THERMODYNAMICS OF SOLUTIONS OF ACETONITRILE WITH 2-METHYL-l-PROPANOL

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ABSTRACT

The thermodynamic properties of solutions containing 2-methyl-1-propanol and acetonitrile have been analysed using an association model, which assumes the self-association and solvation of 2-methyl-1-propanol and acetonitrile, and includes a non-polar interaction term given by the NRTL equation. Vapour-liquid and liquid-liquid equilibria and excess molar enthalpy data for ternary mixtures including 2-methyl-1-propanol, acetonitrile and a non-associating component have been predicted by the association model using binary parameters only.

INTRODUCTION

The thermodynamic properties of binary and ternary solutions containing acetonitrile and simple aliphatic alcohols, namely methanol, ethanol [l], propanols [2] and l-butanol [3], have been predicted over the entire concentration range using an association model based on mole fraction statistics. The association of 2-methyl-l-propanol has been examined in the accurate prediction of the thermodynamic properties of binary and ternary mixtures of the alcohol and hydrocarbons [4]. In this study the thermodynamic properties of binary and ternary mixtures containing acetonitrile and 2-methyl-1-propanol are analysed by the association model. Experimental data for ternary mixtures containing 2-methyl-1-propanol and acetonitrile were obtained from recent publications as follows: the vapour-liqiud equilibria of 2-methyl-1-propanol + acetonitrile + benzene at 60° C from ref. 5, the liquid-liquid equilibria of 2-methyl-1-propanol + acetonitrile + cyclohexane, 2-methyl-1-propanol + acetonitrile + n -hexane and 2-methyl-1-propanol + $\text{acetonitrile} + n\text{-heptane}$ at 25° C from ref. 6 and excess molar enthalpy of 2-methyl-1-propanol + acetonitrile + benzene at 25° C from ref. 7.

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This paper shows how well the vapour-liquid and liquid-liquid equilibria and excess molar enthalpy data of mixtures containing acetonitrile and 2-methyl-1-propanol can be predicted using the association model with binary parameters only.

ASSOCIATION MODEL

We consider a ternary mixture formed by 2-methyl-1-propanol, acetonitrile and benzene. A denotes the alcohol, B denotes acetonitrile and C represents benzene. The association of the alcohol and acetonitrile and the solvation between all of the components in the mixture can be characterized by the following equilibrium constants defined in terms of the mole fractions of chemical species present in the mixture.

(1) The alcohol forms linear open polymers as well as cyclic polymers by hydrogen bonding.

$$
K_2 = x_{A_2}/x_{A_1}^2 \text{ for } A_1 + A_1 = A_2 \tag{1}
$$

$$
K_3 = x_{A_3} / x_{A_2} x_{A_1} \text{ for } A_1 + A_2 = A_3
$$
 (2)

$$
K = x_{A_{i+1}} / x_{A_i} x_{A_1} \text{ for } A_1 + A_i = A_{i+1}, i \ge 3
$$
 (3)

$$
K_{cy} = \theta / i = A_i(cyclic) / A_i(open) \text{ for } A_i(open) = A_i(cyclic), i > 4
$$
 (4)

(2) Acetonitrile forms cyclic dimers and linear polymers.

$$
K'_{\mathbf{B}} = x_{\mathbf{B}_2}(\text{cyclic}) / x_{\mathbf{B}_1}^2 \text{ for } \mathbf{B}_1 + \mathbf{B}_1 = \mathbf{B}_2(\text{cyclic})
$$
 (5)

$$
K_{\mathbf{B}} = x_{\mathbf{B}_{i+1}} / x_{\mathbf{B}_{i}} x_{\mathbf{B}_{1}} \text{ for } \mathbf{B}_{1} + \mathbf{B}_{i} = \mathbf{B}_{i+1}, i \ge 1
$$
 (6)

(3) The alcohol linear polymers, acetonitrile polymers and benzene solvate each other.

$$
K_{A_i, B} = x_{A_i, B}/x_{A_i}x_{B_i} \text{ for } A_i + B_1 = A_i, B \text{ and } i \ge 1
$$
 (7)

$$
K_{A_i, B_j} = x_{A_i, B_j} / x_{A_i} x_{B_j} \text{ for } A_i + B_j = A_i, B_j, i \ge 1, j \ge 1
$$
 (8)

$$
K_{A,C} = x_{A,C}/x_{A,i}x_{C_1} \text{ for } A_i + C_1 = A_i, i \ge 1
$$
 (9)

$$
K_{BC} = x_{BC_1}/x_{B_1}x_{C_i} \text{ for } B_1 + C_1 = BC
$$
 (10)

These equilibrium constants change with temperature according to the van't Hoff relation.

 $\frac{\partial \ln K_2}{\partial} = -\frac{h_2}{h_1}$ $\partial(1/T)$ *R* $\frac{\partial \ln K}{\partial t} = -\frac{h_A}{\partial t}$ $d(1/I)$ **K** $\partial \ln K_3$ 2 $h_A - h_2$ $\partial(1/T)$ *R* $\frac{\partial \ln \theta}{\partial t} = -\frac{h_A}{h}$ $\partial(1/T)$ R

$$
\frac{\partial \ln K'_{\mathbf{B}}}{\partial (1/T)} = -\frac{h'_{\mathbf{B}}}{R} \qquad \frac{\partial \ln K_{\mathbf{B}}}{\partial (1/T)} = -\frac{h_{\mathbf{B}}}{R} \qquad (11)
$$
\n
$$
\frac{\partial \ln K_{\mathbf{A},\mathbf{B}}}{\partial (1/T)} = -\frac{h_{\mathbf{A},\mathbf{B}}}{R} \qquad \frac{\partial \ln K_{\mathbf{A},\mathbf{B}_j}}{\partial (1/T)} = -\frac{h_{\mathbf{A},\mathbf{B}_j}}{R}
$$
\n
$$
\frac{\partial \ln K_{\mathbf{A},\mathbf{C}}}{\partial (1/T)} = -\frac{h_{\mathbf{A},\mathbf{C}}}{R} \qquad \frac{\partial \ln K_{\mathbf{B}\mathbf{C}}}{\partial (1/T)} = -\frac{h_{\mathbf{B}\mathbf{C}}}{R} \qquad \frac{\partial (1/T)}{\partial (1/T)} = -\frac{h_{\mathbf{B}\mathbf{C}}}{R}
$$

Furthermore, the non-polar interactions between all the components are given by the NRTL equation [8].

The activity coefficients of each component in the ternary mixtures, derived from the model assumptions, are expressed by

$$
\ln \gamma_I = \ln \frac{x_{I_1}}{x_{I_1}^* x_I} + \frac{\sum_{J=A} \tau_{JI} G_{JI} x_J}{\sum_{K=A} G_{KI} x_K} + \sum_{J=A} \frac{x_J G_{IJ}}{\sum_{K=A} G_{KI} x_K} \left(\tau_{IJ} - \frac{\sum_{R=A} x_R \tau_{RJ} G_{RJ}}{\sum_{K=A} G_{KJ} x_K} \right)
$$
(12)

where

$$
\tau_{JI} = \left(\frac{g_{JI} - g_{II}}{\rho}\right) / T \tag{13}
$$

$$
G_{JI} = \exp(-\alpha_{JI}\tau_{JI}) \tag{14}
$$

 α_{JI} (= α_{IJ}) is the non-randomness parameter set as 0.3 and $x_{C_1} = 1$ for benzene.

The monomer mole fractions x_{A_1} , x_{B_1} , and x_{C_1} are obtained by solving eqns. (15)-(19) simultaneously. The mass balance equations are related to the mole fractions in terms of the equilibrium constants.

$$
x_{A} = \left\{ \left[1 + K_{A,B}x_{B_{1}} + K_{A,C}x_{C_{1}} + \frac{K_{A,B_{1}}K_{B}x_{B_{1}}^{2}}{(1 - w)} \right] \times \left[x_{A_{1}} + 2K_{2}x_{A_{1}}^{2} + \frac{K_{2}K_{3}x_{A_{1}}^{3}(3 - 2z)}{(1 - z)^{2}} \right] + \frac{K_{2}K_{3}K^{2}\theta x_{A_{1}}^{5}}{(1 - z)^{2}} \right\} / S \qquad (15)
$$

$$
x_{B} = \left\{ \left[K_{A,B}x_{B_{1}} + \frac{K_{A,B_{1}}K_{B}x_{B_{1}}^{2}(2 - w)}{(1 - z)^{2}} \right] \left[x_{A_{1}} + K_{2}x_{A_{1}}^{2} + \frac{K_{2}K_{3}x_{A_{1}}^{3}}{(1 - z)^{2}} \right] \right\}
$$

$$
\left| \left[\begin{array}{cc} \mathbf{A}_{\mathbf{A}_{i}} \mathbf{B} \mathbf{A}_{\mathbf{B}_{i}} + (1-w)^{2} & \left| \begin{array}{cc} \mathbf{A}_{\mathbf{A}_{i}} + \mathbf{A}_{2} \mathbf{A}_{\mathbf{A}_{i}} + (1-z) \end{array} \right| \right. \\ + 2 K_{\mathbf{B}}' x_{\mathbf{B}_{i}}^{2} + \frac{x_{\mathbf{B}_{i}}}{\left(1-w\right)^{2}} + K_{\mathbf{B} \mathbf{C}} x_{\mathbf{B}_{i}} x_{\mathbf{C}_{i}} \right) / S \tag{16}
$$

$$
x_{\rm C} = \left\{ K_{\rm A,C} x_{\rm C_1} \left[x_{\rm A_1} + K_2 x_{\rm A_1}^2 + \frac{K_2 K_3 x_{\rm A_1}^3}{(1-z)} \right] + K_{\rm BC} x_{\rm B_1} x_{\rm C_1} + x_{\rm C_1} \right\} / S \tag{17}
$$

where $z = Kx_{A_1}$, $w = K_Bx_{B_1}$ and the stoichiometric sum S is given by

$$
S = \left[1 + K_{A,B}x_{B_1} + K_{A,C}x_{C_1} + \frac{K_{A,B_1}K_Bx_{B_1}^2}{(1 - w)} \right]
$$

\n
$$
\times \left[x_{A_1} + 2K_2x_{A_1}^2 + \frac{K_2K_3x_{A_1}^3(3 - 2z)}{(1 - z)^2} \right]
$$

\n
$$
+ \frac{K_2K_3K^2\theta x_{A_1}^5}{(1 - z)} + \left[K_{A,B}x_{B_1} + K_{A,C}x_{C_1} + \frac{K_{A,B_1}K_Bx_{B_1}^2(2 - w)}{(1 - w)^2} \right]
$$

\n
$$
\times \left[x_{A_1} + K_2x_{A_1}^2 + \frac{K_2K_3x_{A_1}^3}{(1 - z)} \right]
$$

\n
$$
+ 2K'_Bx_{B_1}^2 + \frac{x_{B_1}}{(1 - w)^2} + 2K_{BC}x_{B_1}x_{C_1} + x_{C_1}
$$
 (18)

The total sum of mole fractions of the species present in the mixture must be equal to unity.

$$
\left[1 + K_{A,B}x_{B_1} + K_{A,C}x_{C_1} + \frac{K_{A,B_j}K_Bx_{B_1}^2}{(1 - w)}\right]\left[x_{A_1} + K_2x_{A_1}^2 + \frac{K_2K_3x_{A_1}^3}{(1 - z)}\right]
$$

$$
-\frac{K_2K_3\theta}{K^3}\left[\ln(1 - z) + z + \frac{z^2}{2} + \frac{z^3}{3} + \frac{z^4}{4}\right]
$$

$$
+ K'_Bx_{B_1}^2 + \frac{x_{B_1}}{(1 - w)} + K_{BC}x_{B_1}x_{C_1} + x_{C_1} = 1
$$
(19)

The monomer mole fractions of the alcohol and acetonitrile in the pure liquid state are obtained from eqns. (20) and (21), respectively.

$$
x_{A_1}^* + K_2 x_{A_1}^{*^2} + \frac{K_2 K_3 x_{A_1}^{*^3}}{(1 - z^*)} - \frac{K_2 K_3 \theta}{K^3}
$$

$$
\times \left[\ln(1 - z^*) + z^* + \frac{z^{*^2}}{2} + \frac{z^{*^3}}{3} + \frac{z^{*^4}}{4} \right] = 1
$$
 (20)

$$
K'_{\mathbf{B}} x_{\mathbf{B}_1}^{*^2} + \frac{x_{\mathbf{B}_1}^{*}}{(1 - w^*)} = 1
$$
 (21)

The ternary excess molar enthalpy of the mixture is expressed by

$$
H^{\rm E} = H_{\rm chem}^{\rm E} + H_{\rm phys}^{\rm E} \tag{22}
$$

where $H_{\text{chem}}^{\text{E}}$ and $H_{\text{phys}}^{\text{E}}$ are given by eqns. (23) and (26).

$$
H_{\text{chem}}^{E} = \left\{ \left[1 + K_{A,B}x_{B_{1}} + K_{A,C}x_{C_{1}} + \frac{K_{A,B_{1}}K_{B}x_{B_{1}}^{2}}{(1-w)} \right] \right\}
$$

\n
$$
\times \left[h_{2}K_{2}x_{A_{1}}^{2} + \frac{h_{A}K_{2}K_{3}x_{A_{1}}^{3}(2-z)}{(1-z)^{2}} \right] + \frac{h_{A}K_{2}K_{3}K^{2}\theta x_{A_{1}}^{5}}{(1-z)}
$$

\n
$$
+ \left[1 + K_{A,B_{1}} \left(x_{A_{1}} + K_{2}x_{A_{1}}^{2} + \frac{K_{2}K_{3}x_{A_{1}}^{3}}{(1-z)} \right) \right] \frac{h_{B}K_{B}x_{B_{1}}^{2}}{(1-w)^{2}} + h_{B}'K_{B}'x_{B_{1}}^{2}
$$

\n
$$
+ \left[x_{A_{1}} + K_{2}x_{A_{1}}^{2} + \frac{K_{2}K_{3}x_{A_{1}}^{3}}{(1-z)} \right] \left[h_{A,B}K_{A,B}x_{B_{1}} + h_{A,C}K_{A,C}x_{C_{1}} \right]
$$

\n
$$
+ \frac{h_{A,B_{1}}K_{A,B_{1}}K_{B}x_{B_{1}}^{2}}{(1-w)} \right] + h_{BC}K_{BC}x_{B_{1}}x_{C_{1}} \Bigg\} / S
$$

\n
$$
- x_{A} \left[h_{2}K_{2}x_{A_{1}}^{*^{2}} + \frac{h_{A}K_{2}K_{3}x_{A_{1}}^{*}(2-z^{*})}{(1-z^{*})^{2}} + \frac{h_{A}K_{2}K_{3}K^{2}\theta x_{A_{1}}^{*^{2}}}{(1-z^{*})} \right] / S_{A}^{*}
$$

\n
$$
- x_{B} \left[h_{B}'K_{B}'x_{B_{1}}^{*^{2}} + \frac{h_{B}K_{B}x_{B_{1}}^{*^{2}}}{(1-w^{*})^{2}} \right] / S_{B}^{*}
$$

\n(23)

where $z^* = Kx_{A_1}^*$ and $w^* = K_Bx_{B_1}^*$; S_A^* and S_B^* are given by

$$
S_A^* = x_{A_1}^* + 2K_2 x_{A_1}^{*^2} + \frac{K_2 K_3 x_{A_1}^{*^3} (3 - 2z^*)}{(1 - z^*)^2} + \frac{K_2 K_3 K^2 \theta x_{A_1}^{*^5}}{(1 - z^*)}
$$
(24)

$$
S_{\rm B}^* = 2K'_{\rm B}x_{\rm B_1}^2 + \frac{x_{\rm B_1}^*}{\left(1 - w^*\right)^2} \tag{25}
$$

$$
H_{\text{phys}}^{\text{E}} = R \sum_{I=A} x_I \left[\frac{\sum_{J=A} x_J \frac{\partial (\tau_{JI} G_{JI})}{\partial (1/T)}}{\sum_{K=A} G_{KI} x_K} - \frac{\sum_{J=A} \tau_{JI} G_{JI} x_J \sum_{K=A} x_K \frac{\partial (G_{KI})}{\partial (1/T)}}{\left(\sum_{K=A} G_{KI} x_K\right)^2} \right] \tag{26}
$$

The temperature dependence of the energy parameters is assumed to be given by

$$
g_{JI} - g_{II} = C_{JI} + D_{JI}(T - 273.15) \tag{27}
$$

DATA REDUCTION

Vapour-liquid equilibrium data reduction was performed using the thermodynamic relation

$$
\Phi_I y_I P = \gamma_I x_I \Phi_I^s P_I^s \exp\left[v_I^L (P - P_I^s) / RT\right]
$$
\n(28)

where y is the vapour-phase mole fraction and P is the total pressure. P^s is the vapour pressure of the pure component; this was taken from the original references or was calculated using the Antoine equation whose constants are available from the literature [9]. The fugacity coefficients Φ were estimated from the volume-explicit virial equation of state truncated after the second term. The second virial coefficients were calculated by the method of Hayden and O'Connell [10]. The liquid molar volume v^L was calculated from the modified Rackett equation [11]. Values of the related parameters for the evaluation of these properties are given in the compilation of Prausnitz et al. [12].

Liquid-liquid equilibria were evaluated from the simultaneous solution of the following equations

$$
\left(\gamma_I x_I\right)^{\text{I}} = \left(\gamma_I x_I\right)^{\text{II}}\tag{29}
$$

$$
\left(\sum_{I=A} x_I\right)^1 = 1, \qquad \left(\sum_{I=A} x_I\right)^1 = 1 \tag{30}
$$

where the superscripts I and II indicate the equilibrated liquid phases.

CALCULATED RESULTS

The association constants and enthalpies of dimerization and polymerization for 2-methyl-1-propanol and acetonitrile were taken from previous papers: for 2-methyl-1-propanol, $K_2 = 30$, $K_3 = 85$, $K = 30$ and $\theta = 70$ at 25 ° C, $h_2 = -21.2$ kJ mol⁻¹ and $h_A = -23.5$ kJ mol⁻¹ [4,7]; for acetonitrile $K_B = 8.35$ and $K_B = 2.1$ at 45°C, $h_B' = -8.9$ kJ mol⁻¹ and $h_B = -6.7$ kJ mol^{-1} [1-3,13]. The solvation constants and enthalpy of complex formation between 2-methyl-1-propanol, acetonitrile and benzene were set as follows [7]: for the alcohol polymer and acetonitrile, $K_{A,B} = 25$ at 60°C and $h_{A,B} = -22$ kJ mol⁻¹; for the alcohol polymer and acetonitrile polymer, $K_{A_i B_j} = 20$ at 60°C and $h_{A_i B_j} =$ and benzene, $K_{A,C}$ -16.8 kJ mol⁻¹; for the alcohol polymer $= 2.5$ at 25 °C and $h_{A,C} = -8.2$ kJ mol⁻¹; for acetonitrile and benzene, $K_{BC} = 0.2$ at 45^oC and $h_{BC} = -5.2$ kJ mol⁻¹. All *h* values were assumed to be independent of temperature.

The calculated results are given in Table 1 together with the energy parameters obtained on fitting the association model to the experimental vapour-liquid equilibria of binary mixtures. The energy parameters were

TABLE 1

Binary parameters and absolute arithmetic mean deviations as obtained from vapour-liquid equilibrium data reduction

 $^{\circ}$ MS = mutual solubility data.

obtained from the minimization of the sum-of-squares of relative deviations in pressure plus the sum-of-squares of deviations in vapour-phase mole fraction using the simplex method [21]. Figures $1(a)$ and $1(b)$ illustrate two

TABLE 2

Binary parameters and absolute arithmetic mean deviations as obtained from excess enthalpy data reduction at 25°C

Fig. 1. Vapour-liquid equilibria for (a) 2-methyl-1-propanol(A) + acetonitrile(B) at 60 °C and (b) 2-methyl-1-propanol(A) + benzene(B) at 45° C. Calculated (-----). Experimental (\bullet): (a) data of Nagata [5]; (b) data of Brown and Smith [14].

Fig. 2. Excess molar enthalpies for two binary systems at 25° C. Calculated ($---$). Experimental: (A) 2-methyl-1-propanol + acetonitrile, data of Nagata and Tamura [7] (\bullet); (B) 2-methyl-1-propanol + benzene, data of Nagata and Tamura [7] (\circ), data of Brown et al. [14] (A).

Fig. 3. Ternary liquid-liquid equilibria at 25°C. Calculated (-----). Experimental tie-line **(O---O) data of Nagata [6]: (A) acetonitrile+ 2-methyl-1-propanol+cyclohexane; (B) acetonitrile + 2-methyl-1-propanol+ n-hexane; (C) acetonitrile + 2-methyl-l-propanol+ n-heptane.**

examples which compare the calculated vapour-liquid equilibria with the experimental results. Table 2 lists the results of the excess molar enthalpy determined from the binary correlations for the three binary systems constituting the 2-methyl-1-propanol + acetonitrile + benzene system at 25° C. Figure 2 compares the experimental excess molar enthalpy with the calculated results from the association model.

Ternary calculations were carried out using the association model with the binary parameters obtained from Table 2. The absolute arithmetic mean diviations between the experimental and predicted values for the vapour-liquid equilibria of 2-methyl-1-propanol(A) + acetonitrile(B) + benzene(C) at 60° C were $\Delta y_A = 0.0057$, $\Delta y_B = 0.0076$ and $\Delta y_C = 0.0050$ for the vapour-mole fractions, which were calculated from $\Delta v_r = \sum_{i=1}^{N} v_r / J$, $\exp(t) - v_t$ (*J, calc)* $|/N$, where N is the number of data points. The absolute arithmetic mean deviations in the pressure were 0.640 kPa, and the average relative deviation was 0.12%. The ternary liquid-liquid equilibrium predictions for the three mixtures are shown in Fig. 3. The absolute arithmetic mean deviation of the excess molar enthalpy was 23.8 J mol⁻¹ for the 70 data points of 2-methyl-1-propanol + acetonitrile + benzene at 25° C.

Finally, we can conclude that the association model predicts well the thermodynamic properties of binary and ternary mixtures containing acetonitrile and 2-methyl-1-propanol.

LIST OF SYMBOLS

Greek letters

Subscripts

Superscripts

- L liquid
- *S* saturated
- * pure liquid

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