Model of C Violation in Semistrong Interactions*

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A simple model of C violation in semistrong interactions is proposed. The basic semistrong interaction is an "equal" mixture of C-invariant and -noninvariant parts. The former part is the coupling between two neutral vector mesons (Sakurai's interaction), and the latter is the coupling between neutral vector and scalar mesons. The nature of this particular interaction and the possible experimental consequences are discussed. The small $K_2^0 \to 2\pi$ decay rate can be consistent with our model. Also the $\rho\eta\pi$ coupling and the C-violating effect in the $\Sigma^0 \to \Lambda e^+e^-$ decay are calculated.

I. INTRODUCTION AND SUMMARY

It has been suggested[1,2] that the CP violation observed in the $K_2^0 \to 2\pi$ decay[3] may come from the C violation in a rather strong interaction, which is estimated to be of the same order or even stronger than electromagnetic interaction. Under the assumption that the electromagnetic interaction has a large C-violating part, several authors suggested simple forms of the basic interaction.[4,5] Another suggestion has been made that the C violation occurs in the semistrong interaction for which charge independence and parity conservation are assumed valid, but the $SU(3)$ symmetry is violated.[1] In this case, however, it seems to have been considered difficult to formulate in a simple way the nature of this important physical law, probably because only little is known about the semistrong interaction itself. Also explained is the discrepancy between the strength of the C-violating interaction estimated simply from the mass splitting of the particles in an $SU(3)$ multiplet, and that estimated from the $K_2^0 \to 2\pi$ rate.[6] The latter gives a much smaller value, and some kind of dynamical mechanism is needed to explain this discrepancy. Furthermore there are many possible interactions which violate C, but only few attempts have been made to consider them in a unified scheme.[7]

Prentki and Veltman[1] have considered the Yukawa interactions between baryons and pseudoscalar mesons. These are assumed to be invariant under the parity and isotopic spin rotations, to violate C, and to transform as an isosinglet member of an octet in $SU(3)$. In this theory only one coupling constant ($\Sigma\Lambda\pi$) is allowed to be complex resulting in C violation. On the basis of this interaction, however, it is rather difficult to calculate C-violating quantities which are amenable to practical experimental tests. Most of these tests are concerned with the decays of mesons.[1,5–9]

There are many possible semistrong couplings among mesons which are forbidden only by C invariance. Among them, the interaction

$$-L=\tfrac{1}{2}g(\eta\partial_\mu\boldsymbol{\pi}-\boldsymbol{\pi}\partial_\mu\eta)\cdot\boldsymbol{\rho}_\mu \qquad (1.1)$$

may be the simplest.[6,7,10] The coupling constant g is dimensionless, and there is only one derivative. Starting from (1.1) we can derive other C-violating couplings like $\eta\to(3\pi)_{I=0}$, $\eta\to\pi^0e^+e^-$,[5,7,11] which are more important from an experimental point of view, but are less convenient for a theoretical analysis than (1.1), because of their complicated structure (many derivatives and dimensions of the effective coupling constants). The interaction (1.1) can also be considered as a part of the more general one,[10]

$$-L=g(\bar{K}K_\mu^*+\bar{K}_\mu^*K+\boldsymbol{\pi}\cdot\boldsymbol{\rho}_\mu)\partial_\mu\eta\,, \qquad (1.2)$$

which has the same transformation properties in the broken $SU(3)$ scheme as those conjectured by Prentki and Veltman.[1] One can, thus, even consider (1.1) or (1.2) as a basic interaction of the whole C-violating effect.

From a theoretical point of view, however, a still simpler interaction may be looked for. In this connection, we shall emphasize the following two points.

First, the scalar and vector fields have a particular property which the pseudoscalar and axial-vector fields do not possess. These two fields can be coupled to, say, the baryon fields in two different ways, in which the effects of the C conjugation are different (the "normal" coupling, i.e., scalar coupling of scalar field and vector

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[1] J. Prentki and M. Veltman, Phys. Letters 15, 88 (1965).
[2] T. D. Lee and L. Wolfenstein, Phys. Rev. 138, B1490 (1965).
[3] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964). W. Galbraith, G. Manning, A. E. Taylor, B. D. Jones, J. Malos, A. Astbury, N. H. Lipman, and T. G. Walker, *ibid.* 14, 383 (1965).
[4] F. Salzman and G. Salzman, Phys. Letters 15, 91 (1965).
[5] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. (to be published).
[6] Y. Fujii and G. Marx, Phys. Letters 17, 75 (1965).
[7] M. Nauenberg [Phys. Letters 17, 329 (1965) and B. Barrett, M. Jacob, M. Nauenberg, and T. N. Truong [Stanford Linear Accelerator Center, 1965 (unpublished)] calculated the $\pi^+\pi^-$ asymmetry of η decay and the $\eta\to\pi^0e^+e^-$ based on a single interaction $\rho\eta\pi$.
[8] S. L. Glashow and C. M. Sommerfield, Phys. Rev. Letters 15, 78 (1965).
[9] D. Cline and R. M. Dowd, Phys. Rev. Letters 14, 530 (1965). F. A. Berends, Phys. Letters 16, 118 (1965); J. Prentki and M. Veltman, Phys. Letters 17, 77 (1965).
[10] G. Marx, Phys. Rev. (to be published).
[11] L. R. Price and F. S. Crawford, Phys. Rev. Letters 15, 123 (1965).

and tensor couplings of vector field, on the one hand, and the "abnormal" coupling, i.e., vector coupling of scalar field and scalar coupling of vector field, on the other). One of these alternatives is ruled out if C invariance is required. Therefore, we must expect that scalar and vector fields should play an important role in the problem of C conjugation. It is also encouraging to note that there have been many indications of the possible existence of various kinds of scalar mesons (0^+), though none of them have been yet established by direct observations.[12–14]

Second, according to Sakurai,[15] the mixing between two vector mesons (unitary octet and singlet) is very important in the semistrong interaction. He considered the interaction[16]

$$-L=m^2 f U_\mu V_\mu, \tag{1.3}$$

where U_μ and V_μ are both isosinglet neutral vector fields; one of them is a unitary singlet and the other is a unitary octet. Then (1.3) transforms as an isosinglet member of an octet under $SU(3)$, and one can consider the interaction (1.3) to be the cause of the principal mass splittings in each $SU(3)$ multiplet as described by the Gell-Mann–Okubo mass formula, as well as the deviation from this mass formula in the case of vector mesons.

Taking these remarks into account, it seems natural to assume the coupling

$$-L=mf' U_\mu \partial_\mu W, \tag{1.4}$$

where U_μ is a neutral vector field, and W is a neutral scalar field. We assume that U_μ and W are coupled to other fields through the strong interactions, and these interactions are C-invariant if one assigns "normal" C parities to U_μ and W (i.e., -1 and $+1$, respectively). Then it is easy to see that the whole Lagrangian including (1.4) does not allow one to assign any definite C parities to these fields. We can also assume at present that (1.4) has the same $SU(3)$ transformation property as (1.3). There are two possibilities: U_μ is a unitary singlet and W is an isosinglet member of an octet; or vice versa.[17]

The most attractive choice of the magnitude of f' may be $f'=f$. This enables us to combine (1.3) and (1.4) into a single form

$$-L=m^2 f(V_\mu+m^{-1}\partial_\mu W)U_\mu, \tag{1.5}$$

where m is the mass of any of U_μ, V_μ, or W (assumed to be nearly equal for simplicity). The magnitude of f can be obtained from the phenomenological analysis of the ω-ϕ mixing and from the approximate calculation of the mass splitting of, say, the octet baryons (Sec. II). Thus the interaction (1.5) does not contain any arbitrary parameter so that the magnitude of C violation is completely fixed. It will be one of the simplest forms by which an "equal" mixture of the C invariant and noninvariant couplings is formulated. Assuming (1.5) as a basic interaction of the whole semistrong interaction, we shall calculate various quantities exhibiting C violation.[18]

One may calculate the C-violating matrix elements by first-order perturbation with respect to the interaction (1.4). A better approximation can be obtained if we apply a similar technique as in the ω-ϕ mixing problem, to the coupled system of U_μ and W. As will be considered in Sec. III, the sum of the free Lagrangians for U_μ and W and the off-diagonal Lagrangian (1.4) with ($f'=f$) can be diagonalized in terms of the new fields $\tilde{U}_\mu$ and $\tilde{W}$ defined by

$$\begin{aligned}\tilde{U}_\mu&=U_\mu-(f/m)\partial_\mu W,\\ \tilde{W}&=(1-f^2)^{1/2}W.\end{aligned} \tag{1.6}$$

In order to see how $\tilde{U}_\mu$ and $\tilde{W}$ transform under C, we consider the Lagrangian for the strong interaction of U_μ and W;

$$-L=J_\mu^{(-)}U_\mu+J^{(+)}W, \tag{1.7}$$

where the source functions $J_\mu^{(-)}$ and $J^{(+)}$ have odd and even C parities, respectively. Using the inverse relation

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[13] L. Durand, III and Y. T. Chiu, Phys. Rev. Letters **14**, 329 (1965); I. Derado, V. P. Kenney, J. A. Poirier, and W. D. Shephard, *ibid.* **14**, 872 (1965).

[14] L. M. Brown, Phys. Rev. Letters **14**, 836 (1965); E. Ferrari, CERN report (unpublished); S. Coleman and S. L. Glashow, Phys. Rev. **34**, B671 (1964); M. Suzuki, Progr. Theor. Phys. (Kyoto) **31**, 1073 (1964).

[15] J. J. Sakurai, Phys. Rev. **132**, 434 (1963).

[16] If one or both of U_μ and V_μ is the gauge particle as considered by the present author [Y. Fujii, Progr. Theoret. Phys. (Kyoto) **21**, 232 (1959); and Ref. 24], the mixing Lagrangian should have the form

$$-L=\tfrac{1}{2}f(\partial_\mu U_\nu-\partial_\nu U_\mu)(\partial_\mu V_\nu-\partial_\nu V_\mu). \tag{A}$$

Then, only the transverse components of U_μ and V_μ are mixed. See also the paper by G. Feldman and P. T. Matthews [Phys. Rev. **132**, 823 (1963)]. Following the technique developed in this paper and Ref. 23, we obtain the diagonalizing matrix given by

$$(m_\phi^2-m_\omega^2)^{-1/2}\begin{pmatrix}(m_\phi^2-m_1^2)^{1/2} & -(m_\phi^2-m_8^2)^{1/2}\\ (m_1^2-m_\omega^2)^{1/2} & (m_8^2-m_\omega^2)^{1/2}\end{pmatrix},$$

where m_1 and m_8 are the masses of pure unitary singlet and octet mesons, respectively. However, the numerical values

$$\begin{pmatrix}0.90 & -0.64\\ 0.43 & 0.77\end{pmatrix}$$

are not greatly different from the conventional orthogonal matrix with an angle $\approx 50°$. This is because the masses of two vector mesons are rather close to each other, and the energy dependence in (A) is not very important. In view of this fact we use the simple form (1.3) in the present paper.

[17] According to (1.4), the current of U_μ has a part $mf'\partial_\mu W$ which is not conserved. This implies that U_μ cannot be a gauge particle. Therefore, if we assume that the unitary singlet vector meson is a gauge particle associated with the strictly conserved baryon number, we must choose the second alternative, i.e., U_μ belongs to a unitary octet.

[18] One need not consider that (1.4) or (1.5) is a fundamental interaction in its literal sense. If there is any other interaction which violates C invariance, then (1.4) will emerge any way. Also (1.4) may be a result of a "spontaneous breakdown" of C invariance assumed for a starting theory. For such an idea, see the papers by G. Marx [Phys. Rev. Letters **14**, 334 (1965) and Ref. 10].

to (1.6),

$$U_\mu = \tilde{U}_\mu + (f/(1-f^2)^{1/2}) m^{-1}\partial_\mu W\,,\qquad W=(1-f^2)^{-1/2}\tilde{W}\,, \tag{1.8}$$

(1.7) can be put into the form

$$-L=\tilde{J}_\mu \tilde{U}_\mu + \tilde{J}\tilde{W}. \tag{1.9}$$

Here, the new source functions are given by

$$\tilde{J}_\mu = J_\mu^{(-)}\,,\qquad \tilde{J}=(1-f^2)^{-1/2}J^{(+)}-(f/(1-f^2)^{1/2})m^{-1}\partial_\mu J_\mu^{(-)}. \tag{1.10}$$

One thus finds that $\tilde{J}_\mu$ and accordingly $\tilde{U}_\mu$ have a definite C parity (odd), while $\tilde{J}$ and accordingly $\tilde{W}$ are mixtures of two parts with different C parities, if $J_\mu^{(-)}$ is not conserved. This gives an important feature of the present model, that C-violating effects occur only through the interaction of $\tilde{W}$, and only if $J_\mu^{(-)}$ is not conserved.[19]

The most probable decay mode of $\tilde{W}$ may be the decay into two pions with even C parity. According to the above argument $\tilde{W}$ should have other decay modes with odd C parity. The branching ratio is given essentially by $f^2(\approx\frac{1}{4})$, apart from the ratios of some strong interaction coupling constants and phase-space densities. It turns out, however, that only heavy-mass states (probably heavier than $\omega 2\pi$) contribute to the nonconserved part of $J_\mu^{(-)}$. If, therefore, the mass of $\tilde{W}$ is lighter than the sum of the masses of ω and 2π (1063 MeV), we cannot expect to observe the most direct result of our model. Careful analyses of some production processes of $\tilde{W}$ will be necessary. The C-violating effects will occur also in the processes involving a virtual $\tilde{W}$. Of such examples, the $K_2^0\to 2\pi$ decay (Sec. IV), the $\rho\eta\pi$ coupling constant (Sec. V), and the $\Sigma^0\to\Lambda e^+e^-$ decay (Sec. VI) will be calculated in the "one-$\tilde{W}$ approximation."

The calculations will be carried out by assuming simplified mechanisms for those processes, especially by assuming the existence of several kinds of scalar mesons. It turns out that a very crude estimate gives the $K_2^0\to 2\pi$ rate almost correctly. The small ratio $\Gamma(K_2^0\to 2\pi)/\Gamma(K_1^0\to 2\pi)$ is due to the fact that the K_1^0 decay rate is greatly enhanced by a scalar meson lying close to the K-meson mass (e.g., σ meson[12]). The calculated $\rho\eta\pi$ coupling constant depends on the various unknown coupling constants of the strong interaction. None the less we can still say that the recent estimate $g^2/4\pi\lesssim 10^{-1}$ from a number of available data[20] seems to be naturally expected from our model with a reasonable choice of the coupling constants.

The C-violating effect can be observed in the $\Sigma^0\to\Lambda e^+e^-$ decay.[5,21] We should observe a nonzero Λ polarization normal to the plane spanned by the momenta of electron and positron. It turns out that there is certainly a simple process to give such an effect, but unfortunately the kinematical factor is so small as to make the experimental test difficult.

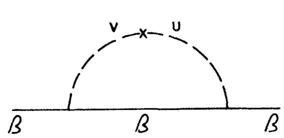

FIG. 1. The simplest diagram for the mass splitting among the octet baryons.

II. ESTIMATE OF f

The simplest estimate of the coupling constant f is obtained by assuming that (1.3) is the only interaction which causes the discrepancy between the actual mass levels of the vector mesons and those of the Gell-Mann–Okubo mass formula[15];

$$m^2 f=[(m_\phi^2-m_\omega^2)^2-(m_1^2-m_8^2)^2]^{1/2}, \tag{2.1}$$

where $m_\phi=(1020$ MeV) and $m_\omega(=780$ MeV) are the observed masses of ϕ and ω mesons, and m_8 is the mass of the isosinglet member of the octet expected from the simple-minded Gell-Mann–Okubo mass formula (930 MeV), and m_1 is the mass of a purely singlet vector meson, given by $(m_\phi^2+m_\omega^2-m_8^2)^{1/2}$. Taking m simply as the mass of the nucleon, we have

$$f=0.5. \tag{2.2}$$

Another estimate comes from the mass splitting among the octet baryons. The self-energy diagram in Fig. 1 gives

$$M_\Xi - M_N \approx \bar{M}(f/\pi)\sqrt{3}(g_1 g_8/4\pi)\,, \tag{2.3}$$

where g_1 and g_8 are the coupling constants for the singlet meson-baryon and octet meson-baryon interactions, respectively. We made an approximation by assuming a pure F-type coupling of the octet meson, correspond-

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[19] From a phenomenological point of view, one might start just from considering the "physical" particle $\tilde{W}$ and its source function $\tilde{J}$ given by the second equation of (1.10), avoiding a more restrictive, but heuristic discussion about the "bare" particle W. For the discussion of this point the author should thank Professor T. D. Lee.

[20] Assuming the process $\eta\to\pi^0+\rho^0\to\pi^0+\gamma\to\pi^0+e^++e^-$, we can calculate this decay rate in terms of $g_{\rho\eta\pi}$. If we further assume $\Gamma(\eta\to 2\gamma)=(m_\eta^3/3m_\pi^3)\Gamma(\pi^0\to 2\gamma)$ from $SU(3)$, and use the observed decay branching ratio of η, we have

$$r\equiv\Gamma(\eta\to\pi^0e^+e^-)/\Gamma(\eta\to\pi^+\pi^0\pi^-)\cong 0.066(g_{\rho\eta\pi}^2/4\pi).$$

(See footnote of Nauenberg's paper, Ref. 7.) The upper limit $r=0.7\%$ quoted in Ref. 11 gives $g_{\rho\eta\pi}^2/4\pi\lesssim 0.11$. The average $\pi^+\pi^-$ asymmetry in the $\eta\to\pi^+\pi^0\pi^-$ decay defined in Ref. 7 is related to r by

$$\Delta\cong\left(6.5\times10^{-9}\frac{g_{\rho\eta\pi}^2/4\pi}{\Gamma(\eta\to\pi^+\pi^0\pi^-)m_\pi^{-1}}\right)^{1/2}\approx 0.5\sqrt{r}.$$

The value $r=0.7\%$ gives $\Delta\cong 4.1\%$. See also Refs. 7 and 10. Also according to these papers, the absence of the decay $X^0\to\rho\pi\to 3\pi$ implies a severe upper limit to the C violating coupling constant $gX_{\rho\pi}^2/4\pi\lesssim 10^{-4}$. However, this coupling should vanish if we assume that X^0 is a pure unitary singlet (without mixing with η), and that the C violating interaction Lagrangian transforms like an isosinglet of an octet.

[21] S. Barshay, Phys. Letters 17, 78 (1965).

ing to the neglect of a small mass difference between Λ and Σ. $\bar{M}$ is the average of the baryon masses $[=\frac{1}{2}(M_N+M_\Xi)$ in the above approximation]. The integral was made finite by introducing a Feynman cutoff factor with the nucleon mass. Further assuming[22]

$$g_1{}^2/4\pi \approx g_8{}^2/4\pi \approx 2\,, \tag{2.4}$$

we have

$$f \approx 0.3\,. \tag{2.5}$$

This is consistent with the previous estimate (2.2).

In the remaining part of this section we shall prove that f', the coupling constant of the C-violating interaction (1.4), does not affect the first-order correction to the mass splitting. By denoting the strong and semistrong interaction Hamiltonians by $H(x)$ and $H'(x)$, respectively, the self-mass of, say, baryon B, is given by the sum of terms like

$$\mathfrak{M}_{nm} = \int dx_1 \cdots \int dx_{n+m} P\langle B|H(x_1)\cdots H(x_n) \times H'(x_{n+1})\cdots H'(x_{n+m})|B\rangle\,. \tag{2.6}$$

Now, the mass of a baryon B should be equal to that of an antibaryon $\bar{B}$, because of the CPT theorem. Therefore, (2.6) must be equal to the similar quantity,

$$\bar{\mathfrak{M}}_{nm} = \int dx_1 \cdots \int dx_{n+m} \times P\langle \bar{B}|H(x_1)\cdots H'(x_{n+m})|\bar{B}\rangle\,. \tag{2.7}$$

Using the charge-conjugation operator C, which commutes with the strong interaction Hamiltonian, we find that the self-mass is given by the sum of the following terms:

$$\tfrac{1}{2}(\mathfrak{M}_{nm}+\bar{\mathfrak{M}}_{nm}) = \frac{1}{2}\int dx_1 \cdots \int dx_{n+m} \times P\langle B|H(x_1)\cdots[H'(x_{n+1})+C^{-1}H'(x_{n+1})C]\cdots|B\rangle\,. \tag{2.8}$$

Then, to the first order in the semistrong interaction, the contribution comes only from the part which commutes with C.

III. MIXING BETWEEN U_μ AND W

The second terms of (1.5) gives rise to mixing between U_μ and W. In order to clarify the nature of this particular mixing, we consider the Lagrangian which consists of the free parts of U_μ and W with the masses m and n, respectively, and the interaction part

$$-L_{\text{mix}} = mfU_\mu \partial_\mu W\,. \tag{3.1}$$

[22] The magnitude of g_8 was obtained from the observed width of ρ, by assuming that ρ couples universally to the isotopic spin current.

The following equations of motion are derived from this Lagrangian:

$$\partial_\nu(\partial_\nu U_\mu - \partial_\mu U_\nu) - m^2 U_\mu + mf\partial_\mu W = 0\,, \tag{3.2}$$

$$(\Box - n^2)W - mf\partial_\mu U_\mu = 0\,. \tag{3.3}$$

The Green's functions (in the momentum representation) for these coupled equations are given by

$$\begin{aligned} G_{\mu\nu} &= \left(\delta_{\mu\nu}+\frac{k_\mu k_\nu}{m^2}\right)\frac{1}{k^2+m^2}+\frac{k_\mu k_\nu}{m^2}\frac{f^2}{1-f^2}\frac{1}{k^2+\tilde{n}^2}\,,\\ G_{\mu W} &= -G_{W\mu} = i\frac{k_\mu}{m}\frac{f}{1-f^2}\frac{1}{k^2+\tilde{n}^2}\,,\\ G_{WW} &= (1-f^2)^{-1}(k^2+\tilde{n}^2)^{-1}\,, \end{aligned} \tag{3.4}$$

where $\tilde{n}$ is defined by

$$\tilde{n}^2 = n^2(1-f^2)^{-1}\,. \tag{3.5}$$

This change of the mass of W field can be seen most easily in the following way: Differentiating (3.2) with respect to x_μ, we have

$$\partial_\mu U_\mu = (f/m)\Box W\,, \tag{3.6}$$

which can be substituted into (3.3), to give

$$(\Box - n^2)W - f^2\Box W = 0\,, \tag{3.7}$$

or by dividing by $1-f^2$,

$$(\Box - \tilde{n}^2)W = 0\,. \tag{3.8}$$

The mass m of U_μ, remains unchanged because only the longitudinal part of U_μ couples to W through L_{mix}, on the one hand, and the "observed" mass of U_μ is essentially that of the transverse part, on the other hand.

We introduce the mixed fields $\tilde{U}_\mu$ and $\tilde{W}$ by[23]

$$\begin{aligned} \tilde{U}_\mu &= c_{11}U_\mu + c_{12}\partial_\mu W\,,\\ \tilde{W} &= c_{21}\partial_\mu U_\mu + c_{22}W\,, \end{aligned} \tag{3.9}$$

where the c_{ij} are to be chosen so that the Green's functions for these fields have diagonalized forms:

$$\begin{aligned} \tilde{G}_{\mu\nu} &= (\delta_{\mu\nu}+k_\mu k_\nu/m^2)(k^2+m^2)^{-1}\,,\\ \tilde{G}_{\mu W} &= \tilde{G}_{W\mu} = 0\,,\\ \tilde{G}_{WW} &= (k^2+\tilde{n}^2)^{-1}\,. \end{aligned} \tag{3.10}$$

The simplest way to get c_{ij} is to introduce a_{ij}, which form the inverse matrix to c_{ij}, and to express the G's in terms of the $\tilde{G}$'s:

$$\begin{aligned} G_{\mu\nu} &= a_{11}{}^2\tilde{G}_{\mu\nu} + a_{12}{}^2 k_\mu k_\nu \tilde{G}_{WW}\,,\\ G_{\mu W} &= -ik_\nu a_{11}a_{21}\tilde{G}_{\mu\nu} + a_{12}a_{22}ik_\mu \tilde{G}_{WW}\,,\\ G_{WW} &= a_{21}{}^2 k_\mu k_\nu \tilde{G}_{\mu\nu} + a_{22}{}^2\tilde{G}_{WW}\,. \end{aligned} \tag{3.11}$$

Substituting from (3.10) and comparing the result with

[23] T. Kaneko, Y. Ohnuki, and K. Watanabe, Progr. Theoret. Phys. (Kyoto) 30, 521 (1963).

(3.4), we have

$$a_{ij}=\begin{pmatrix}1 & (f/(1-f^2)^{1/2})m^{-1}\\ 0 & (1-f^2)^{-1/2}\end{pmatrix}, \tag{3.12}$$

or

$$c_{ij}=\begin{pmatrix}1 & -f/m\\ 0 & (1-f^2)^{1/2}\end{pmatrix}, \tag{3.13}$$

which are equivalent to (1.8) and (1.6), respectively. Neither the matrix c_{ij} nor a_{ij} is orthogonal and they cannot be given by an "angle" as in a simpler case.[15] Still the nonvanishing off-diagonal element $f/(1-f^2)^{1/2}$ or $-f$ can be quite big according to our previous estimate $f\approx\frac{1}{2}$. Therefore large effects can be generally expected in the decay or production processes of U_μ and W.

We define the source function $J_\mu^{(-)}$, to which U_μ couples, and which has an odd C parity. [See Eq. (1.7).] The isosinglet 3π state is the lightest state contributing to $J_\mu^{(-)}$. Likewise we define $J^{(+)}$, to which W couples, and which has an even C parity. The isosinglet 2π state is the lightest contributing state. The new source functions given by

$$\begin{aligned}\tilde{J}_\mu&=a_{11}J_\mu^{(-)}-a_{21}\partial_\mu J^{(+)},\\ \tilde{J}&=-a_{12}\partial_\mu J_\mu^{(-)}+a_{22}J^{(+)},\end{aligned} \tag{3.14}$$

couple to the fields $\tilde{U}_\mu$ and $\tilde{W}$, respectively. [Explicit forms of (3.14) are given by (1.10).] One finds that, because $a_{21}=0$, the particle $\tilde{U}_\mu$ decays only to states with odd C parity (corresponding to $J_\mu^{(-)}$), while $\tilde{W}$ decays into states with even C parity, as well as states with odd C parity, if $J_\mu^{(-)}$ is not conserved. The branching ratio is a product of f^2 and the ratio of the strong-interaction coupling constants involved in $J_\mu^{(-)}$ and $J^{(+)}$, and the phase-space densities. As will be seen from the later discussion, this will be the only observable effect in which the coupling constant $f^2(\approx\frac{1}{4})$ appears without any small multiplying factor [e.g., $(4\pi)^{-1}$].

Now the problem is to find the neutral current $J_\mu^{(-)}$ which is not conserved. We are mainly interested in the part consisting of a number of mesons. The 2π current is the same as the ordinary electromagnetic current of a pion, and has $I=1$. Therefore the transition through this current will be suppressed due to the selection rule $|\Delta I|=1$. Moreover this current is exactly conserved on the mass shells. The 3π current

$$J_\mu^{(-)}=(F_{3\pi}/m^3)\epsilon_{\mu\nu\lambda\sigma}\partial_\nu\pi^+\partial_\lambda\pi^0\partial_\sigma\pi^- \tag{3.15}$$

is obviously conserved, simply as a consequence of the Lorentz invariance. There are some nonconserved 4π currents, but they will be neglected because of the $|\Delta I|=1$ rule. The 5π current is also conserved if there is no final-state interaction.[24] A special version of the 5π current which is not conserved is interesting:

$$J_\mu^{(-)}=F_{\omega 2\pi}\omega_\mu\boldsymbol{\pi}^2+mF_{\rho\xi}\boldsymbol{\rho}_\mu\cdot\boldsymbol{\xi}, \tag{3.16}$$

where ω_μ and $\boldsymbol{\rho}_\mu$ denote the wave functions of ω and ρ mesons, respectively, and $\boldsymbol{\xi}$ stands for an isotriplet scalar meson[14] (its predicted mass is around 1 BeV). From the above considerations one finds that it will be extremely difficult to observe the abnormal decay of $\tilde{W}$, unless its mass is much heavier than the sum of the masses of ω and 2π (1063 MeV).[25] This is not the case if $\tilde{W}$ is identified with ϵ (isosinglet scalar meson of mass ≈ 760 MeV).[13]

It is well known that charge independence or $SU(3)$ symmetry gives a severe restriction on the types of strong-interaction phenomena in which C-violating effects occur.[26] Here we have discovered that the spectrum of the states contributing to a nonconserved part of $J_\mu^{(-)}$ begins at a relatively high mass. This seems to provide another reason why the possible effects of C violation might be deeply hidden from most of the world which we know only through relatively low-energy phenomena. If this spectrum does begin at a relatively high mass, making C-violating effects relatively unimportant in decays, one could search for some production processes involving $\tilde{W}$. As an example, the $p\bar{p}$ annihilation process in which $\tilde{W}$ would be produced together with two more pions. The annihilation from $^3S_1$ $p\bar{p}$ state would go mainly through ω or ϕ, and ρ mesons, as shown in Fig. 2. In process (a), the cross section is proportional to $f^2F^2_{\omega 2\pi}$, and the two pions produced together with $\tilde{W}$ are in an isosinglet state ($2\pi^0$ is possible). The process (b) is a "normal" one coming from a part of $J^{(+)}$, and the two pions are in an isotriplet state (no $2\pi^0$). Therefore we must expect a number of $\tilde{W}2\pi^0$ events comparable with the $\tilde{W}\pi^+\pi^-$ events.

In the following sections we shall consider the proc-

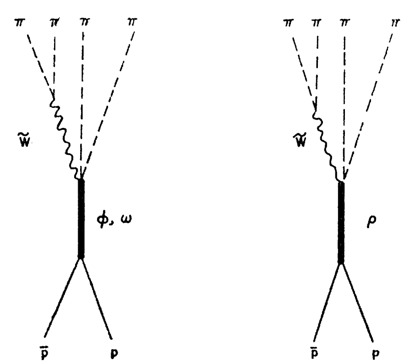

FIG. 2. $p\bar{p}$ annihilation with production of $\tilde{W}$ and two pions.

[24] See also Sec. IV of the paper; Y. Fujii, Phys. Rev. **138**, 423 (1965).

[25] This abnormal decay is equivalent to the C-invariant decay of an isosinglet scalar meson with $C=-1$ (0^{+-}). The decay into $(2\pi)_{I=0}\gamma$ suggested by V. G. Grishin and G. I. Kopylov [Nuovo Cimento **37**, 962 (1965)] is forbidden if γ is real. Only the decay into $(2\pi)_{I=0}e^+e^-$ is allowed.

[26] See, for example, N. Cabibbo, Phys. Rev. Letters **14**, 965 (1965).

esses involving virtual $\tilde{W}$ mesons. In the "one-$\tilde{W}$ approximation" the C violation should come from the product of the "normal" and "abnormal" terms of $\tilde{J}$ in (3.14), i.e., $-a_{12}a_{22}J^{(+)}\partial_\mu J_\mu^{(-)}$. Using the explicit form (1.10), the C-violating term in the S-matrix elements is given by the sum of terms like

$$-i\frac{1}{m}\frac{f}{1-f^2}\frac{(-i)^n}{n!}\int dx_1\cdots\int dx_n \times\Delta_F(x_1-x_2;\tilde{n}^2)P\langle J^{(+)}(x_1)\partial_\mu J_\mu^{(-)}(x_2) \times H(x_3)\cdots H(x_n)\rangle. \quad (3.17)$$

Here $\Delta_F(x_1-x_2;\tilde{n}^2)$ is the propagator of $\tilde{W}$ with the mass $\tilde{n}$, and $H(x)$ denotes the Hamiltonian of interactions not involving $\tilde{W}$.

So far we have simplified the discussion by omitting the mixing interaction between two vector mesons. The whole discussion, however, can be easily generalized to include (1.3). The sole effect of (1.3) is well known.[15] The sum of the free Lagrangians of U_μ and V_μ and the mixing term (1.3) can be diagonalized by introducing the fields ω_μ and ϕ_μ defined by

$$\begin{aligned}\omega_\mu&=U_\mu\cos\lambda-V_\mu\sin\lambda,\\ \phi_\mu&=U_\mu\sin\lambda+V_\mu\cos\lambda,\end{aligned} \quad (3.18)$$

where λ is given by

$$\lambda=\tfrac{1}{2}\tan^{-1}[2m^2f/(m_V^2-m_U^2)], \quad (3.19)$$

with m_U, m_V being the masses of U_μ and V_μ, respectively. Now the straightforward calculation shows that the sum of the Lagrangians of U_μ, V_μ and W and the interaction Lagrangian (1.5) can be diagonalized in terms of the fields given by

$$\begin{aligned}\tilde{\omega}_\mu&=\omega_\mu-f\cos\lambda m^{-1}\partial_\mu W,\\ \tilde{\phi}_\mu&=\phi_\mu-f\sin\lambda m^{-1}\partial_\mu W.\end{aligned} \quad (3.20)$$

The masses of $\tilde{\omega}_\mu$ and $\tilde{\phi}_\mu$ remain the same as those of ω_μ and ϕ_μ, respectively, and the mass $\tilde{n}$ of $\tilde{W}$ is still given by (3.5). The source functions for the mixed fields are given by

$$\begin{aligned}\tilde{J}_\mu^{(\omega)}&=J_\mu^{(\omega)},\\ \tilde{J}_\mu^{(\phi)}&=J_\mu^{(\phi)},\\ \tilde{J}&=(1-f^2)^{-1/2}J^{(+)} -(f/(1-f^2)^{1/2})m^{-1}\partial_\mu J_\mu^{(-)(U)}.\end{aligned} \quad (3.21)$$

Therefore, there is no essential change in the conclusions about C violation by taking V_μ into account.

IV. $K_2^0\to 2\pi$ DECAY

In this section we shall present a simplified discussion of the observed C-violating transition $K_2^0\to 2\pi$, to show how it is possible to explain its characteristics in terms of the suggested basic interaction (1.5).

For simplicity we shall calculate only the mass operator of the K_1^0, K_2^0 system according to the discussion in Ref. 2. This will be justified by smallness of the obtained result. The off-diagonal element of the matrix mass operator is given by

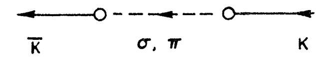

FIG. 3. The transition $K\rightleftarrows\bar{K}$.

$$M_{\bar{K}K}=M_{K\bar{K}}^*=\langle\bar{K}^0|\mathcal{H}|K^0\rangle +\sum_n P\frac{\langle\bar{K}^0|\mathcal{H}|n\rangle\langle n|\mathcal{H}|K^0\rangle}{m_n-m_K}+\cdots, \quad (4.1)$$

where $\mathcal{H}$ is the weak-interaction Hamiltonian including the modification due to C violation. The intermediate state n has zero strangeness. The simplest such states are the one-scalar-meson state (called σ, $I=0$, 0^{++}) and one-pion state as shown in Fig. 3. The relevant weak-interaction Lagrangian is written in the following form;

$$-L=m^2[(ih_\sigma\sigma+h_\pi\pi^0)K^0+(ih_\sigma^*\sigma-h_\pi^*\pi^0)\bar{K}^0 -\sqrt{2}(h_\pi\pi^-K^+-h_\pi^*\pi^+K^-)]. \quad (4.2)$$

This is not invariant under C and P separately, but is invariant under CP if h_σ and h_π are real. The part containing charged mesons is added only to show explicitly the validity of the $|\Delta I|=\frac{1}{2}$ rule. The magnitudes of h_σ can be determined if we assume that the σ meson has the mass $\sim$400 MeV,[12] and that the $K_1^0\to 2\pi$ decay is dominated by a process $K_1^0\to\sigma\to 2\pi$. The decay rate Γ_1 of K_1^0 is given by

$$\Gamma_1=3\frac{g_{\pi\sigma}^2}{4\pi}|h_\sigma|^2\left(\frac{m_\sigma}{m_K}\right)^2 \times(m_K^2-4m_\pi^2)^{1/2}\left(\frac{m^2}{m_K^2-m_\sigma^2}\right)^2, \quad (4.3)$$

where $g_{\pi\sigma}$ is the coupling constant of σ to 2π.[27] The magnitude of h_π could then be determined from the mass difference between K_1^0 and K_2^0, but in view of the uncertainty of its sign we shall leave h_π undetermined.

If CPT and CP invariances are valid, $M_{\bar{K}K}$ is real and is given by

$$M_{\bar{K}K}=\tfrac{1}{2}(m_1-m_2), \quad (4.4)$$

where m_1 and m_2 are the masses of K_1^0 and K_2^0, respectively. Owing to small CP violation, $M_{\bar{K}K}$ gets an imaginary part. Neglecting the higher order terms in CP violation, we have the following formula for the $K_2^0\to 2\pi$ decay:

$$|\epsilon|\equiv\left(\frac{\Gamma(K_2^0\to 2\pi)}{\Gamma(K_1^0\to 2\pi)}\right)^{1/2} \approx\frac{|\mathrm{Im}M_{\bar{K}K}|}{[(m_1-m_2)^2+\frac{1}{4}(\Gamma_1-\Gamma_2)^2]^{1/2}}. \quad (4.5)$$

(See Secs. II and III of Ref. 2.)

[27] The interaction Hamiltonian is given by $H=\frac{1}{2}m_\sigma g_{\pi\sigma}\pi^2\sigma$. The width $\sim$95 MeV (Ref. 12) corresponds to $g_{\pi\sigma}^2/4\pi\approx 0.9$.

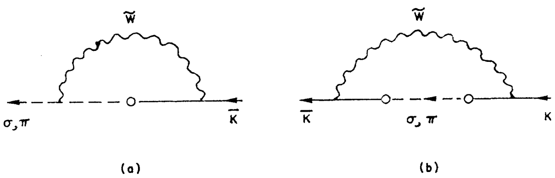

FIG. 4. (a) Corrections to $h_\sigma^{(0)}$ and $h_\pi^{(0)}$, (b) Irreducible correction to $K \to \bar{K}$ transition. Weak vertices $h_\sigma^{(0)}$ $h_\pi^{(0)}$ are indicated by O.

In the one-$\tilde{W}$ approximation, there are two kinds of corrections to $M_{\bar{K}K}$. One is the correction through the correction to $h_\alpha(\alpha=\sigma,\pi)$, as shown in Fig. 4(a). The other is the "irreducible" correction shown in Fig. 4(b).[28] We are considering the part of $J_\mu^{(-)}$ and $J^{(+)}$ given by

$$\begin{aligned} J_\mu^{(-)} &= iF_{KK}[\bar{K}\partial_\mu K-(\partial_\mu\bar{K})K], \\ J^{(+)} &= \tfrac{1}{2}mG_{\pi\pi}\boldsymbol{\pi}^2+\tfrac{1}{2}mG_{\sigma\sigma}\sigma^2+mG_{KK}\bar{K}K. \end{aligned} \tag{4.6}$$

Substituting (4.2) with $h_\alpha=h_\alpha^{(0)}=$real (the unperturbed coupling constants) and (4.6) into (3.17), and using a Feynman cutoff with nucleon mass, the correction to h_α is calculated:

$$\delta h_\alpha \approx ih_\alpha^{(0)}\frac{1}{12\sqrt{3}}\frac{f}{1-f^2}\frac{F_{KK}G_{\alpha\alpha}}{4\pi}. \tag{4.7}$$

The integral in the irreducible diagram is convergent. These add up to give

$$\delta M_{\bar{K}K}=i\,\mathrm{Im}M_{\bar{K}K}\approx -i\frac{1}{12\sqrt{3}}\frac{f}{1-f^2}\frac{F_{KK}}{4\pi}\frac{m^2}{m_K}$$
$$\times\left[[h_\sigma^{(0)}]^2\left(\frac{m^2}{m_K^2-m_\sigma^2}G_{\sigma\sigma}+G_{KK}\right)\right.$$
$$\left.-[h_\pi^{(0)}]^2\left(\frac{m^2}{m_K^2-m_\pi^2}G_{\pi\pi}+G_{KK}\right)\right]. \tag{4.8}$$

Note that the σ and π states contribute with different signs. This comes from the difference in phases (i) of the coupling constants in (4.2).

In order to get a simple numerical estimate we tentatively take $[h_\sigma^{(0)}]^2\approx[h_\pi^{(0)}]^2$, and set all the strong-interaction coupling constants F and G equal, and take $m=M$. We have then

$$\mathrm{Im}M_{\bar{K}K}\approx 0.53(f/(1-f^2))(G^2/4\pi)[h_\sigma^{(0)}]^2(\mathrm{BeV}). \tag{4.9}$$

On the other hand, the denominator of (4.5) is nearly equal to

$$\Gamma_1/\sqrt{2}\approx 0.77(g_{\pi\sigma}^2/4\pi)[h_\sigma^{(0)}]^2\times 10^2(\mathrm{BeV}), \tag{4.10}$$

where use was made of (4.3). Substituting (4.9) and (4.10) into (4.5), we have

$$|\epsilon|\approx 0.96(f/(1-f^2))(G^2/g_{\pi\sigma}^2)\times 10^{-2}, \tag{4.11}$$

or by taking $f\approx\frac{1}{2}$ and $G^2\approx g_{\pi\sigma}^2$,

$$|\epsilon|\approx 6\times 10^{-3}. \tag{4.12}$$

This is to be compared with the observed value 2.2×10^{-3}.[3] The smallness of the result comes mainly from the fact that Γ_1 in (4.10) is "enhanced" because m_σ is close to m_k in (4.3).[29] Although the above estimate is a very crude one, one may expect that there is certainly a dynamical explanation for a small $K_2^0\to 2\pi$ decay in terms of the present model.

V. $\rho\eta\pi$ COUPLING CONSTANT

As emphasized in Sec. I, the $\rho\eta\pi$ interaction (1.1) is very important in a theoretical analysis of C-violating effects in semistrong interactions. In this section we shall derive this interaction from our basic interaction. The simplest diagrams are those shown in Fig. 5(a) and (b). Here we assumed the existence of an isotriplet scalar meson ξ,[14] with the strong interaction

$$-L=mg_{\xi\eta\pi}(\boldsymbol{\pi}\cdot\boldsymbol{\xi})\eta. \tag{5.1}$$

The relevant parts of $J_\mu^{(-)}$ an $J^{(+)}$ are

$$\begin{aligned} J_\mu^{(-)} &= mF_{\rho\xi}(\boldsymbol{\xi}\cdot\boldsymbol{\rho}_\mu), \\ J^{(+)} &= \tfrac{1}{2}mG_{\pi\pi}\boldsymbol{\pi}^2+\tfrac{1}{2}mG_{\eta\eta}\eta^2. \end{aligned} \tag{5.2}$$

The integrals are fortunately convergent. One obtains the effective $\rho\eta\pi$ coupling constant defined by (1.1):

$$g\approx(24\pi^2)^{-1}(f/(1-f^2))F_{\rho\xi}g_{\xi\eta\pi}(G_{\pi\pi}+G_{\eta\eta}). \tag{5.3}$$

The result depends critically on the magnitudes of the coupling constants $g_{\xi\eta\pi}$, $F_{\rho\xi}$, $G_{\pi\pi}$ and $G_{\eta\eta}$, about which

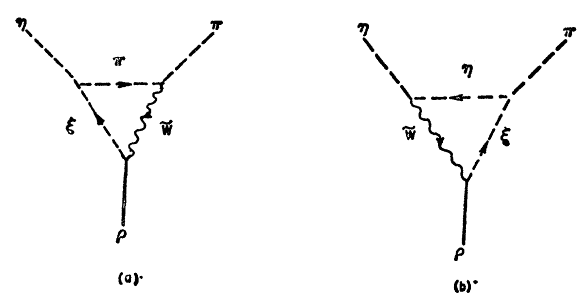

FIG. 5. Diagrams for $\rho\eta\pi$ coupling.

[28] The following calculation can also be made by first eliminating the "two-body interaction" (4.2) by the suitable diagonalizations of K_1^0 and σ, and of K_2^0 and π. The source functions of $\tilde{W}$ given by (4.6) change correspondingly. The results, however, turn out to be the same as those in the text, if we expand them with respect to the weak coupling constants (h's), and keep only the second-order terms.

[29] The result changes only little if we change the mass and width of the scalar meson to 490 and 110 MeV, respectively, according to a more recent analysis by P. G. Thurnauer [Phys. Rev. Letters 14, 985 (1965)]. The only essential fact is that there is a strong S-wave final-state interaction in the isosinglet 2π system.

nothing is known. However, $G_{\pi\pi}{}^2/4\pi$ cannot be much larger than unity, otherwise $\tilde{W}$ with the assumed mass about nucleon mass would have an extremely large width (>340 MeV). Also these coupling constants should be compared with $g_{\rho\pi\pi}{}^2/4\pi\approx 2$, which is the only known coupling constant of the three-boson interaction. Thus, by assuming simply that all the coupling constants in (5.3) are equal to G, with

$$G^2/4\pi = 2-1\,, \tag{5.4}$$

we have a very crude estimate:

$$\frac{g^2}{4\pi}=\frac{1}{9\pi^2}\left(\frac{f}{1-f^2}\right)^2\left(\frac{G^2}{4\pi}\right)^3\approx 4\times(10^{-2}-10^{-3})\,, \tag{5.5}$$

for $f=\frac{1}{2}$. It seems unlikely that a larger value of g can be obtained by considering more complicated processes. It should also be noted that the upper limit quoted in (5.5) is very close to

$$\frac{1}{4\pi}\left(\frac{f}{1-f^2}\right)^2\approx 3\times 10^{-2}\,,\quad (f=\tfrac{1}{2}) \tag{5.6}$$

which may be considered as a "natural" maximum for $g^2/4\pi$ in our model, being independent of the detailed nature of the process. The most recent estimate from the analysis of the decays $\eta\rightarrow(3\pi)_{I=0}$ and $\eta\rightarrow\pi^0e^+e^-$ gives $g^2/4\pi\lesssim 10^{-1}$.[20] An even smaller value seems to be consistent with the present model, in which an equal mixture of C-invariant and -noninvariant interactions is assumed for the basic semistrong interaction.

VI. THE $\Sigma^0\rightarrow\Lambda e^+e^-$ DECAY

In this section we shall consider the process $\Sigma^0\rightarrow\Lambda+e^++e^-$, which has been suggested as a test of C violation.[5,21] It seems reasonable to assume that this decay occurs mainly through a process involving a virtual ρ meson, as illustrated in Fig. 6. The interaction Lagrangian for $\Sigma\Lambda\rho$ coupling is given by

$$-L=if_1\bar{\Lambda}\gamma_\mu\mathbf{\Sigma}\cdot\boldsymbol{\varrho}_\mu+f_2\bar{\Lambda}\sigma_{\mu\nu}\mathbf{\Sigma}\cdot(\partial_\mu\boldsymbol{\varrho}_\nu-\partial_\nu\boldsymbol{\varrho}_\mu)+f_3\bar{\Lambda}\mathbf{\Sigma}\cdot\partial_\mu\boldsymbol{\varrho}_\mu+\text{H.c.} \tag{6.1}$$

The coupling constants f_1, f_2, f_3 are real if we assume C invariance in (6.1). The effective interaction Lagrangian between ρ^0 and photon can be written as follows:

$$\begin{aligned}-L&=ef_4(\partial_\mu\rho_\nu{}^0-\partial_\nu\rho_\mu{}^0)(\partial_\mu A_\nu-\partial_\nu A_\mu)\\&=-2ef_4\rho_\mu{}^0 j_\mu\,.\end{aligned} \tag{6.2}$$

Here f_4 is related to $\langle r^2\rangle$, the squared radius of the isovector electromagnetic form factors by

$$f_4=(m_\rho{}^2/24g_{\rho NN})\langle r^2\rangle\,. \tag{6.3}$$

To obtain the second line of (6.2) we performed a partial integration and used the equation of motion for photon

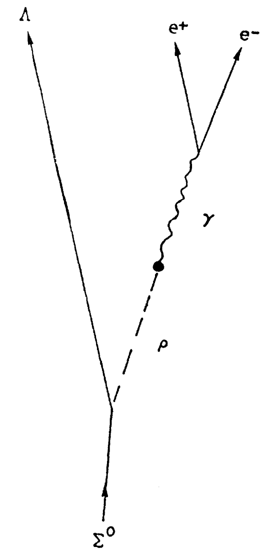

FIG. 6. Diagram for $\Sigma^0\rightarrow\Lambda e^+e^-$.

field A_μ;

$$\partial_\nu(\partial_\nu A_\mu-\partial_\mu A_\nu)=j_\mu\,. \tag{6.4}$$

As easily seen in the first line of (6.2), only the transverse component of $\rho_\mu{}^0$ has an effective coupling, so that the term in f_3 in (6.1) does not contribute to the process in question.

From the interactions (6.1) and (6.2) we can calculate the differential decay distribution for which the initial Σ^0 is at rest and unpolarized, the final Λ has a polarization $\boldsymbol{\sigma}_\Lambda$, and the positron and electron have the momentum $\mathbf{q}_\pm$, and the energy $\epsilon_\pm$, respectively, in the rest frame of Σ^0. The calculated decay distribution is proportional to

$$\begin{aligned}f_4{}^2\{&-2|f_1|^2(\mathbf{q}_+\cdot\mathbf{q}_-+\epsilon_+\epsilon_-)+4|f_2|^2(\mathbf{q}_++\mathbf{q}_-)^2(q_+q_-)\\&-2\,\mathrm{Im}(f_1f_2{}^*)((\epsilon_+{}^2-\epsilon_-{}^2)/M_\Lambda)[(\mathbf{q}_+\times\mathbf{q}_-)\cdot\boldsymbol{\sigma}_\Lambda]\}\,,\end{aligned} \tag{6.5}$$

where, in each of the three terms in (6.5), we neglected the higher order terms in $M_\Lambda{}^{-1}$. The last term giving the Λ polarization perpendicular to $\mathbf{q}_+$ and $\mathbf{q}_-$ is a typical result of the violation of time reversal invariance, and also violates C invariance in this decay. Because the second term in (6.5) can be neglected compared to the first term, the ratio of the C-violating term to the C-invariant term is given by a ratio R,

$$R=\frac{\mathrm{Im}(f_1f_2{}^*)}{|f_1|^2}\,\frac{\epsilon_+{}^2-\epsilon_-{}^2}{M_\Lambda}\,. \tag{6.6}$$

Now we shall calculate the imaginary parts of f_1 and f_2 in our model. The simplest diagram is shown in Fig. 7(a). The relevant parts of $J_\mu{}^{(-)}$ and $J^{(+)}$ are given by

$$\begin{aligned}J_\mu{}^{(-)}&=F_{\Sigma\Sigma}\bar{\Sigma}i\gamma_\mu\Sigma+F_{\Lambda\Lambda}\bar{\Lambda}i\gamma_\mu\Lambda\,,\\J^{(+)}&=G_{\Sigma\Sigma}\bar{\Sigma}\Sigma+G_{\Lambda\Lambda}\bar{\Lambda}\Lambda\,.\end{aligned} \tag{6.7}$$

The integrals are logarithmically divergent. By using Feynman cutoff factors with nucleon mass and neglecting the Σ-Λ mass difference, we have

$$f_\alpha \approx f_\alpha^{(0)}\left(1 - i\frac{1}{4\pi}\frac{f}{1-f^2}\frac{F_{\Sigma\Sigma}G_{\Lambda\Lambda}-F_{\Lambda\Lambda}G_{\Sigma\Sigma}}{4\pi}\right), \quad (\alpha=1,2) \qquad (6.8)$$

where $f_\alpha^{(0)}$ is the unperturbed real coupling constant.[30] Note that both of f_1 and f_2 get the same phase factor, to give no contribution to the interference term $\mathrm{Im} f_1 f_2^*$.

The next simplest processes are the ones shown in Fig. 7(b) and (c), where the $\rho\eta\pi$ coupling is effective. As easily found, these contributions are very similar to the 2π part of the isovector electromagnetic form factors of the nucleon. The only essential difference is the appearance of $\frac{1}{2}g$ in (1.1) in place of ie in the pion electromagnetic current. Neglecting the mass difference between π and η, and Σ and Λ in the intermediate states and assuming $SU(3)$ symmetry for the coupling constants,

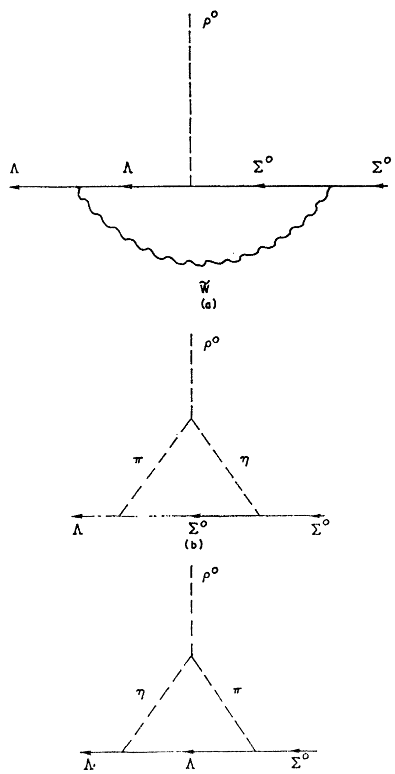

FIG. 7. Corrections to $\Sigma\Lambda\rho$ coupling.

we obtain the contributions of δf_1 and δf_2 given by

$$\delta f_\alpha \approx -i\tfrac{2}{3}gF_\alpha^{(V,2\pi)}(0), \qquad (6.9)$$

where $F_\alpha^{(V,2\pi)}(0)$ is the 2π part of the isovector electromagnetic form factor at zero momentum transfer:

$$F_1^{(V,2\pi)}(0) = \tfrac{1}{2}\alpha, \qquad F_2^{(V,2\pi)}(0) \approx \mu_V/2M = 1.85/2M. \qquad (6.10)$$

Here $1-\alpha$ represents the "core part" of $F_1^{(V)}(q^2)$, and μ_V is the half of the difference between proton and neutron anomalous magnetic moments.[31] We add (6.9) to the real parts of f's, $f_1^{(0)}$ and $f_2^{(0)}$. Further assuming $f_2^{(0)} \approx (\mu_V/M) f_1^{(0)}$, we have

$$f_1 \approx f_1^{(0)} - i\tfrac{1}{3}g\alpha, \qquad f_2 \approx (\mu_V/M)(f_1^{(0)} - i\tfrac{1}{3}g), \qquad (6.11)$$

and finally

$$\frac{\mathrm{Im}(f_1 f_2^*)}{|f_1|^2} \approx -\frac{1}{3}\frac{g}{f_1^{(0)}}\frac{\mu_V}{M}(1-\alpha). \qquad (6.12)$$

The presently available data do not show any significant core term $1-\alpha$ in the isovector form factor of the nucleon.[32] This may suppress the value of (6.12). Furthermore the remaining factor $(\epsilon_+^2 - \epsilon_-^2)/M_\Lambda$ in (6.6) is very small. The maximum of this factor is given by

$$\frac{M_\Sigma + M_\Lambda}{2M_\Sigma M_\Lambda}(M_\Sigma - M_\Lambda)^2 \cong 0.5\times 10^{-2} M. \qquad (6.13)$$

Therefore, by using the estimate of g in (5.5) and $f_1^{(0)} \approx (2\times 4\pi)^{1/2} \approx 5$, we have a rather small upper limit of R given by

$$|R| \lesssim 2.2\times(1-\alpha)\times 10^{-3}. \qquad (6.14)$$

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[30] If W is a unitary singlet and U_μ belongs to an octet, we have $F_{\Lambda\Lambda} = -F_{\Sigma\Sigma} \equiv -F$, $G_{\Lambda\Lambda} = G_{\Sigma\Sigma} \equiv G$, which give $F_{\Sigma\Sigma}G_{\Lambda\Lambda} - F_{\Lambda\Lambda}G_{\Sigma\Sigma} = 2FG$. This result remains almost the same in the alternative assignment of U_μ and W.

[31] Gauge invariance requires that $F_1^{(V)}(0)$ is equal to $\frac{1}{2}$ whatever the contribution $F_1^{(V,2\pi)}(0)$ from a 2π state may be. The simplest estimate of α is given by the formula

$$F_1^{(V)}(q^2) = \tfrac{1}{2}[1 - \alpha + (\alpha m_\rho^2/(q^2 + m_\rho^2))].$$

On the other hand, there is no such restriction for $F_2^{(V)}(0)$ $(=\mu_V/2M)$, and to a good approximation we may assume that most of μ_V comes from the 2π contribution.

[32] See, for example, R. R. Wilson and J. S. Levinger, Ann. Rev. Nucl. Sci. 14, 135 (1964).