange of SU_3 octet meson states, a new sum rule relating meson-nucleon total cross sections is derived:

$$\sigma_t(K^-p)-\sigma_t(K^+p)=\sigma_t(K^-n)-\sigma_t(K^+n)+\sigma_t(\pi^-p)-\sigma_t(\pi^+p).$$

Comparison of the sum rule with experiment indicates substantial agreement from 10 to 18 BeV/*c*. The f/d ratio for the charge coupling of the vector-meson octet to the baryon octet is also determined from ratios of total cross-section differences.

ACCUMULATING evidence supports the assignment of the observed low-mass meson states to either octet or singlet (or nonet) representations of SU_3. The classification of the known mesons and meson resonances in a 0^- octet and singlet $[\pi,K,\eta,X^0]$, a 1^- nonet $[\rho,K^*,\varphi,\omega]$ and a 2^+ nonet[1] $[A_2,\hat{K}(1430),\hat{f}^0(1525),f^0]$ fairly well exhausts the meson mass spectrum, provided that certain other enhancements $[A_1,B,K^{**}(1175)]$ prove to be of kinematic origin.[2] In any case since these other enhancements have parity assignments $(-1)^{J+1}$, and thus are not coupled to a pseudoscalar-meson pair, they are not relevant to our subsequent analysis of pseudoscalar meson-nucleon scattering. The recently completed 2^+ nonet is presumably the physical manifestation of Pignotti's conjectured SU_3 octet and singlet of Regge poles $[R,Q,P',P]$ implied by bootstrap dynamics.[3]

The occurrence of only the 1 and 8 representations of SU_3 for the observed bosons suggests a picture of high-energy elastic amplitudes dominated by exchanges of unitary singlet and octet states in the crossed $1+\bar{1}\to 2+\bar{2}$ channel. In this article we derive a sum rule for meson-nucleon total cross sections at high energies based on that assumption. The Regge-pole hypothesis provides a natural framework for this picture but is by no means an essential part of this analysis.

If the dominant underlying mechanism of high-energy elastic scattering is the exchange of singlet and octet meson states of arbitrary number and spin, then the elastic meson-baryon scattering amplitudes (MB) may be written as

$$\begin{pmatrix}(K^-p)\\(K^+p)\\(K^-n)\\(K^+n)\\(\pi^-p)\\(\pi^+p)\end{pmatrix}=\begin{pmatrix}1&\frac{2}{3}&-2&0&0\\1&\frac{2}{3}&2&0&0\\1&-\frac{1}{3}&-1&-1&1\\1&-\frac{1}{3}&1&-1&-1\\1&-\frac{1}{3}&-1&1&-1\\1&-\frac{1}{3}&1&1&1\end{pmatrix}\times\begin{pmatrix}1\\8_{ss}\\8_{aa}\\8_{sa}\\8_{as}\end{pmatrix},\quad(1)$$

where the s, a subscripts label the symmetric and antisymmetric octet representations of the coupling to the exchanged mesons. Applying the optical theorem to these amplitudes, we find directly a sum rule relating the meson-nucleon total cross sections

$$\Delta_{Kp}=\Delta_{Kn}+\Delta_{\pi p},\quad(2)$$

where we have introduced the notation

$$\Delta_{MB}=\sigma_t(M^-B)-\sigma_t(M^+B).$$

Comparison of the sum rule with experiment[4] in Table I indicates substantial agreement from 10 to 18 BeV/*c*. The sum rule appears to be in quantitatively better agreement with the data than the Johnson-Treiman relations[5]:

$$\Delta_{Kp}=2\Delta_{\pi p}=2\Delta_{Kn}.\quad(3)$$

TABLE I. Comparisons of the sum rule $\Delta_{Kp}=\Delta_{Kn}+\Delta_{\pi p}$ and the Johnson-Treiman relations $\Delta_{Kp}=2\Delta_{\pi p}=2\Delta_{Kn}$ with experiment. (Data from Ref. 4).

$P_{\rm LAB}$ (BeV/*c*)	Total cross section differences (mb)			
	Δ_{Kp}	$\Delta_{\pi p}+\Delta_{Kn}$	$2\Delta_{\pi p}$	$2\Delta_{Kn}$
6	7.0±0.3	6.7±0.7	4.6±0.8	8.8±1.1
8	6.3±0.2	4.5±0.7	4.8±0.8	4.2±1.1
10	5.2±0.2	4.8±0.7	3.4±0.8	6.2±1.1
12	4.3±0.2	4.3±0.7	3.4±0.8	5.2±1.1
14	4.1±0.2	4.1±0.7	3.0±0.8	5.2±1.1
16	4.3±0.4	4.6±0.8	3.4±0.8	5.8±1.4
18	3.9±0.8	4.2±1.2	3.0±0.8	5.4±2.3

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Contributions to the cross-section differences Δ_{MB} are entirely due to octets whose neutral members are odd under charge conjugation. Consequently the observed 2^+ meson octet is not relevant to the relations in Eqs. (2) and (3). If we make a further assumption that only the one vector-meson octet V is responsible for the Δ_{MB}, then the f/d ratio for the $VB\bar{B}$ charge coupling can be calculated from the experimental total cross sections (the VMM coupling is pure f-type, and the $VB\bar{B}$ magnetic coupling vanishes for forward scattering). The f/d ratio can be calculated from the following expressions:

$$\begin{aligned} f/d &= \Delta_{Kp}/(2\Delta_{\pi p}-\Delta_{Kp}) \\ &= \Delta_{Kp}/(\Delta_{Kp}-2\Delta_{Kn}) \\ &= (\Delta_{\pi p}+\Delta_{Kn})/(\Delta_{\pi p}-\Delta_{Kn}) \end{aligned} \tag{4}$$

which are equivalent according to Eq. (2). Pure f-type coupling yields the Johnson-Treiman relations as previously noted by Sawyer.[6] The determination of the f/d ratio from the data[4] through Eq. (4) is given in Table II. The results indicate a mean value somewhere between $f/d\approx-3$ and $f/d\approx-5$, showing an appreciable deviation from the universality prediction $d=0$.[7] The errors on the cross sections are sufficiently large to make a precise determination of f/d difficult.

TABLE II. Determinations of the vector meson-nucleon charge coupling f/d ratio from experiment. (Data from Ref. 4.)

P_{LAB} (BeV/c)	f/d ratio for Vector Meson-Baryon charge coupling: $\frac{f}{d}=\frac{\Delta_{Kp}}{(2\Delta_{\pi p}-\Delta_{Kp})}$	$\frac{f}{d}=\frac{\Delta_{Kp}}{(\Delta_{Kp}-2\Delta_{Kn})}$	$\frac{f}{d}=\frac{(\Delta_{\pi p}+\Delta_{Kn})}{(\Delta_{\pi p}-\Delta_{Kn})}$
6	−2.9	−3.9	−3.2
8	−4.2	+3.0	15.0
10	−2.9	−5.2	−3.4
12	−4.8	−4.8	−4.8
14	−3.7	−3.7	−3.7
16	−4.8	−2.9	−3.8
18	−4.3	−2.6	−3.5

In any event we emphasize that the sum rule of Eq. (2) is dependent only on the general octet-dominance property of the $MM\to\bar{B}B$ channel and not upon these further detailed considerations.

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$SU(6)$ Predictions for s-Wave Baryon-Baryon Scattering*

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The predictions of the $SU(6)$-symmetry model for s-wave baryon-baryon scattering are derived and compared with some low-energy experimental data.

THE application of $SU(6)$ symmetry to meson-baryon scattering yielded a number of relations that can be compared with experiment.[1] Such comparisons are of interest in determining the validity of $SU(6)$ symmetry and the nonrelativistic limit of its relativistic extensions.[2] In this paper the $SU(6)$ predictions for s-wave baryon-baryon scattering are tabulated and compared with some low-energy experimental data.

For the s-wave baryon-baryon scattering process

$$B(4)+B(2)\to B(3)+B(1),$$

the $SU(6)$-invariant scattering operator S, which incorporates the generalized Pauli principle, can be written in terms of the 56-dimensional $SU(6)$ baryon wave function[3] as

$$\begin{aligned} S=A\{&\bar{\psi}^{ABC}(1)\psi_{ABC}(2)\bar{\psi}^{DEF}(3)\psi_{DEF}(4) \\ &-\bar{\psi}^{ABC}(1)\psi_{ABC}(4)\bar{\psi}^{DEF}(3)\psi_{DEF}(2)\} \\ &+(C/81)\{\bar{\psi}^{ABC}(1)\psi_{ABD}(2)\bar{\psi}^{EFD}(3)\psi_{EFC}(4) \\ &-\bar{\psi}^{ABC}(1)\psi_{ABD}(4)\bar{\psi}^{EFD}(3)\psi_{EFC}(2)\}. \end{aligned} \tag{1}$$

The result of Eq. (1) for BB reactions of experimental interest are listed in Table I. Only amplitudes which are isospin-independent are included. The quantities α and β of Table I are scalar products of Pauli spinors,

$$\begin{aligned} \alpha&=\chi^i(1)\chi_i(2)\chi^j(3)\chi_j(4), \\ \beta&=\chi^i(1)\chi_i(4)\chi^j(3)\chi_j(2). \end{aligned} \tag{2}$$

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