Photon-Induced Beta Decay in Stellar Interiors*

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Because of the astrophysical importance of beta-decay lifetimes in stellar interiors, we have calculated the rate of photon-induced beta decay ("photobeta" decay). In the photobeta process the photon can be considered to decay virtually into an electron-positron pair, with the positron being absorbed by the nucleus. The photobeta process is in competition with normal beta decay (if energetically possible), excited-state beta decay, and free-positron capture. An infrared divergence problem for exothermic photobeta decay is discussed, although the exothermic photobeta rate is too small to enhance most spontaneous beta decays. Applications are discussed in the driven decay of stable nuclei. As an example, the photobeta lifetime of a nucleus stable by 200 keV drops from 3×10^{10} yr at 3×10^{8} °K, to 10^{5} yr at 1.2×10^{9} °K for a nuclear transition matrix element characterized by log ft=6. The competition between the photobeta process and excited-state beta decay is discussed.

I. INTRODUCTION

THE rates of weak nuclear reactions (e.g., beta decay) in stellar interiors appear to be intimately related to the time scales for many important astrophysical processes. These weak reactions range from the familiar nuclear beta decay to a host of theoretically predicted neutrino-producing reactions, too slow to be observed in the laboratory, which nonetheless proceed at a physically significant rate in the environment of an evolved stellar interior. At least three different applications of weak reaction rates are astrophysically relevant: (1) to the rate of evolution of highly evolved stars, (2) to the time scale and environment of heavy-element nucleosynthesis, and (3) to the abundances of rare nuclear species.

The application to terminal rates of stellar evolution comes about because neutrino emission becomes the dominant mechanism of energy transfer when the central temperature of stars approaches 10^{9} °K. An extensive summary of these neutrino-producing reactions and their consequences on the pre-supernova stage has been presented by Fowler and Hoyle.[1] Suffice it to repeat here that the star becomes an elaborate machine for converting gravitational work into neutrinos, the collapse time therefore being governed by the weak-reaction rates.

A phenomenological application of the weak interactions occurs in the analysis of the neutron capture chains responsible for the elements heavier than iron. The operation of these chains was systematically presented by Burbidge, Burbidge, Fowler, and Hoyle,[2] who concluded that the nuclear lifetimes against the capture of free neutrons could be inferred from the beta-decay lifetimes of several key nuclei. This correlation was enriched by Cameron's[3] observation that certain beta-decay lifetimes are significantly shortened in stellar interiors by decay from thermally populated excited states of the nuclear species. The extraction of the astrophysical information requires accurate calculations of the dependence of beta-decay lifetimes upon temperature, and therefore a consideration of *all* relevant weak-interaction processes.

The abundances of many relatively rare proton-rich nuclear species have remained somewhat of a puzzle. But at least one such species, Er^{164}, seems to owe its existence to the beta decay of a *terrestrially stable* nuclear species, and Reeves[4] has suggested that many of these species may be the result of the capture of free positrons present at high temperatures in stellar interiors. Although the problem of the proton-rich abundances is presently unsolved, it does seem likely that the charge-increasing weak reactions at temperatures of 10^{9} °K and greater are intimately involved.

With the above possibilities in mind, we have calculated the rate of direct photon-induced beta decay, both endothermic and exothermic, in a high-temperature environment. Here the photon virtually dissociates into an electron-positron pair and the nucleus absorbs the positron; the over-all effect is that of ordinary beta decay except that a photon is absorbed (thereby providing an additional source of energy). In this "photobeta"[5] process the nuclear transition proceeds directly from a state of the nucleus (Z,A) to a state of the nucleus $(Z+1, A)$. The photobeta rate is inversely proportional to the ft value which characterizes the nuclear matrix element connecting those two states, and proportional to the fine-structure constant and the photon density. The rate from a given level is independent of

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[1] William A. Fowler and F. Hoyle, Astrophys. J. Suppl. **91**, 201 (1964).
[2] E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, Rev. Mod. Phys. **29**, 547 (1957).
[3] A. G. W. Cameron, Astrophys. J. **130**, 452 (1959).
[4] Hubert Reeves and Pierre Stewart, Astrophys. J. **141**, 1432 (1965). These authors erroneously state that excited-state beta decay (which they call photo-beta reaction) is a virtual positron-capture process. The photobeta reaction which we calculate in this paper does not involve the intermediate excited state, and includes virtual positron capture.
[5] In Ref. 3, Cameron called excited-state beta decay by the name *photobeta reactions*. Excited-state beta decay is the generally accepted terminology, however, which frees the adjective *photobeta* for the more appropriate direct process described in this paper.

the level structure of the nucleus (Z,A), although the total photobeta rate for the entire nucleus (Z,A) will depend upon the fraction of the nuclei existing in each state. Typically only one state of the initial and final nuclei will dominate the process. The photobeta process competes with the process of excited-state beta decay, which is essentially a two-stage process of real excitation followed by beta decay. One of our principal objectives is to delineate the relative importance of the photobeta reaction and the excited-state beta decay.

II. CALCULATION OF THE PHOTOBETA REACTION RATE

A. Endothermic Case

We first discuss the reaction rate for the process[6]

$$\gamma+(Z,A)\rightarrow(Z+1,A)+e^-+\bar{\nu} \tag{1}$$

subject to the restriction

$$\Delta=M_{Z+1}+m-M_Z\geq 0, \tag{2}$$

where M_Z, M_{Z+1} are nuclear masses, m is the electron mass, and Δ is the *atomic* mass difference. The restriction to endothermic reactions means, of course, that only photons of energy greater than Δ produce reaction (1). Although the reaction rate for the limiting case $\Delta=0$ is finite, an extension of the rate to the exothermic domain $(\Delta<0)$ must be done carefully, in principle, to obtain a finite result. The proper treatment of the exothermic case will be taken up in the second part of this section.

The Feynman diagrams relevant to the endothermic case are indicated in Fig. 1. In particular, the photon-nucleus reactions in Fig. 1(b) and 1(c) include, in principle at least, a variety of real and virtual intermediate states, such as excited (Z,A) and $(Z+1,A)$ states, $(Z,A-1)$ plus neutron states, etc. The production of these intermediate states involves the interaction of the electromagnetic field with the *internal* nuclear currents, and although these interactions are of paramount importance in establishing the population of internal states assumed in excited-state beta decay, they are, nevertheless, negligible compared to the photon-electron reaction in Fig. 1(a) (of the order of the electron-nucleon mass ratio). The approximation of neglecting internal nuclear currents must, however, be made in a gauge invariant manner. To this end we shall ignore all processes included in Figs. 1(b) and 1(c) *except* for those required by gauge invariance. This requirement can be met by retaining only the convective part of the nuclear current (i.e., just that due to the over-all nuclear charge, approximated to be a point charge)

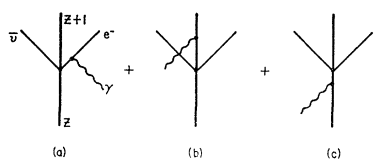

FIG. 1. Feynman diagrams for the endothermic photobeta process.

$$j_\mu^{(c)}=Ze(P_a+P_b)_\mu, \tag{3}$$

where Ze is the nuclear charge and $P_{b\mu}$ and $P_{a\mu}$ are the nuclear four-momenta before and after the absorption of a photon, respectively. The transition amplitude for reaction (1) is calculated according to the standard Feynman rules, and with the approximation expressed in Eq. (3), the amplitude is given by

$$F=\sqrt{2}eN_\mu\left\{\left[Z\frac{\epsilon\cdot(2P+k)}{(P+k)^2-M_Z^2}+(Z+1)\frac{\epsilon\cdot(2\bar{P}-k)}{(\bar{P}-k)^2-M_{Z+1}^2}\right]\bar{u}(\mathbf{p})\gamma_\mu a v(\mathbf{q})-\bar{u}(\mathbf{p})\epsilon\cdot\gamma\frac{\gamma\cdot(p-k)+m}{(p-k)^2-m^2}\gamma_\mu a v(\mathbf{q})\right\}, \tag{4}$$

where we use the notation

P_μ = four-momentum of the initial (charge Z) nucleus $(P^2=M_Z^2)$,
$\bar{P}_\mu$ = four-momentum of the final (charge $Z+1$) nucleus $(\bar{P}^2=M_{Z+1}^2)$,
p_μ = four-momentum of the electron $(p^2=m^2)$,
q_μ = four-momentum of the antineutrino $(q^2=0)$,
k_μ = four-momentum of the absorbed photon $(k^2=0)$,
ϵ_μ = transverse polarization vector of the photon $(k\cdot\epsilon=0)$,
$a=\frac{1}{2}(1+i\gamma_5)$ with $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$,
N_μ = nuclear beta-decay matrix element,
$u(\mathbf{p})$ = electron Dirac wave function $[\gamma\cdot pu(\mathbf{p})=mu(\mathbf{p})]$,
$v(\mathbf{q})$ = antineutrino Dirac wave function $[\gamma\cdot qu(\mathbf{q})=0]$,

and we use natural units with $\hbar=c=1$.

The gauge invariance of Eq. (4) is easily verified by expanding the propagator denominators, using the anti-commutation relation

$$\gamma\cdot\epsilon\gamma\cdot p+\gamma\cdot p\gamma\cdot\epsilon=2\epsilon\cdot p$$

and noting that F vanishes when ϵ_μ is replaced by k_μ. Since we shall make the static approximation M_Z, $M_{Z+1}\rightarrow\infty$ (with $M_{Z+1}-M_Z$ held fixed) the amount of calculation can be reduced by choosing the particular gauge

$$\epsilon_\mu=(0,\boldsymbol{\epsilon}). \tag{5}$$

[6] The analogous process of positron emission is surely unimportant since it must compete with electron capture, a process that requires 1.02 MeV less energy.

Then only diagram 1(a) survives and we have

$$F=\sqrt{2}eN_\mu\bar{u}(\mathbf{p})\epsilon\cdot\gamma\frac{\gamma\cdot(p-k)+m}{2p\cdot k}\gamma_\mu a v(\mathbf{q}). \quad (6)$$

The rate of reaction (1) is obtained by integrating the absolute square of Eq. (6) (with spin and polarization sums) over the Planck spectrum and over the final density of states. The result before taking the trace is

$$R=\frac{e^2\langle N_\mu N_\nu{}^*\rangle}{2(2\pi)^8}\int\frac{d^3k}{2\omega}f(\omega)\frac{d^3p}{2E}\frac{d^3q}{2q_0}$$
$$\times\delta(M_Z+\omega-M_{Z+1}-q_0-E)(p\cdot k)^{-2}$$
$$\times\sum_\epsilon \mathrm{Tr}\{(\gamma\cdot p+m)\epsilon\cdot\gamma[\gamma\cdot(p-k)+m]$$
$$\times\gamma_\mu aq\cdot\gamma\gamma_\nu[\gamma\cdot(p-k)+m]\epsilon\cdot\gamma\}, \quad (7)$$

where we have introduced "projection operators" $\gamma\cdot p+m$ and $\gamma\cdot q$ for the electron and antineutrino, respectively. The brackets on the nuclear parts indicate the appropriate averaging and summing over spin states. The function $f(\omega)$ is the Planck spectral distribution function and is given by

$$f(\omega)=(e^{\omega/\tau}-1)^{-1}, \quad (8)$$

where τ is the absolute temperature multiplied by the Boltzmann constant, and ω is the photon energy.

We shall consider the weak nuclear matrix elements in the allowed approximation only, for which

$$N_0=g_V\int 1 \quad \text{(Fermi transition)},$$

$$N_k=g_A\int\sigma_k \quad \text{(Gamow-Teller transition)},$$

where g_V and g_A are the vector and axial-vector coupling constants, respectively. For pure transitions, the form of the integrand in Eq. (7) depends on the particular type of transition only through an angular correlation term. The angular correlation term vanishes, however, when the average is carried out over the antineutrino direction, and over the final nuclear spin states. Hence Eq. (7) actually has the same form for both Fermi and Gamow-Teller transitions, and, after the trace and polarization sums are carried out, the rate is given by

$$R=2e^2\frac{\langle|N|^2\rangle}{(2\pi)^8}\int\frac{d^3k}{2\omega^3}f(\omega)\frac{d^3p}{2E}d^3q\delta(M_Z+\omega-M_{Z+1}-E-q)$$
$$\times\left[\frac{m(\omega-E)}{(E-p\cos\theta)^2}+\frac{(\omega-E)^2+E^2}{(E-p\cos\theta)}+\omega-E\right], \quad (9)$$

where p, q now stand for the magnitude of the electron and antineutrino three-momenta, respectively, $E=(p^2+m^2)^{1/2}$ is the electron energy, θ is the angle between the electron and the photon, and $\langle|N|^2\rangle$ is the averaged square of the weak nuclear matrix element (either Fermi or Gamow-Teller).

After the integration over angles is carried out, the rate becomes

$$R(\tau,\Delta)=\frac{\ln 2}{\pi}\frac{\alpha}{ft}\int_\Delta^\infty\frac{d\omega}{\omega}f(\omega)G(\omega,\Delta), \quad (10)$$

where

$$G(\omega,\Delta)=\int_1^{\omega-\Delta+1}dE(\omega-E-\Delta+1)^2[2(\omega-E)(E^2-1)^{1/2}$$
$$+(\omega^2-2\omega E+2E^2)\ln\{E+(E^2-1)^{1/2}\}] \quad (11)$$

and τ, energies, and masses are expressed in units of the electron mass. In Eq. (10) the nuclear matrix element has been expressed in terms of the nuclear ft value from the relation

$$ft=2\pi^3\ln 2/m^5\langle|N|^2\rangle$$

and the electric charge in terms of the fine structure constant

$$\alpha=e^2/4\pi\simeq 1/137.$$

The rate is most conveniently expressed as a rapidly convergent series of Maxwell-Boltzmann rates

$$R(\tau,\Delta)=\sum_{n=1}^\infty r\left(\frac{\tau}{n},\Delta\right), \quad (12)$$

where $r(\tau/n,\Delta)$ is obtained from $R(\tau,\Delta)$ by replacing $f(\omega)$ in Eq. (10) by $\exp(-n\omega/\tau)$. By using the integral representations for the modified Bessel functions of the second kind and order zero and one, namely,

$$K_0(x)=x\int_1^\infty e^{-Ex}\ln\{E+(E^2-1)^{1/2}\}dE,$$
and
$$K_1(x)=x\int_1^\infty e^{-Ex}(E^2-1)^{1/2}dE, \quad (13)$$

the function $r(\tau,\Delta)$ can be written

$$r(\tau,\Delta)=\frac{2\ln 2}{\pi}\frac{\alpha}{ft}\Bigg(e^{-\Delta/\tau}\{\tau^4[\bar{K}_1(1/\tau)$$
$$+(\Delta-1)\bar{K}_0(1/\tau)]+4\tau^5\bar{K}_0(1/\tau)\}$$
$$-2\int_{1/\tau}^\infty dx\,\bar{K}_0(x)e^{-\Delta x}\left[\frac{3}{x^6}+\frac{\Delta-1}{x^5}\right]\Bigg), \quad (14)$$

where $\bar{K}_\nu(x)=e^xK_\nu(x)$. Since the value of $1/\tau=5.934\times10^9/T$ is considerably greater than unity for all except very high values of the temperature, the modified Bessel functions in Eq. (14) can be represented to a good

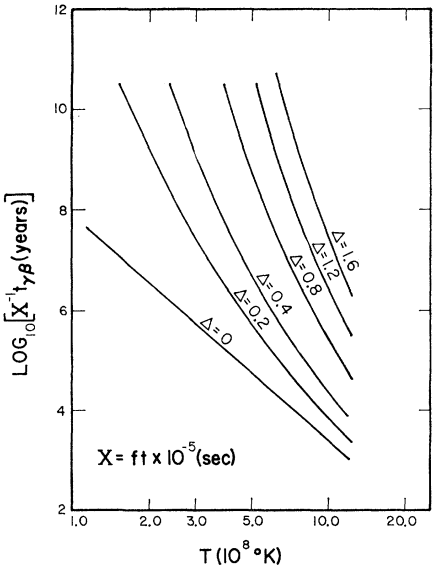

FIG. 2. Photobeta lifetime of a nuclear state as a function of temperature for various values of atomic mass differences. The quantity Δ is measured in units of the electron rest energy.

approximation by their asymptotic expansions

$$\bar{K}_0(x)=\left(\frac{\pi}{2x}\right)^{1/2}\left[1-\frac{1}{8x}+\frac{9}{128x^2}+\cdots\right],$$
$$\bar{K}_1(x)=\left(\frac{\pi}{2x}\right)^{1/2}\left[1+\frac{3}{8x}-\frac{15}{128x^2}+\cdots\right]. \quad (15)$$

The rate $r(\tau,\Delta)$ then becomes

$$r(\tau,\Delta)=\frac{2\ln 2}{\pi}\frac{\alpha}{ft}\tau^{9/2}e^{-\Delta/\tau}H(\tau,\Delta), \quad (16)$$

where

$$H(\tau,\Delta)=\Delta+2(1-\Delta)\bar{E}_{11/2}(\Delta/\tau)+\tfrac{1}{4}\tau[18-\Delta+(\Delta-25)\bar{E}_{13/2}(\Delta/\tau)]+\cdots. \quad (17)$$

The functions $\bar{E}_\nu(x)$ are related to generalized exponential integrals and are defined by

$$\bar{E}_\nu(x)=e^x\int_1^\infty e^{-xt}t^{-\nu}dt\simeq\frac{x+\nu}{(x+\nu)^2-\nu}, \quad (18)$$

where the approximation is good to within 1% for all values of $x\geq 0$ and for values of ν needed in Eq. (17) (i.e., $\nu\geq 11/2$). Since the function $H(\tau,\Delta)$ is of order unity for $\tau<0.3$ and $\Delta<2$, the logarithm (base 10) of the rate R can be approximated within these limits by

$$\log_{10}R(\tau,\Delta)\simeq\log_{10}r(\tau,\Delta)\simeq\log_{10}\left[\frac{2\ln 2}{\pi}\frac{\alpha}{ft}\tau^{9/2}e^{-\Delta/\tau}\right], \quad (\tau<0.3,\ \Delta<2) \quad (19)$$

with an accuracy of 10%. For $\tau\gg 1$, Δ, on the other hand, the rate (10) is dominated by large photon and electron energies, and can be approximated by

$$R\simeq(\alpha/\pi)[5.78\tau^5\ln\tau+3.10\tau^5+\cdots]. \quad (\tau\gg 1,\ \Delta). \quad (20)$$

We have plotted the logarithm (base 10) of the photobeta lifetime $t_{\gamma\beta}$ (defined as the reciprocal of the rate R) in Fig. 2 as a function of T for fixed Δ and in Fig. 3 as a function of Δ for fixed T. The scale factor for the lifetime (in years) is given by the nuclear ft value (in seconds) multiplied by a factor of 10^{-5}. Thus for an ft value of 10^5 sec and $\Delta=0$, $t_{\gamma\beta}\simeq 10^3$ years at $T=1.2\times 10^9$ °K. In constructing Figs. 2 and 3, we have used Eq. (19) as an approximation. Since any discussion of the photobeta lifetimes must be made in comparison with excited-state beta-decay lifetimes, we shall postpone a discussion of the results until Sec. III.

B. Exothermic Case

From a practical point of view any detailed calculation of the exothermic rate is unnecessary since spontaneous beta decay occurs if Δ is negative. In general, the spontaneous decay rate will dominate unless $-\Delta$ is rather small. A reasonable approximation for small values of Δ would then be to use the value of the endothermic rate at $\Delta=0$. However, for the sake of completeness and in view of the fact that the exothermic rate calculation contains an interesting feature that is absent in the endothermic rate calculation, we shall present a detailed treatment.

As was mentioned previously, the reaction rate (10) gives a convergent result when the limiting case $\Delta=0$ is reached, even though the denominator of the electron propagator vanishes at zero photon frequency. The

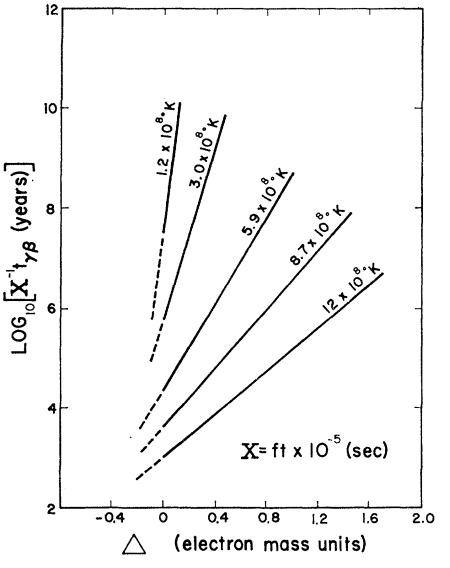

FIG. 3. Photobeta lifetime of a nuclear state as a function of atomic mass difference for various temperatures. The dotted lines are linear extrapolations into the exothermic region.

absence of divergence can be traced to the electron being at rest (hence no electromagnetic current). As soon as the system becomes slightly exothermic ($\Delta<0$), however, the integrand in Eq. (10) diverges as ω^{-2}, the electron no longer being at rest at zero frequency.

The divergence problem encountered here is a slight variation of the usual infrared divergence problem considered in great detail by numerous authors.[7] In general, low-frequency divergences will occur in the transition probability for a given process when radiative corrections due to the emission and absorption of virtual photons are taken into account. Infrared divergences in transition probabilities also occur when additional real soft photons appear in the final state for the given process. The solution to the usual problem is realized by the exact cancellation (to all orders of perturbation theory) of the infrared divergences from both real and virtual photons. The cancellation requires that transition probabilities for the emission of different numbers of real soft photons be added together, and the justification for such a procedure hinges upon the fact that finite experimental resolution precludes a knowledge of the number of soft photons in the final state.

The above analysis has been applied to second-order electromagnetic corrections to beta decay.[8] The infrared divergence that occurs when beta decay is accompanied by spontaneous emission (inner-bremsstrahlung) is exactly cancelled by an infrared divergence that appears in the transition probability for beta decay with virtual photon emission. The divergence in the latter transition probability arises from an interference between the amplitude for beta decay without radiative corrections and the (infrared divergent) amplitude for beta decay corrected (to second order) for virtual photons. The spontaneous problem is of no concern to us except to note that the electromagnetic corrections are of the order of several percent and are temperature-independent.

In the presence of thermal radiation, infrared divergences also appear in the transition probability for absorption and induced emission. These divergences are not cancelled, however, by virtual photon effects, but rather by interaction between the charged particles and the thermal radiation. In particular, a photon of momentum k_μ can be absorbed by one charged particle while an identical photon is emitted by another (or the same) particle. The transition amplitudes for all such processes must be added coherently to the simple beta-decay amplitude, since the initial and final states are the same in all cases. For the simple beta decay, the photons do not enter the decay process but remain part of the background in which the decay occurs. The amplitude for the coherent interaction with the radiation background is infrared divergent and the interference between it and the amplitude for beta decay cancels the infrared divergences in absorption and induced emission.

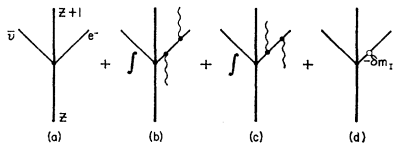

FIG. 4. Feynman diagrams for coherent processes that give rise to a second-order interference term in the beta-decay rate in an exothermic situation. The integral sign before diagrams (b) and (c) indicates a summation over the Planck spectrum.

One result of the interaction between a free particle and the radiation background is the effective increase in the particle's mass above the zero-temperature (i.e., no photons) value, in analogy with the mass shift due to a particle's interaction with its own electromagnetic field. Unlike the self-interaction correction, this mass correction is finite since the Planck spectrum cuts off rapidly at high energy. With the static approximation and the gauge in Eq. (5), the only additional diagrams that need to be considered are given in Fig. 4. It is straightforward to see that the inclusion of nonzero convective nuclear currents due to finite nuclear masses results in a gauge-invariant interference term and leads to a more general cancellation of infrared divergences. It is evident that only a particle's convective current is involved in infrared divergences, and for this reason photobeta reactions involving electrons at rest are free of infrared difficulties.

Before investigating the diagrams in Fig. 4, we give the net rate $R_1(\tau,\Delta)$ for absorption and induced emission. Since the amplitude for emission can be obtained from the amplitude for absorption by the substitution rule[9] $k_\mu \rightarrow -k_\mu$, the net rate can be written down directly from Eq. (10)

$$R_1(\tau,\Delta)=\frac{\ln 2}{\pi}\frac{\alpha}{ft}\left\{\int_{\omega_0}^{\infty}\frac{d\omega}{\omega}f(\omega)G(\omega,\Delta)+\int_{\omega_0}^{-\Delta}\frac{d\omega}{\omega}f(\omega)G(-\omega,\Delta)\right\} \quad (21)$$

with $\Delta<0$ and where $G(\omega,\Delta)$ is defined in Eq. (11). The low-frequency cutoff at $\omega=\omega_0$ is inserted for the moment to ensure the convergence of $R_1(\tau,\Delta)$. The alternative use of a small fictitious photon mass to obtain a Lorentz-invariant cutoff is of no advantage here, since we have already singled out the special frame of reference in which the temperature is defined. When the contributions from Fig. 4 are included, the resultant rate remains finite in the limit $\omega_0 \rightarrow 0$.

[7] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. 13, 379 (1961). This reference contains an extensive bibliography on the infrared divergence problem.

[8] A. Sirlin, in *Lecture Notes on Weak Interactions*, edited by Christian Fronsdal (W. A. Benjamin, Inc., New York, 1963), p. 173. This reference contains a bibliography of radiative corrections for beta decay.

[9] J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1955), p. 162.

As a first step in calculating the contribution from the diagrams in Fig. 4, we calculate the effective mass increase for the electron due to its interaction with the thermal radiation bath. (For present purposes we shall no longer use the electron mass as the unit of mass). Following the usual procedure,[10] the sum of the diagrams in Fig. 5 gives the modified electron propagator (to second order)

$$S_F'(p)=\frac{i}{p\cdot\gamma-m_0-\Sigma(p)-I(p)}, \tag{22}$$

where m_0 is the "bare" electron mass, $\Sigma(p)$ is the second-order self-energy part, and $I(p)$ is defined analogously as the "induced-energy part" given by the coherent summation,[11] over the Planck spectrum, of the forward-Compton-scattering amplitude:

$$I(p)=\sum_\epsilon e^2\int\frac{d^3k}{2\omega}\frac{f(\omega)}{(2\pi)^3}\times\epsilon\cdot\gamma\left[\frac{1}{\gamma\cdot(p+k)-m_0}+\frac{1}{\gamma\cdot(p-k)-m_0}\right]\epsilon\cdot\gamma. \tag{23}$$

The "experimental mass" $m_{\rm exp}$ is then determined by the position of the pole in Eq. (22), and in particular, the "induced mass" contribution is given by

$$\delta m_I=\tfrac{1}{2}\,\mathrm{Tr}\left[\frac{(\gamma\cdot p+m)}{2m}I(p)\right]=\tfrac{1}{3}\pi\alpha(\tau/m)^2m, \tag{24}$$

where α is the fine-structure constant and m is the zero-temperature electron mass. By renormalizing the experimental mass to include the contribution (24) for a free electron in the thermal bath, we introduce in the usual way an induced-mass "counter term" as indicated in Fig. 4(d). The diagrams in Fig. 4 can then be added together following Dyson's method,[10] for example, to give the result

$$-\sqrt{2}e^2N_\mu\bar{u}(\mathbf{p})\gamma_\mu a v(\mathbf{q})\sum_\epsilon\int\frac{d^3k}{2\omega}\frac{f(\omega)}{(2\pi)^3}\times\left[\left(\frac{\epsilon\cdot p}{k\cdot p}\right)^2+\frac{1}{2m^2}\right], \tag{25}$$

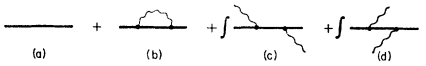

FIG. 5. Feynman diagrams for the electron propagator corrected to second order for virtual photons and for the interaction of the thermal photons.

where the integral is again cut off (when necessary) to avoid an infrared divergence. The amplitude in Eq. (25) must now be added to the simple beta-decay amplitude, squared, and integrated over the electron-antineutrino spin and phase space. When the interference term arising from such an operation is added to Eq. (21) we obtain for the corrected total photobeta rate (excluding the simple beta-decay rate)

$$R_2(\tau,\Delta)=\frac{\ln 2}{\pi}\frac{\alpha}{ft}\left\{\int_{\omega_0}^{\infty}\frac{d\omega}{\omega}f(\omega)[G(\omega,\Delta)-2G(0,\Delta)]+\int_{\omega_0}^{-\Delta}\frac{d\omega}{\omega}f(\omega)G(-\omega,\Delta)\right\}-R_\beta\delta m_I+\left(\frac{dR_\beta}{dm}\right)_{m=1}\delta m_I, \tag{26}$$

where R_β is the simple beta-decay rate in the absence of both induced mass and Coulomb corrections. The last term in Eq. (26) represents the induced-mass correction to the simple beta decay and has been included in the rate R_2 in addition to the contributions referred to above.

To see that the rate $R_2(\tau,\Delta)$ is finite in the limit $\omega_0\to 0$ we isolate the ω_0-dependent part in the form

$$\int_{\omega_0}^{-\Delta}\frac{d\omega}{\omega}f(\omega)\{G(\omega,\Delta)+G(-\omega,\Delta)-2G(0,\Delta)\}. \tag{27}$$

If, for fixed Δ, we apply Taylor's formula (with remainder) to the function $G(\omega,\Delta)$, it is evident that the bracketed part of the integrand in Eq. (27) is of order ω^2 and hence the integral converges. Furthermore, the rate may be written in the form convenient for small negative Δ

$$R_2(\tau,\Delta)=\frac{\ln 2}{\pi}\frac{\alpha}{ft}\int_0^{\infty}\frac{d\omega}{\omega}\left[f(\omega)\sum_{n=0}^{2}\Delta^n\frac{1}{n!}\left(\frac{\partial^nG}{\partial\Delta^n}\right)_{\Delta=0}\right]+\rho(\tau,\Delta), \tag{28}$$

where $\rho(\tau,\Delta)$ is a remainder term of order $(-\Delta)^{5/2}$. The integral term on the right-hand side of Eq. (28) is also contained in an expansion of the endothermic rate for small Δ, and a smooth extrapolation of Eq. (16) across $\Delta=0$ is justified. For $\Delta\ll 0$, the dominant term in Eq. (26) is

$$-\delta m_I R_\beta\sim-|\Delta|^5. \tag{29}$$

This term arises from the destructive interference be-

[10] Silvan S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson and Company, New York, 1961), p. 531 ff.

[11] A more formal approach is to replace the photon propagator as the *vacuum* expectation value of a time-ordered product of field operators by the expectation value in the presence of the thermal radiation. The photon propagator in momentum space is then given, in a particular gauge, by

$$D_{\mu\nu}(k^2)=-i\delta_{\mu\nu}\left[\frac{1}{k^2+i\epsilon}-2\pi i\delta(k^2)f(\omega)\right],$$

where $k^2=k_4^2-\omega^2$, $\delta(k^2)$ is the Dirac δ function, and $\delta_{\mu\nu}$ is the metric tensor with $\delta_{11}=\delta_{22}=\delta_{33}=-\delta_{44}=-1$.

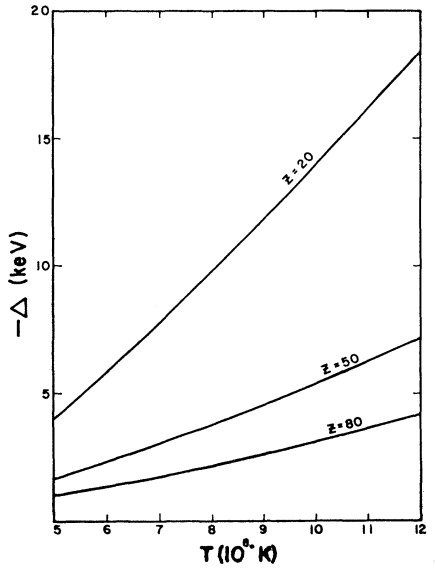

FIG. 6. Tracks of equal beta-decay and photobeta-decay rates for an exothermic reaction. Region above track for a given Z is beta dominant. Region below track is photobeta dominant. The Z dependence of the equal-lifetime tracks arises from Coulomb corrections to the simple beta-decay rate.

tween the noninfrared part of Eq. (25) and the simple beta-decay amplitude, and gives a small temperature-dependent *reduction* (order $-\delta m_I$) of the simple beta-decay rate.

To give an idea of the relative importance of the photobeta- and beta-decay rates, we have plotted in Fig. 6 curves of equal photobeta and beta rates in the Δ, τ plane for several choices of Z. In plotting the curves the photobeta rate was evaluated at $\Delta=0$. Figure 6 shows that the photobeta rate can exceed the spontaneous rate of an exothermic decay only for high temperature and very small end-point energy.

III. DISCUSSION

An assessment of the importance of the photobeta process requires a survey of nuclear energy level schemes, and in a preliminary search[12] the following three criteria were invoked to select likely candidates among nuclei: (1) spontaneous lifetimes should be in excess of 10^4 years for exothermic cases, (2) nuclei should be stable by no more than a few hundred keV for endothermic cases (see Fig. 3 for lifetime versus stability), and (3) excited-state beta decay should be unfavored.

The third criterion needs some amplification because the beta decay of thermally populated excited states provides an important increase in beta-decay rates in stars. The rate of excited-state beta decay will be suppressed, however, if all excited states capable of favorable transitions lie at large excitation energies above the ground state. To obtain a general idea of the relative importance of the photobeta- and excited-state beta-decay rates we have indicated in Fig. 7 the circumstances under which the two rates are equal. In Fig. 7 we have assumed that the transformation $(Z,A)\rightarrow(Z+1,A)$ is dominated by one excited-state beta-decay and one photobeta transition to a common final state. For simplicity, the transitions are characterized by equal ft values, equal initial-state spins (the initial state for the photobeta reaction being the ground state), and nuclear charge $Z=50$. In constructing Fig. 7 we have used Eq. (19) for the photobeta rate at temperatures less than 3×10^9 °K and Eq. (20) at temperatures greater than 3×10^{10} °K. A smooth extrapolation was used for intermediate temperatures. With these approximations the curves of equal lifetimes are essentially independent of the parameter Δ. At temperatures for which Eq. (19) is applicable the factor $\exp(-\Delta/\tau)$ in the photobeta rate is cancelled by part of the Boltzmann factor in the excited-state beta-decay rate. The remaining part of the factor in the latter rate is $\exp(-W/\tau)$, where W is the energy difference, in electron mass units, between the excited state of (Z,A) and the particular state of $(Z+1,A)$. A similar cancellation would occur if the initial state involved in the photobeta reaction were not the ground state, since the amount of excitation would appear only in a Boltzmann factor in both the photobeta- and excited-state beta-decay rates. For a given value of W, Fig. 7 shows that the photobeta reaction will dominate at low temperatures since, at these temperatures, the thermal population of the excited state will be relatively small. At higher temperatures the excited-state beta decay becomes dominant because of the increased thermal population of the excited state. As the temperature is increased still further the excited state and the ground state become

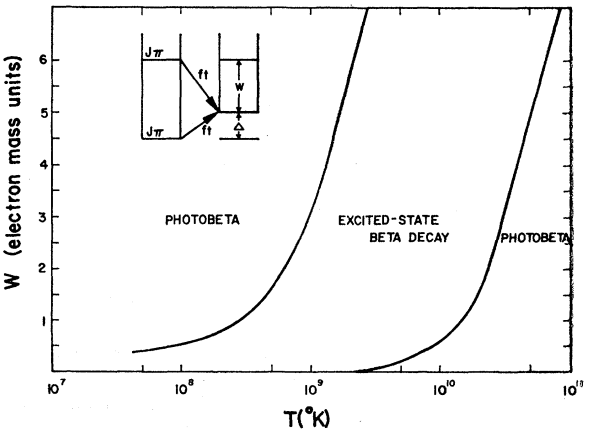

FIG. 7. Tracks of equal excited-state beta-decay and photobeta-decay rates for ground and excited states with same spin, parity, and ft values, and $Z=50$. Nuclear level diagram defines energies involved. The tracks take into account only the two transitions indicated.

[12] Nuclear Data Group, *Nuclear Data Sheets* (Oak Ridge National Laboratory, Oak Ridge, Tennessee, 1964).

equally populated (in the limit $T \rightarrow \infty$). The photobeta reaction, however, depends on the photon density whereas excited-state beta decay does not. Since the photon density increases without bound for increasing temperature, the photobeta reaction would again dominate at high temperature, provided beta decay from states of higher excitation (but increased thermal population) remains unimportant as originally assumed. The requirement that the photobeta reaction proceed at a significant rate, demands relatively high temperatures (see Fig. 2). Figure 7 indicates, however, that at those temperatures the energy W must be large in order that the photobeta rate exceed the excited-state beta-decay rate (we exclude the region to the right of and below the right-hand equal-lifetime track).

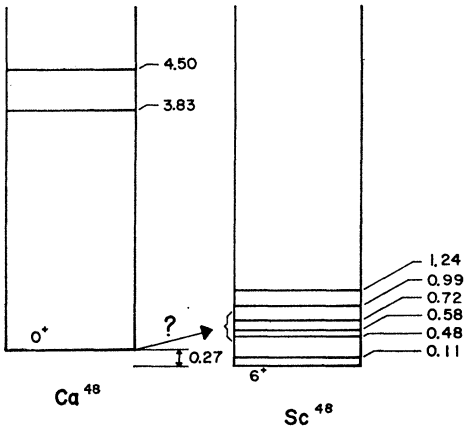

FIG. 8. Nuclear energy levels of Ca^{48} and Sc^{48}. Natural beta decay of Ca^{48} to ground or first excited state of Sc^{48} is highly forbidden. If the transition to any of the next few excited states of Sc^{48} is allowed, Ca^{48} is photobeta unstable at high temperature.

A good example of a photobeta candidate having essentially no competition from excited-state beta decay is Ca^{48}, shown in Fig. 8. The beta decay of Ca^{48} to Sc^{48} has remained undetected because of the high degree of forbiddenness of a transition from the 0^+ ground state of Ca^{48} to the first two states of Sc^{48}. Depending on the spins and parities of the next few excited states of Sc^{48}, it is quite possible that Ca^{48} is photobeta unstable at high temperature. A similar situation occurs for Zr^{96}.

Perhaps the details of the competition of the photobeta process with the excited-state beta decay is best illustrated by an actual example. Figure 9 shows the relative energy level diagram of Te^{125} and I^{125}. Although Te^{125} is stable in the laboratory, it can be transformed to I^{125} at high temperature both by excited-state decay and by photobeta decay. The photobeta process will proceed via the allowed transition ($\log_{10} ft = 4.9$) from Te^{125} (0.0353 MeV) which is heavily populated thermally at temperatures in excess of 10^8 °K, to the ground state of I^{125}. That transition is endothermic by 115 keV. The excited-state decay will probably proceed via Te^{125} (0.462 MeV). The lifetimes against both processes are

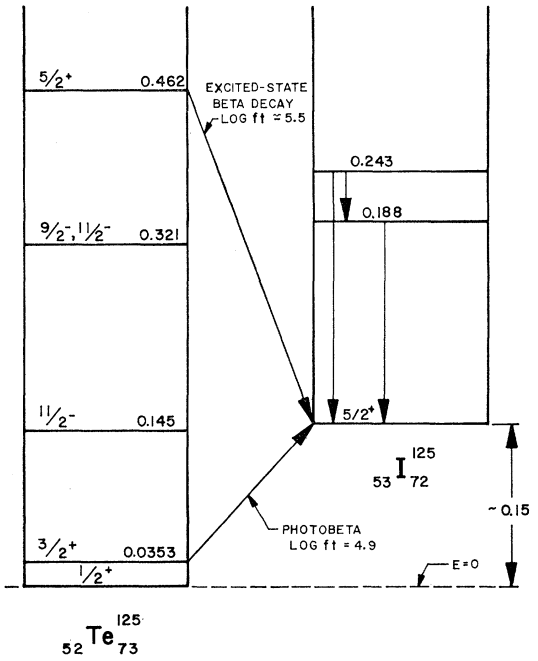

FIG. 9. Nuclear energy levels of Te^{125} and I^{125}. In the laboratory I^{125} decays by electron capture to the $\frac{3}{2}^+$ state of Te^{125} with $\log ft = 4.9$. At high temperature, Te^{125} may be unstable, either by the endothermic photobeta process or by excited-state beta decay. Energies are in MeV. The $\log ft \approx 5.5$ for decay from the 0.462-MeV state has been estimated from neighboring analog transitions.

plotted in Fig. 10. It is evident from this figure that the photobeta rate is the faster of the two only for temperatures less than about 1.7×10^8 °K, for which the lifetimes are greater than about 10^{10} yr. At the higher temperatures where the lifetime is much shorter, the rate is dominated by the excited-state beta decay. We also see, from Fig. 10, that such conclusions are exceedingly sensitive to the assumed $\log ft$ value.

There are at least three important natural radioactivities that are speeded up by the photobeta process at high temperature. The $Re^{187}(\beta^-)Os^{187}$ decay is a

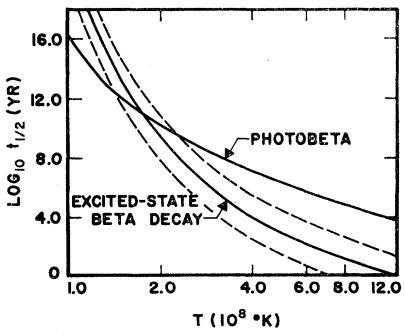

FIG. 10. The lifetimes of Te^{125} against the photobeta process and against excited-state beta decay. The photobeta decay dominates only for $T < 1.7 \times 10^8$; i.e., for $t_{1/2} > 10^{11}$ yr. The upper (lower) dashed line shows the effect of increasing (decreasing) the estimated $\log ft$ value for decay from the 0.462-MeV state of Te^{125} to 7.0 (4.0) illustrating the sensitivity on that (unknown) quantity.

first-forbidden unique decay with an endpoint energy of only 2.6 keV and a half-life of about 4×10^{10} years.[13] This decay has extreme astrophysical importance, moreover, since it provides a galactic clock.[14] The decay $Rb^{87}(\beta^-)Sr^{87}$ with a half-life of 4.7×10^{10} yr may be enhanced by the allowed photobeta transition to the first excited state of Sr^{87}. The same statement applies to the decay $Pd^{107}(\beta^-)Ag^{107}$, with a half-life of 7×10^6 yr.

The above examples are but a few of the astrophysically interesting possibilities suggested from a preliminary examination of the Nuclear Data Sheets.[12] The photobeta process (together with excited-state beta decay) provides a mechanism for transforming a stable (Z,A) nucleus to an unstable $(Z+1, A)$ nucleus and thence to another stable $(Z+2, A)$ nucleus (e.g., $Ca^{48} \rightarrow Sc^{48} \rightarrow Ti^{48}$). On the other hand the cycle illustrated by $\gamma+Te^{125} \rightarrow I^{125}+e^-+\bar{\nu}$, $e^-+I^{125} \rightarrow Te^{125}+\nu$ is essentially the catalytic conversion of a photon into a neutrino-antineutrino pair.

It is not possible to make any general statement concerning the relative importance of the photobeta process and excited-state beta decay: Such evaluation requires a knowledge of the level structure and *ft* values for decay *from* excited states (which only rarely can be determined experimentally). On the other hand the photobeta process is largely independent of such considerations (except for photobeta to excited states as in Ca^{48}) and, since it does not interfere with excited-state beta decay, one can always obtain a *lower limit* to rates for the processes $Z \rightarrow Z+2$ and $\gamma \rightarrow \nu+\bar{\nu}$ as discussed above. For example, from Fig. 10 we see that at 12×10^8 °K every Te^{125} nucleus produces about 0.3 MeV in neutrinos *at least* as rapidly as once every 10^4 years, regardless of the excited-state beta-decay rate.

[13] R. L. Brodzinski and D. C. Conway, Phys. Rev. **138**, B1368 (1965).
[14] Donald D. Clayton, Astrophys. J. **139**, 637 (1964).

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Equations of Motion without Infinite Self-Action Terms in General Relativity*

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The role of the Bianchi identity in obtaining the equations of motion in general relativity is further discussed in connection with the problem of the infinite self-action terms. It is shown that because of this identity and under certain assumptions (concerning the type of singularity of the Christoffel symbols near the particle), there is a possibility of obtaining an equation of motion, free of infinite self-action terms, without referring to any renormalization procedure. This is the Infeld equation of motion which describes the motion of a particle of finite mass and in which the time coordinate is taken to be the independent parameter. Besides that, however, the Bianchi identity imposes certain constraints that the field functions have to satisfy in addition to the equation of motion.

INTRODUCTION

ONE of the most important achievements, and one of the most attractive features, of the general theory of relativity since its publication is the discovery by Einstein and Grommer[1] in 1927 that the equations of motion need not be postulated separately in addition to the gravitational field equations; rather, they follow from them.

Indeed, 11 years later, Einstein, Infeld, and Hoffmann[2] succeeded in developing an approximation method by means of which they found the equations of motion of finite-mass particles represented as singularities of the gravitational field. The equation of motion for each particle obtained in this way (Einstein-Infeld-Hoffmann equation) includes a relativistic correction of order $1/c^2$ to the well-known Newton equation and tends to the latter for velocities $v \ll c$, where c is the speed of light in vacuum. This additional force term is the one associated with the advance of the perihelion of the planetary motion, a phenomenon to which only general relativity can give a satisfactory answer.

Following Einstein, Infeld, and Hoffmann's original paper, other works on the subject were devoted to improving the mathematical methods towards better understanding of the problem.[3] Nowadays the Einstein-Infeld-Hoffmann equation has been extended to include

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[1] A. Einstein and J. Grommer, Sitzber. Preuss. Akad. Wiss. Physik. Math. Kl. **2**, 235 (1927).
[2] A. Einstein, L. Infeld, and B. Hoffmann, Ann. Math. **39**, 65 (1938); L. Infeld, Phys. Rev. **53**, 836 (1938); A. Einstein and L. Infeld, Ann. Math. **41**, 455 (1940); A. Einstein and L. Infeld, Can. J. Math. **1**, 209 (1949).
[3] See, for example, J. N. Goldberg, in *Gravitation, An Introduction to Current Research*, edited by L. Witten (John Wiley & Sons, Inc., New York, 1962), Chap. 3.