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WEIGHING WITH STRAIN GAUGES*

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ABSTRACT

Weighing with strain gauges is shown to have some very attractive features. An analysis is given of the limitation of the sensitivity due to various *noise sources*. It is shown that it is possible to use strain gauge balances somewhat below the milligram region.

THEORY

Strain gauges are used for the measurement of dilatations. However, if the elastical properties of the body to which the gauges are attached are known, forces or moments of force causing the dilatations can be calculated. In such a situation, the sensitivity of the force measurement is determined by that elastical body. If it is a thin spring, small forces will already cause large outputs of the strain gauge measurement bridge, the thinner the spring, the larger the sensitivity.

If we want to know the greatest possible sensitivity of such force measurement, we have to deal with the situation where the gauge is not attached to any body at all and where the elastical properties of the strain gauge itself determines the sensitivity. One can even go one step further and use a single bare strain wire, as the substrates and glues used in common strain gauges produce disadvantages with respect to sensitivity and reproducibility.

In the present paper, we shall discuss the question of the utmost sensitivity within reach with such a single wire. We shall restrict ourselves to the most simple situation where the force to be measured acts in the direction of the wire. We shall not discuss the possibilities of using combinations of wires and of mechanical levers.

There are various disturbances which limit the sensitivity of such a simple wire-system. First of all, we have the fundamental sensitivity limit caused by the Nyquist noise. We shall restrict ourselves to the noise caused by the strain wire which implies that it is supposed that a special low noise amplifier is used in the measurement bridge. The other implication is that the other bridge elements do not

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contribute to the noise, this is not always realistic and brings along an uncertainty of a factor of the order of 2 into our estimate, an uncertainty which can easily be corrected; these corrections will depend upon the type of bridge used.

The Nyquist noise is given by:

$$\langle(\Delta U_N)^2\rangle = 4KTRB = 16KT\,\frac{l\rho}{\pi d^2}\,B = N \tag{1}$$

where ΔU_N stands for the voltage fluctuations due to Nyquist noise, R for the resistance of the wire under consideration, B for the bandwith of the amplifier used, l and d for the length and diameter of the strain wire, ρ for its specific resistance and where K is Boltzmann's constant. N is a quantity defined by the last equality for later use.

This noise has to be compared with the signal for which we use

$$U_S = \frac{RG}{EA}\,IF = \frac{16\rho Gl}{\pi^2 Ed^4}\,IF = SIF \tag{2}$$

where G stands for the gauge factor, A for the cross-sectional area of the wire, E for its Youngs' modulus, I for the current through the wire and F for the force to be measured, S is a quantity defined by the last equality for later use. According to eqns (1) and (2) we get for the signal to noise ratio SNR

$$\mathrm{SNR} = \frac{SIF}{N^{1/2}} \tag{3}$$

Equations (3) shows that the SNR increases with the current I and one should conclude that very high values of I should be favourable for the measurements. In reality, eqn (3) is only valid for small values of I as only there the Nyquist noise is the main disturbance. To deal with the other disturbances as well, we shall classify the disturbances with respect to the exponent of I.

From eqn (1), we see that the Nyquist noise is charactarised by this exponent being zero. When the exponent is one, we have to do with errors or fluctuation in the resistance which are not dependent upon the value of I.

$$\Delta U = I\Delta R \tag{4}$$

We shall discuss errors in the resistance caused by temperature fluctuations of the ambient temperature.

The value of ΔT will be dependent upon the quality of the thermostat used. It follows:

$$\Delta U = I\alpha R\Delta T = I\alpha\,\frac{4\rho l}{\pi d^2}\,\Delta T = AI\Delta T \tag{5}$$

Where α stands for the temperature coefficient of the resistance and A is a constant defined by the last equation. In most measurements, the temperature fluctuation will be ruled out to a great extent by using a dummy resistance in the bridge. Equation (5) can than however still be used when using the erroneous difference in ambient temperature between sample and dummy caused by temperature inhomogeneities in the thermostat.

In eqn (5) it has been supposed that the temperature of the wire is equal to the temperature of its environment. In reality, specially at higher currents there will be an extra increase of the temperature of the wire due to the Joule heat. For this temperature increase, we shall use

$$\Delta T_J = \frac{I^2 R}{\pi \mathrm{Nu}\, \lambda l} \tag{6}$$

Where λ stands for the heat conductivity of the surrounding air and Nu for Nusselt's number which relates the heat dissipation from the wire to the velocity of convection currents in the surrounding air[1] by

$$\mathrm{Nu} = 0.34 + 0.65 \left(\frac{Vd}{\nu}\right)^{0.45} \tag{7}$$

Where V stands for the velocity of the convection of the air and ν for the kinematic viscosity of air.

Taking the convection velocities between 0 and $V_{max.}$, and remembering that when using a dummy wire, only the differences in the Nusselt number and thus of the convection velocities will play a role, we get from eqns (6) and (7)

$$\langle(\Delta T_J)^2\rangle^{1/2} = \frac{I^2 R}{(0.34)^2 \pi \lambda l}\, 0.1 \left(\frac{V_{max.} d}{\nu}\right)^{0.45} \tag{8}$$

Remembering that the Joule effect is an extra temperature effect, we can use (8) into (5) and we get for the errors in the voltage due to the Joule effect:

$$\langle(\Delta U_J)^2\rangle^{1/2} = J I^3 \tag{9}$$

where J stands for

$$J = \frac{\alpha \rho^2 l}{\pi^2 d^4 (0.34)^2 \lambda}\, 0.1 \left(\frac{V_{max.} d}{\nu}\right)^{0.45}$$

If we suppose that the different errors are independent, we get for the signal to noise ratio:

$$\mathrm{SNR} = \frac{SIF}{(N + A^2 I^2 \Delta T^2 + J^2 I^6)^{1/2}} \tag{10}$$

From eqn (10) it follows that SNR has a maximum when I satisfies:

$$I_{max.} = \left(\frac{N}{2J^2}\right)^{1/6} \tag{11}$$

For this maximum value we get:

$$(\mathrm{SNR})_{max.} = \frac{SF}{\sqrt{A^2 \Delta T^2 + 3\left(\frac{JN}{2}\right)^{2/3}}} \tag{12}$$

As an example we have used the following specific values:

$l \;\; = 10^{-1}$ m
$d \;\; = 2 \times 10^{-5}$ m
$\rho \;\; = 5 \times 10^{-7}\ \Omega$m (refs. 2, 3)
$E \;\; = 1.5 \times 10^{11}$ Nm^{-2} (refs. 2, 3)
$G \;\; = 2$ (refs. 2, 3)
$\alpha \;\; = 10^{-5}$ K^{-1} (ref. 3)
$\nu \;\; = 16 \times 10^{-6}$ m^2 s^{-1} (30°C)
$\lambda \;\; = 26 \times 10^{-3}$ W m^{-1} K^{-1}
$K \;\; = 1.4 \times 10^{-23}$ W s K^{-1}
$T \;\; = 300$ K
$B \;\; = 1$ s^{-1}
$V_{max.} = 10^{-4}$ m s^{-1} (ref. 4)

This leads to:

$S \;\; = 6.8$
$N \;\; = 2.6 \times 10^{-18}$
$A \;\; = 1.6 \times 10^{-3}$
$J \;\; = 0.47$

Using these data, we get:

$I_{max.} = 1.5 \times 10^{-3}$ A

Using $\Delta T = 10^{-2}$ K we get

$$(SNR)_{max.} = 4.3 \times 10^5 \times F \qquad (13)$$

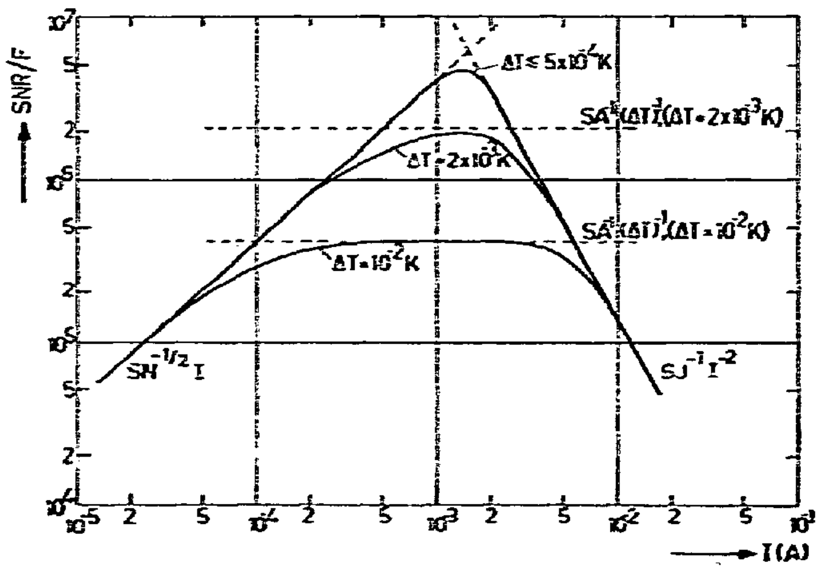

Fig. 1. The signal-to-noise ratio as a function of the current according to eqn (10).

Using $\Delta T = 2 \times 10^{-3}$ K we get

$$(\mathrm{SNR})_{\mathrm{max.}} = 2 \times 10^{6} \times \mathrm{F} \quad (14)$$

Using $\Delta T\ 5 \times 10^{-4}$ K we get

$$(\mathrm{SNR})_{\mathrm{max.}} = 4.8 \times 10^{6} \times \mathrm{F} \quad (15)$$

The results are shown in more detail in Fig. 1.

DISCUSSION

From the estimations above, it follows that for reasonable homogeneity of the temperature in the balance case a maximum sensitivity of 10^{-5} N (= 1 mgf) can be expected. It has been suggested that considerable increase of the sensitivity could be expected by using pulse techniques which would avoid too much heating of the wire.

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