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RESEARCH ON THE FFT METHOD APPLIED TO A DYNAMIC VISCOELASTOMETER *

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ABSTRACT

In this paper the Fast Fourier Transform (FFT) method of data processing is developed for the dynamic viscoelastometer. The FFT method and also the time-domain method currently used for commercially available instruments are analysed and tested on a dynamic viscoelastometer made by the authors. The FFT method is better than the time-domain method in terms of data reproducibility and anti-disturbance, and it can be used to determine the dynamic mechanical frequency spectrum and nonlinear viscoelasticity. A displacement correction routine within the program is also given.

INTRODUCTION

Dynamic mechanical tests are very useful in studying the molecular motion of macromolecular materials. Such tests are sensitive to glass transition, crystallinity, cross-linking, phase separation and so on [1]. They are therefore widely used in work on the theory and application of materials. Dynamic viscoelastometers are among the most commonly used dynamic mechanical testing instruments. Measurements using early instruments required constant operator attention. In order to improve range, accuracy and convenience, the instruments were automated in the late seventies [2,3]. Commercial instruments using microprocessors have been constructed, but their data processing still relies upon the conventional time-domain method. In this paper the Fast Fourier Transform (FFT) is used for data processing in a dynamic viscoelastometer. It is confirmed that the FFT method has significant advantages over the time-domain method.

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DYNAMIC MECHANICAL TEST

For macromolecular material, deformation response to applied force is partly elastic and partly viscous: in other words, the material displays viscoelasticity. If linear mechanical properties of the material are assumed and a sinusoidal stretching stress is exerted on the material

$$\sigma(t) = \sigma_m \sin \omega t = \frac{f_m}{S} \sin \omega t \tag{1}$$

where S is the cross-sectional area of the material, f_m the magnitude of the applied force and ω the angular frequency, its strain response will also be sinusoidal. Because of the viscoelasticity, the strain lags behind the stress

$$\epsilon(t) = \epsilon_m \sin(\omega t - \delta) = \frac{l_m}{L} \sin(\omega t - \delta) \tag{2}$$

where L is the length of the material, l_m the magnitude of the displacement and δ the phase shift between stress and strain. Therefore, the complex Young's modulus of the macromolecular material is given by

$$E = \frac{f_m L}{l_m S} \exp(j\delta) = |E| \exp(j\delta) = |E| \cos \delta + j|E| \sin \delta = E' + jE'' \tag{3}$$

where the real part E' is defined as the elastic modulus and represents the energy stored by the elastic deformation of the material, the imaginary part E'' is defined as the loss modulus and represents the energy dissipated in heat and $\tan \delta$ is called the loss tangent, equal to E''/E', and usually represents the dissipated energy or damping. From the measured spectra of $|E|$, E', E'' and $\tan \delta$ vs. temperature or frequency, the molecular motion of macromolecular materials can be studied.

DATA PROCESSING

At present, the time-domain method of data processing is used in dynamic viscoelastometers; that is, the time-domain displacement and applied force signals of the sample are directly used to calculate the dynamic viscoelasticity. This method is equivalent to evaluating the following integrals

$$IL = \int_0^{\pi/\omega} l(t)\, dt - \int_{\pi/\omega}^{2\pi/\omega} l(t)\, dt = \frac{4 f_m L}{\omega |E| S} \tag{4}$$

$$IF' = \int_0^{\pi/\omega} f(t)\, dt - \int_{\pi/\omega}^{2\pi/\omega} f(t)\, dt = \frac{4}{\omega} f_m \cos \delta \tag{5}$$

$$IF'' = \int_0^{\pi/\omega} f\left(t - \frac{\pi}{2\omega}\right) dt - \int_{\pi/\omega}^{2\pi/\omega} f\left(t - \frac{\pi}{2\omega}\right) dt = \frac{4}{\omega} f_m \sin \delta \tag{6}$$

where

$$l(t) = \frac{f_m L}{|E|S} \sin \omega t \tag{7}$$

is the displacement of the sample and

$$f(t) = f_m \sin(\omega t + \delta) \tag{8}$$

is the force applied.

Obviously the complex Young's modulus can be calculated from the relations

$$|E| = \left[\left(\frac{IF'}{IL}\right)^2 + \left(\frac{IF''}{IL}\right)^2\right]^{1/2} \frac{L}{S} \tag{9}$$

and

$$\tan \delta = \frac{IF''}{IF'} \tag{10}$$

APPLICATION OF FFT

The data processing approach used in dynamic viscoelastometers is the time-domain method but the modulus measured is complex. A complex function of frequency can be obtained by utilizing the Fourier Transform (FT) of a function of time $x(t)$ $(0 < t < T_0)$

$$X(\omega) = \int_0^{T_0} x(t) \exp(-j\omega t)\, dt \tag{11}$$

and $X(\omega)$ is not only complex but also includes components of many frequencies. Provided the FTs of the displacement and applied force are performed, the complex modulus can also be obtained. The displacement and applied force can be expressed as

$$l(t) = \frac{f_m L}{|E|S} \sin \omega_m t \tag{12}$$

and

$$f(t) = f_m \sin(\omega_m t + \delta) \tag{13}$$

respectively. Then the FTs of the displacement and force at ω_m are as follows

$$F_m[l(t)] = \frac{T_0 f_m L}{2|E|S} \exp\left(-j\frac{\pi}{2}\right) \tag{14}$$

and

$$F_m[f(t)] = \frac{T_0}{2} f_m \exp\left[-j\left(\frac{\pi}{2} - \delta\right)\right] \tag{15}$$

The complex Young's modulus at ω_m can then be obtained from

$$E_m = \frac{F_m[f(t)]\,L}{F_m[l(t)]\,S} \tag{16}$$

This method can be used when the driving signal includes two frequencies or more. Since the discrete sequence of length-limited can be calculated and the time needed should be as short as possible using a computer, the Fast Fourier Transform [4] is used rather than eqn. (11).

The FFT data processing method is obviously better than the time-domain method and has some functions that the latter lacks.

According to eqns. (4)–(6), the displacement of the sample and the applied force should be sinusoidal and there should be no error in the integral intervals: otherwise the results will be imprecise. The FFT method can effectively overcome the faults of the time-domain method. It can be used with non-sinusoidal driving signals and is good at resisting the static component and at minimizing disturbance in the signals. The reason is that, after the FFT is performed, the components of different frequencies will occur at different locations in the spectrum; the frequencies of the disturbance and the static component are usually different from the frequency of the driving signal so that the results are still correct. Therefore, the reproducibility of the FFT method is better than that of the time-domain method.

The variables f_m, l_m, $\cos\delta$ and $\sin\delta$ can be obtained by means of the electronic circuits in commercially available instruments but they are complex. If the time-domain method is utilized in a computer, that is, digital integrals are performed to calculate eqns. (4)–(6), the sampling frequency needed is high. It is more appropriate to utilize the FFT method in conjunction with a computer. If the FFT method is used, the sampling frequency needed is only twice as high as the highest frequency in the signal [4], the complex phase and magnitude electronic circuits can be eliminated and the number of electronic components reduced, thus improving the reliability of the instrument.

From the macromolecular physics involved it is known that measurements should be made over a wide range of both temperatures and frequencies [1] in order to exploit the full potentialities of dynamic mechanical tests. If the dynamic viscoelasticity–frequency spectrum is measured using the time-domain method, a complex scanning ultra-low-frequency generator and long measurement times are needed. If the driving signal includes many frequencies and the frequency spectrum analyses of both the displacement and the applied force are performed by FFT, the frequency spectrum can be determined quickly.

In practice, the viscoelasticity of macromolecular material is actually nonlinear or the displacement of the material and the applied force are not sinusoidal at the same time. It is difficult to measure the nonlinear viscoelasticity instrumentally by means of the time-domain method. Provided the FFT method is used, the magnitudes and phases of the displacement and applied force can be obtained at different points within the frequency spectrum. The nonlinear viscoelasticity of the sample may thus be conveniently measured so that it can be further studied.

INSTRUMENTATION

A dynamic viscoelastometer was made by us and a block diagram is shown in Fig. 1.

The temperature of the furnace is controlled by means of a programming temperature controller. The driving signal is produced by a program in the computer and converted into the driving force by the digital-to-analogue (D/A) converter, driving power amplifier and electromagnetic driver in order to improve stability and to make it possible to generate non-sinusoidal waveforms. The displacement of the sample is produced by the applied force. The displacement and force are converted into voltages by the respective detectors. Each voltage is amplified and filtered by the amplifier and band-pass filter and converted into digital form by the analogue-to-digital (A/D) converter. The sampled displacement and applied force signals are stored on the floppy disk of the computer. Next, the sampled data are transferred into memory and the results are calculated in turn by the FFT

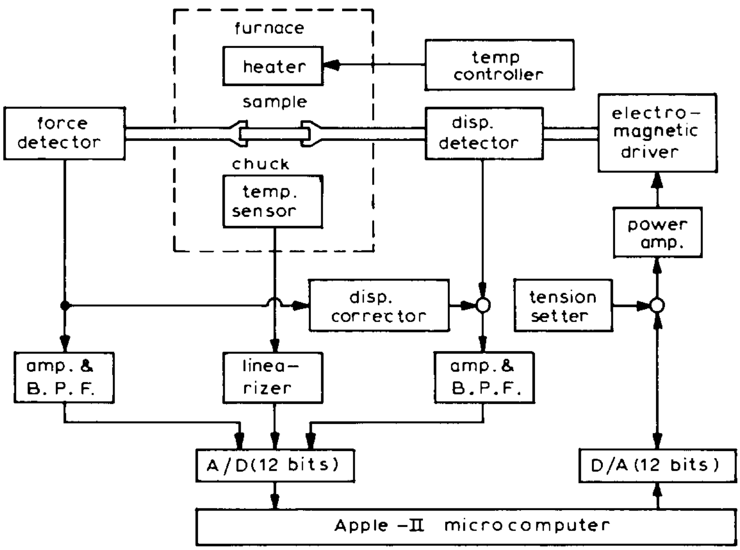

Fig. 1. Block diagram of the instrument: B.P.F., band-pass filter; A/D, analogue-to-digital converter; D/A, digital-to-analogue converter.

and time-domain method programs so that the two methods can be compared.

In the FFT program, the digital smoothing, Hamming weighting and FFT of the displacement and applied force are performed [4,5] and the complex Young's modulus is calculated from

$$E_i = \frac{|F_i[f(t)]| \exp(j\theta_{f_i}) L}{|F_i[l(t)]| \exp(j\theta_{l_i}) S} = |E_i| \exp(j\delta_i)$$

where $|F[f(t)]| \exp(j\theta_f)$ is the FFT of the applied force, $|F[l(t)]| \exp(j\theta_l)$ is the FFT of the displacement and subscript i denotes the frequency i.

In the time-domain program, the zero points of the displacement, $t = 0$, π/ω, $2\pi/\omega$, are sought as the integral intervals and then eqns. (4)–(10) are calculated.

Since the additional displacement of the apparatus itself is included in the detected displacement, the displacement must be corrected or the modulus and loss angle will be incorrect [6]. The displacement-correction system may be composed of electronic circuits, but the phase-shift circuit is in the system and its phase shift is relative to the frequency. The system cannot be used when the driving signal includes two or more frequencies. A suitable correction system has been incorporated in the program and a block diagram is shown in Fig. 2.

In order to correct the displacement exactly, the following equations can be obtained

$$\theta_a = \theta_c \tag{18}$$

and

$$K_e K_l = K_f K_c \tag{19}$$

Because of the location of the phase corrector, the additional phase shift is

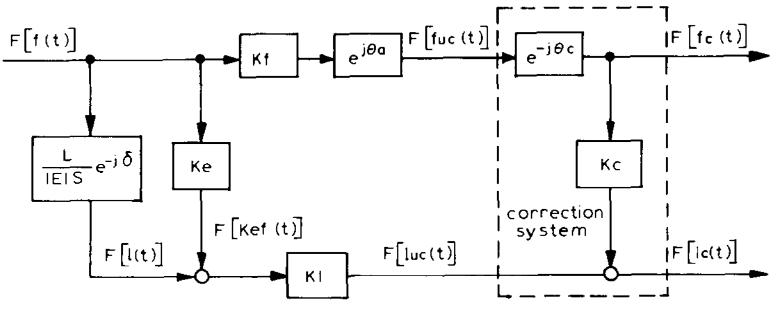

Fig. 2. Block diagram of the displacement correction system: $f(t)$, applied sinusoidal force; $l(t)$, displacement of sample; K_e, additional displacement coefficient of apparatus; $K_e f(t)$, additional displacement; K_f and $\exp(j\theta_a)$, coefficient and phase shift from force detector to FFT; K_l, coefficient from displacement detector to FFT; θ_c, correcting phase angle; K_c, correcting displacement coefficient; subscripts uc denote uncorrected and c corrected.

rectified when the displacement is corrected. θ_c and K_c can be obtained from $F[f_{uc}(t)]$ and $F[l_{uc}(t)]$ in the experiment.

This system can be used when the driving signal includes many frequencies and $F_i[f_{uc}(t)]$ and $F_i[l_{uc}(t)]$ ($i = 1, 2, \ldots, n$) can be rectified by means of the phase shifters $\exp(-j\theta_{ci})$ and displacement correctors K_{ci} ($i = 1, 2, \ldots, n$).

Using a sinusoidal driving signal at 3 Hz, PMMA was tested on this correction system with good results.

EXPERIMENTAL

Using a sinusoidal driving signal at 3 Hz and a furnace heating rate of 1.5 °C min^{-1}, PVC film, PMMA and PET film were tested on the instrument made by the authors.

The experimental results for the PVC film were as follows. The peak temperatures (those at maximum damping) measured by the two methods are listed in Table 1 and the temperature spectra are shown in Figs. 3 and 4. This film was successively tested ten times at room temperature and the mean square deviations of the modulus and loss angle are listed in Table 2. In addition, the mean square deviations were calculated from the peak temperatures: that of the FFT method was 0.71 °C and that of the time-domain method 1.03 °C.

It has been experimentally confirmed that the spectrum measured by the FFT method is compatible with that obtained using the time-domain method; the average deviation of the peak temperatures determined by the two methods was only 1 °C and the reproducibility of the FFT method is better than that of the time-domain method.

TABLE 1

Peak temperatures of PVC

No.	Sample size (mm)	Displacement ($\times 10^3$ mm)	Temperature (°C) [a]	
			FFT	Time-domain
1	48 ×6×0.23	12.67	62.2	61.5
2	48.5×6×0.23	7.31	64.0	64.7
3	48.5×6×0.23	5.65	61.6	63.0
4	49 ×6×0.23	6.25	61.9	63.1
5	49 ×6×0.23	6.95	61.7	63.9
6	48.5×6×0.23	6.65	61.9	62.0
7	49 ×6×0.23	7.18	62.2	62.4
8	49 ×6×0.23	7.06	62.0	62.0
9	48.5×6×0.23	7.18	62.2	62.1

[a] Obtained by interpolation.

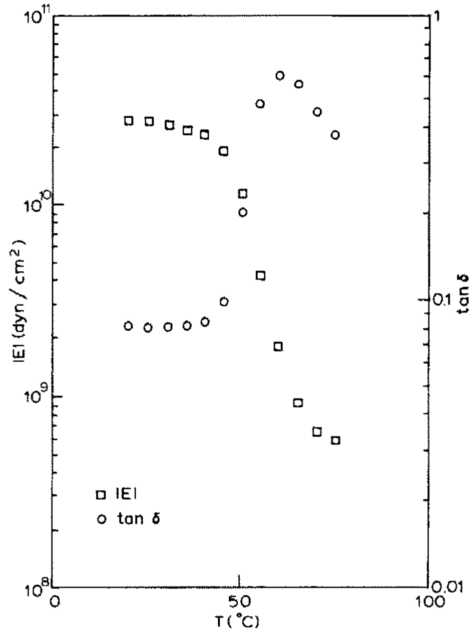

Fig. 3. Spectrum obtained using the FFT method (No. 6).

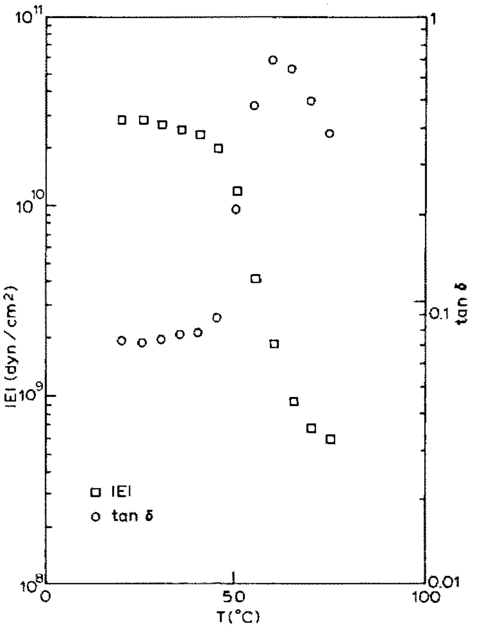

Fig. 4. Spectrum obtained using the time-domain method (No. 6).

TABLE 2

Mean square deviation of $|E|$ and δ

Method	$\sigma_{\|E\|}$ (dyn cm^{-2})	σ_δ (rad)
FFT	1.12×10^8	0.0030
Time-domain	5.29×10^8	0.0066

In order to compare the anti-disturbance ability of the two methods, a simulative calculation was made. When the frequency of the driving signal was 40 Hz, that of the sinusoidal disturbance was 50 Hz and its magnitude was 20% of that of the driving signal; the modulus error of the FFT method was only 0.5% of that of the time-domain method and the loss angle error was only 6.1%. It has therefore been demonstrated that the FFT method is good at anti-disturbance.

CONCLUSIONS

It has been shown that FFT can be used with the dynamic viscoelastometer. The dynamic mechanical spectrum measured by the FFT method is compatible with that obtained using the time-domain method. The FFT method is better than the time-domain method in terms of reproducibility and anti-disturbance. If the FFT method is used, the number of parts can be reduced and the instruments made more reliable. The FFT method may be used for determining the dynamic mechanical frequency spectrum and nonlinear viscoelasticity, but it is relatively difficult to use the time-domain method for the same purposes.

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