$SU(6)$ and Deviations from the $\Delta I=\frac{1}{2}$ Rule in the Nonleptonic Hyperon Decays*

MAHIKO SUZUKI†
National Science Foundation, Summer Institute for Theoretical Physics, University of Wisconsin, Madison, Wisconsin
(Received 30 July 1965)

Deviations from the $\Delta I=\frac{1}{2}$ rule in the nonleptonic hyperon decays are investigated within the framework of $SU(6)$ on the assumption that the weak interaction should be of the current $\times$ current type and CP-invariant. A remarkable result is obtained for parity-violating amplitudes: The deviation turns out to exist only in the Σ decay, not in the Λ nor Ξ decay. Experimental data are in good agreement with this prediction.

THE eightfold way[1,2] predicts a relation[3] among small deviations from the $\Delta I=\frac{1}{2}$ rule of the nonleptonic hyperon decays

$$S(\Lambda_-^0)+\sqrt{2}S(\Lambda_0^0)=-\{S(\Xi_-^-)+\sqrt{2}S(\Xi_0^0)\} \quad (1)$$

on the following assumptions:

(1) The strong interactions are approximately invariant under $SU(3)$, and symmetry-breaking interactions are characterized by T_3^3.

(2) The weak interactions are CP-invariant and arise from bilinear self-interactions of unitary-spin currents.[4,5] The above relation was proved to hold only for parity-violating amplitudes up to T_3^3 corrections to the 27-plet spurion.

Recently, the spin and unitary-spin independence $SU(6)$[6,7] was proposed for the strongly interacting particles. This has achieved a great success in predicting the static properties[8,9] including the parity-violating (s wave) amplitudes of the nonleptonic hyperon decays.[10–12] We report here the consequences of $SU(6)$ in the deviations from the $\Delta I=\frac{1}{2}$ rule of the nonleptonic decays on the corresponding assumptions. The starting assumptions are therefore:

(1) The strong interactions are approximately invariant under $SU(6)$ and symmetry-breaking interactions are characterized by T_3^3 and all other kinds of interactions preserving charge independence.

(2) The weak interactions are CP-invariant and arise from bilinear self-interactions of the $SU(6)$ currents belonging to the 35-dimensional representation.

We shall neglect possible corrections to the $\Delta I=\frac{3}{2}$ amplitudes coming from various symmetry-breaking effects, though the corrections to the $\Delta I=\frac{1}{2}$ amplitudes coming from $SU(6)$ violations preserving charge independence are entirely taken into account. Since, by assumptions, the spurion arises from the symmetric products of the two currents belonging to the identical 35-plet, it must have the transformation property of a 35-, 189-, or 405-dimensional tensor. It is (27, 1)'s in the 189-plet and the 405-plet that contribute to the $\Delta I=\frac{3}{2}$ amplitudes.

Just as in the previous case,[3] the assumptions of the current$\times$current interactions and of the CP invariance impose strong restrictions on the parity-violating amplitude to lead to sum rules among them, but are less restrictive in the parity-conserving amplitude so that no relation can be obtained among them.

We remember that $SU(6)$ has been successful in reproducing the parity-violating amplitudes, but not the parity-conserving amplitudes in which the kinetic-energy spurion seems to play an essential role as a symmetry-breaking interaction. Therefore the following discussions will be confined to the parity-violating amplitudes,[13] where the 35-plet spurion explains very well their $\Delta I=\frac{1}{2}$ amplitudes. Then relevant amplitudes are, in general, written as

$$\begin{aligned}\mathfrak{M}=\sum_{k=m,l=n}\sum_{P,p} & a\{\bar{B}^{\alpha 12,ikl}B_{\beta 13,jmn}M_{\alpha,i}{}^{\beta,j}+(CP)\} \\ & +b\{\bar{B}^{\alpha\beta 1,ijk}B_{\alpha 13,imn}M_{\beta,j}{}^{2,l}+(CP)\} \\ & +c\{\bar{B}^{\alpha 12,ikl}B_{\alpha\beta 1,ijm}M_{3,n}{}^{\beta,j}+(CP)\} \\ & +d\{\bar{B}^{\alpha\beta 1,ijk}B_{\alpha\beta 1,ijm}M_{3,n}{}^{2,l}+(CP)\} \\ & +e\{\delta_p\bar{B}^{\alpha\beta 1,ijk}B_{\alpha\beta 1,ijm}M_{3,n}{}^{2,l}+(CP)\}, \quad (2)\end{aligned}$$

where $\bar{B}$ and B stand for the creation and the annihilation wave functions of the 56-plet baryons, respectively,

* Work supported in part by the National Science Foundation and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

† Present address: California Institute of Technology, Pasadena, California.

[1] M. Gell-Mann, California Institute of Technology Report No. CTSL-20, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962).
[2] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] M. Suzuki, Phys. Rev. **137**, B1602 (1965).
[4] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).
[5] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).
[6] B. Sakita, Phys. Rev. **136**, B1756 (1964).
[7] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, *ibid.* **13**, 175 (1964); F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1964).
[8] T. K. Kuo and T. Yao, Phys. Rev. Letters **13**, 415 (1964); M. A. B. Bég and V. Singh, *ibid.* **13**, 418 (1964).
[9] M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964); B. Sakita, *ibid.* **13**, 643 (1964).
[10] S. P. Rosen and S. Pakvasa, Phys. Rev. Letters **13**, 773 (1964).
[11] M. Suzuki, Phys. Letters **14**, 64 (1965); G. Altarelli, F. Buccella, and R. Gatto, *ibid.* **14**, 70 (1965).
[12] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 51 (1965); K. Kawarabayashi, *ibid.* **14**, 86 (1965); P. Babu, *ibid.* **14**, 166 (1965).

[13] We shall denote the s wave by the symbol S.

while M are the wave functions of the 35-plet mesons.[14] The notations P and p mean permutations of the unitary-spin indices $1\leftrightarrow 2$ and $1\leftrightarrow 3$, and of the spin indices $k\leftrightarrow l$ and $m\leftrightarrow n$, respectively, δ_p being a signature taking the value 1 or -1 according as p is an even or an odd permutation. The term (CP) is a CP conjugate of the first term in each curly bracket. As is easily understood, the last curly bracket is a contribution of the 189-plet spurion while the others are contributions of the 405-plet spurion.[15] If we invoke the $2\leftrightarrow 3$ symmetry which is the consequence of the current$\times$current interactions and of CP invariance in the weak interactions, the following restrictions are obtained:

$$a=0,\quad b=-c,\quad d=0,\quad e=0. \tag{3}$$

We have only one parameter left for the parity-violating amplitudes.

We have three independent amplitudes to compare with experiment, namely

$$\Delta S(\Lambda)=S(\Lambda_-{}^0)+\sqrt{2}S(\Lambda_0{}^0)\,,$$
$$\Delta S(\Sigma)=S(\Sigma_-{}^-)+\sqrt{2}S(\Sigma_0{}^+)-S(\Sigma_+{}^+)\,,$$

and

$$\Delta S(\Xi)=S(\Xi_-{}^-)+\sqrt{2}S(\Xi_0{}^0)$$

which are exactly zero if the $\Delta I=\frac{1}{2}$ rule is exact. Carrying out rather lengthy but straightforward calculations, we find

$$\Delta S(\Lambda)=0\,, \tag{4a}$$
$$\Delta S(\Sigma)=(8/3)b\,, \tag{4b}$$
$$\Delta S(\Xi)=0\,. \tag{4c}$$

One of the striking characteristics is that the deviations from the $\Delta I=\frac{1}{2}$ rule exist only in Σ decay, but not in Λ decay nor in Ξ decay, so far as the parity-violating amplitudes are concerned. As was mentioned above, these relations hold good up to possible corrections to the 405-plet spurion coming from $SU(6)$-breaking interactions.

We turn to comparison of the predictions with the experimental data.[16–18] What is definitely known in experiment is that in Λ decay there is no serious disagreement with the $\Delta I=\frac{1}{2}$ rule nor among experimenters, while discrepancy with the $\Delta I=\frac{1}{2}$ rule is appreciable in Σ decay. The triangular relation in Σ decay is definitely not closed even if one takes the most favorable choice within experimental errors. As regards the Ξ decay, we do not have, unfortunately, any accurate data on the Ξ^0 decay suitable for the test. For Σ decay[19] we have

$$S(\Sigma)=(0.40_{-0.18}{}^{+0.16})S(\Sigma_-{}^-)\,, \tag{5}$$
$$=(-0.27_{-0.13}{}^{+0.11})S(\Sigma_-{}^-)\,.$$

For Λ decay,[20]

$$|\Delta S(\Lambda)|\lesssim 0.03|S(\Lambda_-{}^0)|\,. \tag{6}$$

The 35-plet spurion from the current$\times$current interactions leads to

$$S(\Lambda_-{}^0):S(\Sigma_-{}^-):S(\Xi_-{}^-)=1:\sqrt{\tfrac{2}{3}}:-1 \tag{7}$$

together with $S(\Sigma_+{}^+)=0$. So far as the central values are adopted, therefore,

$$|\Delta S(\Lambda)/\Delta S(\Sigma)|\lesssim\tfrac{1}{10}\,. \tag{8}$$

Even if one takes the most unfavorable choice within the experimental error, the left-hand side is much smaller than unity.

Thus the prediction of $SU(6)$ is in fine agreement with experiment on the $\Delta I=\frac{3}{2}$ amplitudes of the s wave also.

In conclusion, we speculate on the structure of the weak interactions. According to the above discussion, the high accuracy of the $\Delta I=\frac{1}{2}$ rule in Λ decay and probably in Ξ decay is rather accidental, while the $\Delta I=\frac{3}{2}$ amplitudes are of appreciable fractions of the $\Delta I=\frac{1}{2}$ amplitudes in Σ decay. This fact appears to support strongly the enhancement theory of the adjoint representation[21–23] as well as the current$\times$current picture of the weak interactions.[24] Further detailed discussions on this point are found in Ref. 3.

ACKNOWLEDGMENT

The author should like to express his sincere gratitude to Professor Kirk W. McVoy for hospitality extended to him at the Summer Institute in Theoretical Physics at the University of Wisconsin.

[14] For the explicit tensor representations of $\bar{B}$, B, and M, see for example Refs. 6 and 9.

[15] Precisely speaking, Eq. (2) contains a part of the 35-plet spurion, but it does not matter to the following discussions at all.

[16] F. S. Crawford, in *Proceedings of 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (Scientific Information Service, CERN, Geneva, Switzerland), p. 825.

[17] M. L. Stevenson, J. P. Berge, J. R. Hubbard, G. R. Kalbfleisch, J. B. Shafer, F. T. Solmitz, S. G. Wojcicki, and P. G. Wholmut, Phys. Letters **9**, 349 (1964).

[18] R. H. Dalitz, lecture note given at the International School of Physics "Enrico Fermi" on Weak Interactions organized by the Italian Physical Society at Varenna in June, 1964.

[19] We adopt the averaged value in Fig. 6 in Ref. 16. As is well known, two possible solutions exist in $\Sigma_0{}^+$ decay.

[20] We adopt the averaged value in Table I in Ref. 17, combining with the data in Ref. 16.

[21] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

[22] M. Suzuki, Progr. Theoret. Phys. (Kyoto) 31, 1090 (1964); **32**, 166 (1964).

[23] R. F. Dashen and S. C. Frautschi, Phys. Rev. Letters **13**, 449 (1964); R. F. Dashen, S. C. Frautschi, M. Gell-Mann, and Y. Hara, *Proceedings of the International Conference on High-Energy Physics at Dubna, 1964* (Atomizdat, Moscow, 1965).

[24] We can show within the framework of broken $SU(3)$ that the $\Delta I=\frac{3}{2}$ amplitude of the $K\rightarrow 2\pi$ decay is forbidden to all orders of $T_3{}^3$ violation of the octuplet spurion.