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Review

CRITICAL REVIEW OF THE CALORIMETRIC METHOD FOR EQUILIBRIUM CONSTANT DETERMINATION

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INTRODUCTION

For reactions that do not go to completion but rather reach an equilibrium, the equilibrium constant, K, can be determined, within given limits, by a series of calorimetric experiments [1–5]. The great sensitivity of the calorimetric technique, together with the fact that almost all reactions occur with an appreciable change in the heat content, offers great possibilities in the field of simultaneous determination of equilibrium constants and enthalpy changes. A direct calorimetric determination of enthalpy of reaction, ΔH, and K values is to be preferred to an evaluation that makes use of van't Hoff's equation. In fact, since differentiation procedures always result in a loss of precision, the ΔH value obtained by differentiating log K determined at different temperatures, is bound to be less accurately known than log K. This lack of accuracy may sometimes lead to erroneous generalizations. For instance, the first complexation step of UO_2^{2+} with the 'hard' fluoride ion, determined by the van't Hoff equation, turned out to be exothermic ($\Delta H^{\ominus} \cong -2$ kcal mol^{-1}) [6]. This was rather surprising and led people to think of this case as an 'exception'. The same author reinvestigated the system by direct calorimetry and, contrary to the earlier result, demonstrated that the reaction is endothermic ($\Delta H^{\ominus} = 0.49$ kcal mol^{-1}) [7]. This further stresses the unreliability of many values determined by the temperature coefficient of the stability constant. Furthermore, this method requires relatively large amounts of material. The calorimetric technique allows one to overcome this latter obstacle. In fact, the measurements are usually

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performed at one temperature. Moreover, modern microcalorimeters make it possible to run the experiments with limited quantities of material. This is of particular relevance when working with biological material or, more generally, with compounds that are difficult to obtain. This is, for instance, the reason why several authors, as well as one of the co-authors of the present paper (G.A.) have made extensive use of the calorimetric technique for the purpose of determining log K and ΔH values [8–11].

Some reviews concerning the simultaneous determination of K and ΔH values have been reported [12–16]. Specifically, Lamberts [12] dealt with molecular complexes in solution, while Fenby and Hepler [13] were mainly concerned with hydrogen bond and charge transfer complexes; Christensen and co-workers [14–16] had the stated purpose to "acquaint the reader with the general usefulness of titration calorimetry for the determination of equilibrium constants for reactions in solution".

This paper, while reviewing calorimetric methods for the simultaneous determination of K and ΔH values, deals with this subject from a different perspective. In fact, the scope of the present paper is to give the reader an idea of the advantages and limitations of the calorimetric technique, as well as of the conditions that should be obeyed in order to minimize the effect of both random and systematic errors on the simultaneous determination of K and ΔH values.

Throughout the text, the equations have been written as they were in the original papers in order to allow the reader to make an immediate connection with the source reference.

BASIC PRINCIPLES AND EQUATIONS

For a process taking place at constant temperature and pressure, or for very small temperature intervals, we can write

$$-Q = \sum_{i=1}^{n} \Delta H_i \delta n_i \tag{1}$$

if we take the heat (Q) absorbed by the system to be positive. In eqn. (1), ΔH_i represents the enthalpy of formation of the ith species and δn_i is the number of moles involved in the reaction. Equation (1) suggests which information can be obtained through calorimetric measurements. In fact, the calorimetric data, in the form of either temperature or heat change (depending upon the type of calorimeter employed) versus volume of titrant added, can be analysed to obtain information on: (a) the type and/or number of reactions taking place in the calorimetric vessel; (b) the end point of a titration; (c) the concentration of species present; and (d) the ΔH or, in some cases and with the limitations that we will discuss, the ΔH and ΔG

simultaneously. We shall focus on the last case and, in particular, on the possibility of simultaneous determination of equilibrium constants and enthalpic changes through calorimetric measurements. In this context, Sturtevant's work, on the mutarotation of α,β-*d*-glucose, represents a pioneering contribution [17].

However, before presenting and discussing the details of the method, let us examine the advantages and disadvantages of the calorimetric techniques over other methods commonly employed to determine K values.

Firstly, under proper reaction conditions, the calorimetric procedure allows the simultaneous determination of ΔH and K values at a given temperature. Other methods, such as potentiometry, conductivity, spectroscopy, etc., produce only K values. The use of the calorimetric procedure leads to a saving both in time and material if one wishes to know the ΔH value as well as K. Obviously, owing to its precision, the potentiometric procedure is to be preferred when the K value only is sought.

Secondly, unlike spectrophotometric methods, calorimetry does not require transparent systems and, therefore, can be used, for instance, for biological systems. Calorimetry does not require, as spectrophotometry does, that an absorption band exist which changes upon complexation. Even when this condition is met, the equilibrium constants differ by substantially more than the assigned uncertainties [13]. Furthermore it is sometimes possible to obtain ΔH and K under temperature and pressure conditions not easily attainable by other means [18].

However, there are certain disadvantages to calorimetry. First of all, heat changes are non-specific and consequently the data can be interpreted with relative ease only for simple systems. For more complicated systems, ambiguity in the interpretation may arise; in such cases additional information, obtainable through different techniques, is needed before the data can be unambiguously interpreted. Secondly, until recently calorimetry required expensive sophisticated equipment and skilled personnel. In recent years, however, calorimetry has progressed enormously in various respects and this trend can be expected to continue in view of the fact that problems associated with the components of the calorimeter, i.e. size of reaction vessels, constant temperature bath, temperature sensing circuits, etc. have gradually been solved. Furthermore, the equipment, even if still sophisticated, is becoming relatively inexpensive and lends itself well to automation.

As indicated by eqn. (1), the thermogram of a reaction is a function of both ΔH and δn_i. Calculation of n_i values requires that the K for the reaction be known. For reactions with the same ΔH value (Fig. 1), an increased overall curvature of the thermogram is obtained with decreasing values of K. Curves, obtained for systems with increasing K values differ only slightly from one another the higher one goes; therefore an accurate determination of higher K values (see below), turns out to be difficult, if not altogether impossible. Conversely, for reactions with K values within given

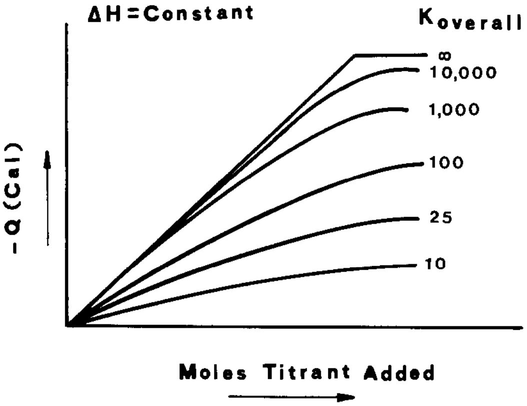

Fig. 1. Thermogram for the reaction A + B = AB, showing the effect that the value of the equilibrium constant has on the shape of the thermogram. (Reprinted with permission from J.J. Christensen, J. Ruckman, D.J. Eatough and R.M. Izatt, Thermochim. Acta, 3 (1972) 203.)

limits (see below), small quantities of species are formed and, consequently, only a little heat is evolved.

The dependence of the heat of reaction on ΔH is obvious; in fact, given the relation between ΔH and Q, for reactions with the same K, the heat change will vary linearly with ΔH, as shown in Fig. 2. Hence, the smaller the constant, the higher the ΔH should be, if reliable quantitative results are sought.

The above remarks indicate that in order to obtain K and ΔH values through calorimetric measurements: (a) the association equilibrium constant, K_a, must be within given general limits ($1 < K_a < 10^3$), as will be discussed below; and (b) the ΔH of reaction must be large enough to generate a temperature change of at least 0.01°C (0.01°C approaches the lower limit of temperature change necessary to generate thermograms reproducible to 0.2%) [14].

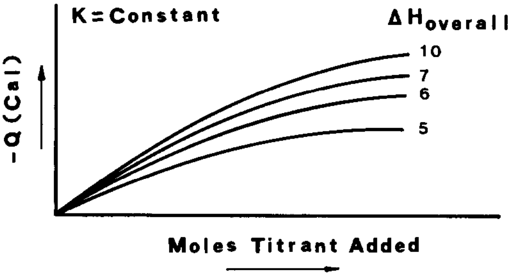

Fig. 2. Thermogram for the reaction A + B = AB, showing the effect that the value of the enthalpy change has on the shape of the thermogram. (Reprinted with permission from J.J. Christensen, J. Ruckman, D.J. Eatough and R.M. Izatt, Thermochim. Acta, 3 (1972) 203.)

For a generic association equilibrium reaction of the type

$$M + nL = ML_n \tag{2}$$

eqn. (1) may be written as

$$Q = -V\sum_{i=1}^{n} \delta[ML_i]\Delta H_i \tag{3}$$

where V is the total volume of the solution and $\delta[ML_i]$ is the change of concentration of the ith species formed. In eqn. (2), subscript n refers to the number of ligands in the complex, whereas superscript n in eqn. (3) refers to the nth species formed. According to eqn. (4)

$$[ML_i] = \beta_i[M][L]^i \tag{4}$$

$[ML_i]$ is a function of the overall stability constant, β_i, and the concentrations of the free metal ion, [M], and the free ligand, [L].

The mass balance equations for the metal ion and the ligand will be given by

$$[M]_T = [M] + \sum_{i=1}^{n} \beta_i[M][L]^i \tag{5}$$

$$[L]_T = [L] + \sum_{i=1}^{n} i\beta_i[M][L]^i$$

As the free concentrations of both the metal ion and the ligand depend on the total concentrations and the overall stability constants, according to eqns. (4) and (5)

$$[ML_i] = f(\beta_1, \beta_2, \ldots, \beta_n, [M]_T, [L]_T) \tag{6}$$

It is therefore clear from eqns. (6) and (3) that

$$Q = f(\Delta H_1, \Delta H_2, \ldots, \Delta H_n, \beta_1, \beta_2, \ldots, \beta_n, [M]_T, [L]_T, V) \tag{7}$$

Because eqn. (7) is not linear, the values of the unknowns cannot be calculated by using a simple least-squares method. This problem has been tackled in different ways, both for simple and for multiple equilibria cases.

CALCULATION TECHNIQUES

Simple case

In the simplest of possible cases, i.e. for a reaction of the type

$$A + B = AB \tag{8}$$

the following equilibrium constant, K_c, can be written

$$K_c = \frac{c}{(c_a - c)(c_b - c)} \tag{9}$$

where c is the equilibrium molar concentration of the species AB and c_a and c_b are the analytical concentrations of the reagents A and B, respectively. This equation may also be written as

$$c^2 - \left(c_a + c_b + K_c^{-1}\right)c + c_a c_b = 0 \tag{10}$$

which, solved with respect to c, yields

$$c = \frac{\left(c_a + c_b + K_c^{-1}\right) - \left[\left(c_a + c_b + K_c^{-1}\right)^2 - 4c_a c_b\right]^{1/2}}{2} \tag{11}$$

Of course, the other root of eqn. (11) cannot be accepted. This equation has been combined in various ways with eqn. (1) or other forms of eqn. (1) such as

$$Q = cV\Delta H \tag{1a}$$

In eqn. (1a), the assumption is made that the heat involved is positive when the reaction is endothermic.

Lamberts and Zeeger-Huyskens [19] and Abello and Pannetier [20] obtain

$$\Delta H = \frac{2Q/V}{\left(c_a + c_b + K_c^{-1}\right) - \left[\left(c_a + c_b + K_c^{-1}\right)^2 - 4c_a c_b\right]^{1/2}} \tag{12}$$

by combining eqn. (11) with eqn. (1a), from which they derive both K_c and ΔH by means of a trial-and-error procedure. From plots of ΔH versus concentration for a given range of arbitrarily chosen K_c values at various base concentrations, the above authors select as the 'best' value of K_c that which gives a constant value of ΔH, i.e. a line of zero slope. This method is illustrated in Fig. 3; among the various K_c values, that corresponding to line c (zero slope) is chosen as the best value.

Bolles and Drago [3], by combining eqns. (11) and (1a), obtain

$$\frac{1}{K_c} = \frac{1}{V}\frac{Q}{\Delta H^{\ominus}} + c_a c_b V \frac{\Delta H^{\ominus}}{Q} - \left(c_a + c_b\right) \tag{13}$$

$\Delta H^{\ominus}$ is calculated for a series of K_c values; K_c reciprocal is then plotted against $\Delta H^{\ominus}$. By means of trial-and-error calculations, a family of straight lines is obtained, which should ideally intersect at one point that represents the unique solution of eqn. (13). However, this is rarely so. The plots obtained are of the type shown in Fig. 4; in other words, the lines usually cross at points which define a series of triangles, the total area of which is an indication of the precision of the experiments. Choosing the average of the different intersections is not straightforward. In fact, if two experiments are run under approximately the same experimental conditions, the slopes of the lines will be fairly close to one another and, therefore, even small errors produce large differences in the intersection points, thus yielding K_c values very different from one another. In the case where, as often happens, more

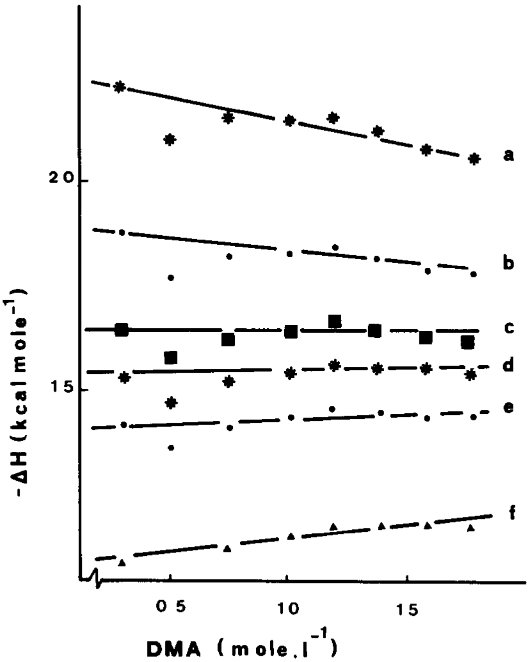

Fig. 3. Lamberts and Zeeger-Huyskens' method of obtaining K and ΔH values for dimethylacetamide–benzene adduct in cyclohexane. K value (l mol^{-1}): a, 0.167; b, 0.198; c, 0.222; d, 0.244; e, 0.260; f, 0.333. Constant enthalpy value for line c: 1.61 ± 0.05 kcal mol^{-1}. (Reprinted with permission from L. Abello and G. Pannetier, Bull. Soc. Chim. Fr., (1967) 3752.)

measurements are performed at each concentration, it is advisable to use their average, instead of drawing more lines. Furthermore, if many more experiments are run at one given concentration rather than at others, the data may tend to be biased. Drago and co-workers suggest choosing the experimental conditions so that the plots of $1/K_c$ versus $\Delta H^{\ominus}$ give lines that differ in slope as much as possible. A statistical method for evaluating the reliability of K values obtained in this way has also been reported [21]. The method outlined above has also been employed for the simultaneous determination of K and the molar absorptivity of the complex from spectroscopic measurements [22].

The method proposed by Neerink et al. [23] uses calorimetric data obtained at two different temperatures. According to van't Hoff's equation

$$\ln\left[K_{c(1)}/K_{c(2)}\right] = \frac{\Delta H^{\ominus}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right) \tag{14}$$

which combined with eqn. (9) gives

$$\Delta H^{\ominus} = \frac{RT_1T_2}{T_1 - T_2}\ln\frac{c_1(c_a - c_2)(c_b - c_2)}{c_2(c_a - c_1)(c_b - c_1)} \tag{15}$$

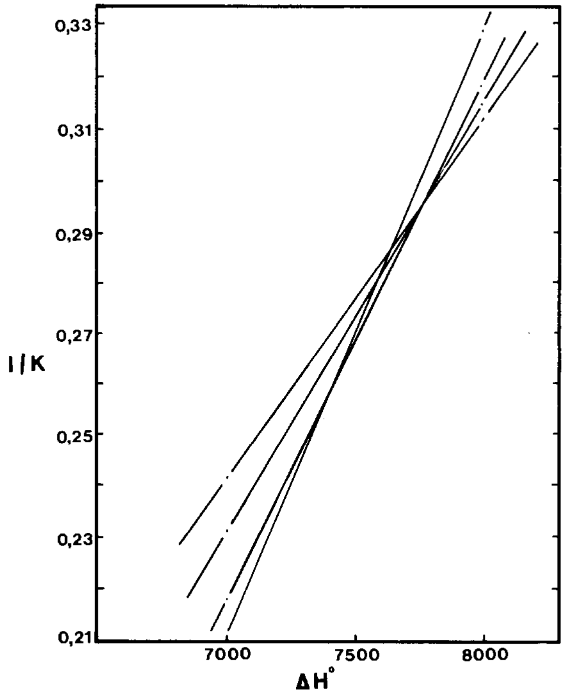

Fig. 4. K^{-1} versus $\Delta H^{\ominus}$ plot for the DMA–$(CH_3)_3SnCl$ system. (Reprinted with permission from T.F. Bolles and R.S. Drago, J. Am. Chem. Soc., 87 (1965) 5015. Copyright 1965, American Chemical Society).

This method implies that: (1) $\Delta H^{\ominus}$ is constant in the temperature range investigated; (2) the thermal expansivity of the solution is negligible [24,25]; and (3) the experiments be arranged so that the molarities, c_a and c_b, are the same at temperatures T_1 and T_2. If these conditions are met, eqns. (15) and (1a) may be combined and the values of K and ΔH can then be obtained by means of trial-and-error. Neerink et al. find that while there is good agreement with the ΔH values published by other authors, remarkable discrepancies are observed for the K_c values.

A method that does not require trial-and-error calculations has been proposed by Fenby and Hepler [13]. If the concentrations, c_a and c_b, are chosen such that

$$\chi = c_a + c_b + \frac{1}{K_c} \tag{16}$$

be constant, then eqn. (13) may be written

$$\frac{Vc_ac_b}{Q} = \frac{Q}{(\Delta H^{\ominus})^2 V} + \frac{\chi}{\Delta H^{\ominus}} \tag{17}$$

According to this equation, a plot of Vc_ac_b/Q versus Q/V gives a straight line with slope $-1/(\Delta H^{\ominus})^2$ and intercept $\chi/\Delta H^{\ominus}$. Once $\Delta H^{\ominus}$ is obtained, K_c can be calculated. Such a method can be used only if $(c_a + c_b)K_c$ is not much greater than unity. As this term becomes considerably higher than unity, the determination of K_c becomes increasingly more difficult.

Multiple equilibria case

The methods illustrated above can only be employed in the simplest cases (1 : 1 reactions). For more complex cases, one has to resort to a method that allows the determination of K and ΔH, keeping in mind that the equation that correlates these two parameters with the experimentally measured quantity (Q) is a non-linear expression (see eqn. (7)). However, this problem can be solved either by means of trial-and-error procedures or by some iterative method. An accurate solution involves the following steps: (1) approximated β_i values are chosen; (2) by making use of these approximated values and of the total concentrations of both ligand and metal ion, [M] and [L] values that satisfy eqn. (5) are calculated; (3) having obtained the free concentrations, the species concentrations are calculated by means of eqn. (4); (4) ΔH is determined by a linear best-fit of eqn. (3). Incidentally, it should be noted that the $\Delta H^{\ominus}$ value determined by a best fit of eqn. (3) does not necessarily coincide with the mean of the $\Delta H^{\ominus}$ values obtained by taking the average of the $\Delta H^{\ominus}$ values calculated at each data point. The 'average' $\Delta H^{\ominus}$ and the $\Delta H^{\ominus}$ obtained by means of a least-squares procedure will be fairly close only if random errors are present. If systematic errors are present in the Q data, the above values may be significantly different; (5) the function

$$U = \sum_{j=1}^{m} \left(Q_j + V \sum_{i=1}^{n} \delta[\mathrm{ML}_i] \Delta H_i \right)^2 \tag{18}$$

is calculated, where m represents the number of measurements; and (6) the minimum for eqn. (18) is searched for. This problem of searching for a minimum has been, and still is, of particular interest. In the past, four methods have essentially been used. We shall briefly discuss these methods without going into detail; for a more detailed description of each the reader is referred to the papers quoted.

1. Schematic mapping of U (eqn. (18)). β_i values are varied over a large domain and $U_{\min}$ is searched for by trial-and-error. This method has been employed for systems involving only one equilibrium [26–28]. For systems involving more than one equilibrium, the computing time may be so large as to render the method impractical.

2. Pit-mapping. A functional relationship is assumed for $U(\beta_i, \Delta H^{\ominus})$ and $U_{\min}$ is found by direct differentiation. This method, proposed by Sillèn [29], has been successfully used by Paoletti et al. for the Ag–pyridine system [30].

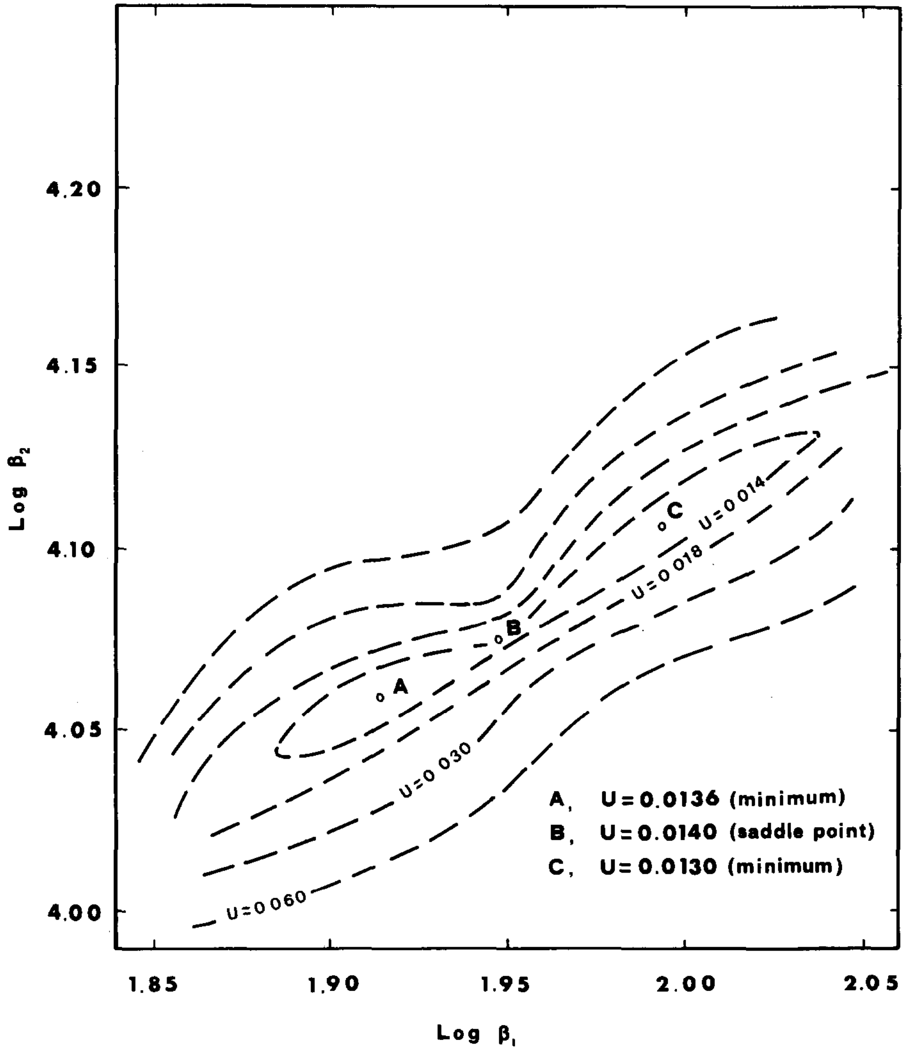

Fig. 5. Schematic map of $U(\beta_i, \Delta H_i^{\ominus})$. (Reprinted with permission from R.M. Izatt, D. Eatough, J.J. Christensen and R.L. Snow, J. Phys. Chem., 72 (1968) 1208. Copyright 1968, American Chemical Society.)

3. Simultaneous solution of equations. If a number of equations greater than the number of parameters to be determined is available, in theory it is possible to solve the equations simultaneously by appropriately combining the equations of the form of eqns. (3)–(5). It is then assumed that the average of the β_i values determined through the possible combinations of the experimental points represents a minimum for U. There is no way to check this assumption. This method has been used in some cases [4].

4. VMM. This method, originally developed at Argonne National Laboratory [31], uses a variable metric method of minimization (VMM) to find

TABLE 1

Log β_i, $\Delta H_i^\ominus$ and $\Delta S_i^\ominus$ values for the interaction of Ag^+ with Py [a]

Reaction	Log β_i	$\Delta H_i^\ominus$ (kcal mol^{-1})	$\Delta S_i^\ominus$ (cal deg^{-1} mol^{-1})
$Ag^+ + Py = AgPy^+$	2.04 ± 0.06 [b]	-4.6 ± 0.2	-6.2 ± 0.8
	2.05 ± 0.05 [c]	-4.6 ± 0.2	-6.0 ± 0.9
	2.05 ± 0.03 [d]	-4.5 ± 0.2	-5.8 ± 0.8
	(2.00) [e]	(−4.83) [e]	(−7.0) [e]
	(2.24) [f]	(−4.77) [f]	(−5.7) [f]
	(2.04) [g]		
	(2.00) [h]		
$Ag^+ + 2Py = AgPy_2^+$	4.09 ± 0.05 [b]	-11.25 ± 0.09	-19.0 ± 0.4
	4.10 ± 0.07 [c]	-11.24 ± 0.12	-18.9 ± 0.4
	4.11 ± 0.05 [d]	-11.21 ± 0.12	-18.8 ± 0.4
	(4.11) [e]	(−11.34) [e]	(−19.2) [e]
	(4.19) [f]	(−11.53) [f]	(−19.5) [f]
	(4.22) [g]		
	(4.11) [h]		

[a] Valid at 25 °C. The uncertainties are expressed as standard deviations.

[b] $\mu = 0$. Determined by the entropy titration procedure using the VMM method with random steps to analyse the data.

[c] $\mu = 0$. Same as b except random steps were not taken. Initial values used were log $\beta_1 = 2.05$ and log $\beta_2 = 4.15$.

[d] $\mu = 0$. Determined by the entropy titration procedure using the pit-mapping method to analyse the data. Initial values were log $\beta_1 = 2.05$ and log $\beta_2 = 4.15$.

[e] Determined by conventional calorimetry using the 'pit-mapping' method to analyse the data. Valid at $\mu = 0.5$.

[f] Determined by incremental calorimetry using an initial slope method to analyse the data. Valid at $\mu = 0.2$.

[g] Determined by potentiometry. Valid at $\mu = 0.6$.

[h] Determined by solubility measurements. Valid at $\mu = 0$.

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relative minima and has been adapted to the problem of least-squares fitting of calorimetric data. After a relative minimum has been located, β_i is randomly varied to explore the region around the minimum and to see whether the minimum found is the only relative minimum in the region.

The study of the silver–pyridine system in different laboratories using these four methods has made their comparison possible [32]. Table 1 shows that there are no substantial differences among the methods proposed, provided allowance is made for the different conditions. It must be pointed out that attempts to calculate β_i and ΔH values using the method of simultaneous solution of equations failed, probably owing to the importance of small errors in the solution of large numbers of independent equations. By way of example a schematic map obtained for this system is shown in Fig. 5. There are two close minima, A and C, separated by a 'saddle' point,

TABLE 2

Comparison of methods of data analysis

Method	Advantages	Disadvantages
Schematic map of $U(\beta_i, \Delta H_i^{\ominus})$	The method is thorough and accurate	Computer time required is long (approximately 1/2 h per run for the Ag^+–Py data)
Pit-mapping	Computer time is short (approximately 1 min/run for the Ag^+–Py data)	Locates saddle points as well as minima. Area of convergence is smaller than in the VMM method
VMM	Only relative minima are located. Area of convergence is much larger than in the pit-mapping method	Requires approximately 25% more computer time than pit-mapping
Simultaneous solution of equations	None	Apparently not applicable to systems involving more than one reaction

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B, which may represent a problem. Table 2 shows advantages and disadvantages of the four methods employed for the silver–pyridine system.

With respect to the value of $U_{\min}$, it should be as close as possible to that calculated from the known precision of the calorimetric system. For example, if in a titration composed of ten points we have a minimum detection level of 0.01 calories, then the function U calculated for the ten points should approach the value

$$U = 10(0.01)^2 = 0.001 \text{ cal}^2 \tag{19}$$

Significant deviations of the $U_{\min}$ value from this estimated value usually indicates that either the reactions assumed to be taking place do not correctly describe the system or that large systematic or random errors are present.

ANALYSIS OF ERRORS

An aspect that deserves particular attention is that a minimum of U does not necessarily mean that the species assumed describe the system correctly; also it does not mean that K and ΔH values calculated represent the 'true' values. In order for $U_{\min}$ to represent the 'best' K and ΔH values, it is mandatory that the experiments be performed in such a way as to avoid, or at least minimize, both random and systematic errors. A detailed treatment of this problem, that will only be summarily discussed here, can be found in ref. 33.

Random errors

To see how random errors influence our data, let us suppose, in a first instance, that our experiment is free from systematic errors and that the C_a, C_b and V values of eqn. (20)

$$K = \frac{(c_b - q_r/V\Delta H)(c_a - q_r/V\Delta H)}{q_r/V\Delta H} \tag{20}$$

which refers to

$$\mathrm{AB} = \mathrm{A} + \mathrm{B} \tag{21}$$

are free from errors. This last assumption is not unlikely, owing to the relatively high concentrations and volumes used in calorimetry. Solving for K, we have

$$K = \frac{q_r}{V\Delta H} + c_a c_b \frac{V\Delta H}{q_r} - (c_a + c_b) \tag{22}$$

which is analogous to eqn. (13).

Even if the ΔH chosen is that characteristic of the system, the corresponding K value will differ from the 'true' value owing to random errors in the determination of q_r. The relation expressing this dependence is given by

$$\left(\frac{\partial K}{\partial q_r}\right)_{c_a,\, c_b} = \frac{1}{V\Delta H} - \frac{c_a c_b V\Delta H}{q_r^2} \tag{23}$$

at constant c_a and c_b. With finite increments we have

$$|\Delta K| = \left(\frac{q_r}{V\Delta H} - \frac{c_a c_b V\Delta H}{q_r}\right)\left|\frac{\Delta q}{q_r}\right| \tag{24}$$

which may also be written as

$$|\Delta K| = -D\left|\frac{\Delta q}{q_r}\right| \tag{25}$$

where

$$D = \left[(K + c_a + c_b)^2 - 4c_a c_b\right]^{1/2} \tag{26}$$

If we are dealing with isoperibolic calorimeters, then, as

$$q_r = \epsilon \Delta \vartheta \tag{27}$$

we can write

$$|\Delta K| = -D\,|\Delta(\Delta\vartheta)/\Delta\vartheta| \tag{28}$$

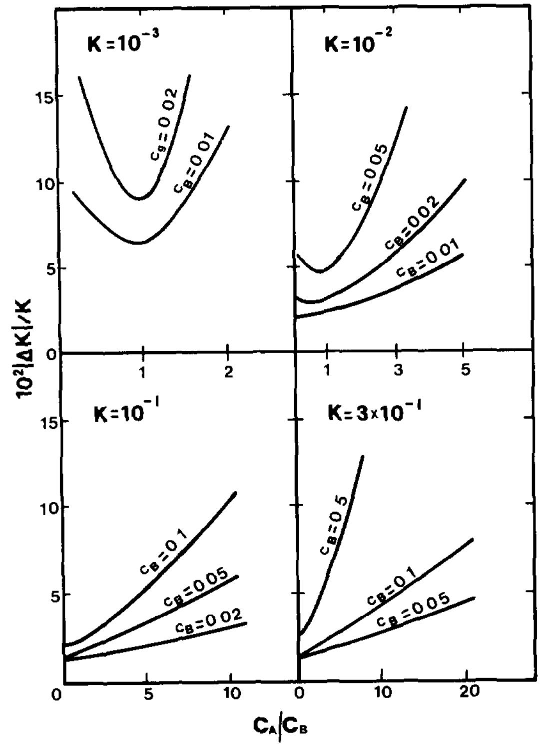

Fig. 6. Error propagation in the calorimetric evaluation of dissociation constants of AB compounds; a constant error $\Delta q/q_r = 0.01$ is assumed. (Reprinted with permission from S. Cabani and P. Gianni, J. Chem. Soc. (A), (1968) 547.)

Figure 6, obtained by assuming an error, $\Delta q/q_r$, equal to 1%, gives an idea of the error in K as a function of c_a/c_b, for four different K values. The figure clearly shows that: (1) dissociation constants lower than 10^{-3} are difficult to determine, or cannot be determined at all by calorimetry; and (2) the precision of dissociation constants with high values strongly depends upon the choice of concentrations.

Analogous conclusions are arrived at by Christensen et al. [34] in a paper in which possible errors in the titrant and titrate concentrations are also taken into account; it is important to note that in this paper a wider range of formation constant, K_a, values is investigated ($10^{-1} < K_a < 10^3$ in ref. 34, whereas $3.3 < K_a < 10^3$ in ref. 33). In Christensen's paper [34] Q_R values are generated for given values of K, ΔH, C_T (titrant concentration), C_S (solution concentration), volume of titrant added and volume of titrate. Then random errors of known magnitude are introduced into Q_R, C_T and C_S; these errors are 0.4, 0.1 and 0.1% respectively. An error in Q_R (ΔQ) of 0.4%, rather than 1% [33], is chosen, in that it is more in keeping with the characteristics of modern calorimetric apparatuses [35]. Keeping the ratio

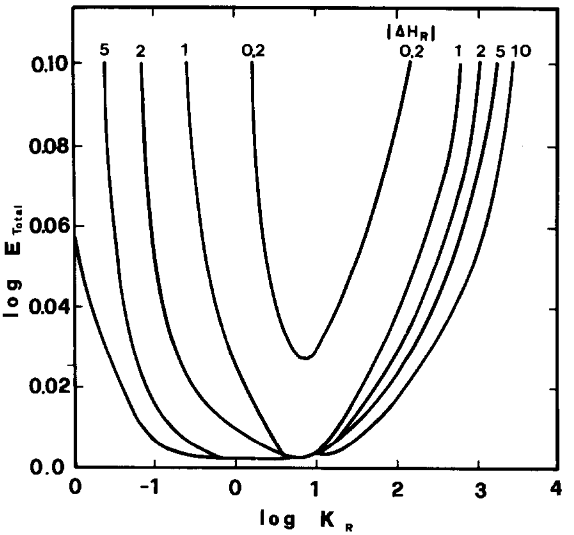

Fig. 7. Plot of log E_{TOTAL} versus log K_{R}. The $|\Delta H_{\mathrm{R}}|$ value (kcal mol^{-1}) for the reaction is designated for each of the curves. $C_{\mathrm{T}} = 0.5$. Errors: ±0.1% in C_{T}, ±0.1% in C_{S} and ±0.4% in Q_{R}. (Reprinted with permission from J.J. Christensen, D.P. Wrathall, J.O. Oscarson and R.M. Izatt, Anal. Chem., 40 (1968) 1713. Copyright 1968, American Chemical Society.)

$C_{\mathrm{T}}/C_{\mathrm{S}}$ constant $(=20)$, approximately 300 log E_{TOTAL} values are calculated. log E_{TOTAL} is defined as

$$\log E_{\mathrm{TOTAL}} = \left[\log {}^2E_{K(C_{\mathrm{T}})} + \log^2 E_{K(C_{\mathrm{S}})} + \log^2 E_{K(Q_{\mathrm{R}})}\right]^{1/2} \tag{29}$$

where log E_K is the absolute value of the difference between the log K_{R} value used to generate the Q_{R} value in the first step and the log K_{R} values obtained in the second step by using the values $Q_{\mathrm{R}} + \Delta Q_{\mathrm{R}}$, $C_{\mathrm{T}} + \Delta C_{\mathrm{T}}$ and $C_{\mathrm{S}} + \Delta C_{\mathrm{S}}$. Figures 7, 8 and 9 show that it is possible to single out 'optimum' experimental conditions under which K and ΔH can be determined simultaneously. However, it is evident that even under the most favourable conditions it is not advisable, or it is not at all possible, to determine log $K_{\mathrm{R}} > 3$. ΔH_{R} can be plotted against the titrant concentration so that, for a given log E_K value, each of the curves in Fig. 10 corresponds to a log E_{TOTAL} value of 0.01. In such a way any combination of ΔH_{R} and C_{T} values with coordinates above and to the right will have a log $E_{\mathrm{TOTAL}} < 0.01$. Such a plot gives an immediate idea of the feasibility of a given experiment. For example, log K_{R} values for reactions with $|\Delta H_{\mathrm{R}}| > 5$ kcal mol^{-1} can be determined with an accuracy of $\pm$ 0.01 log K unit if $-1 < \log K_{\mathrm{R}} < 2$ (see Fig. 10). It is interesting to note that, by properly choosing the titrant, it is possible to calculate equilibrium constants for reactions of the type

$$\mathrm{HB} = \mathrm{B}^- + \mathrm{H}^+ \tag{30}$$

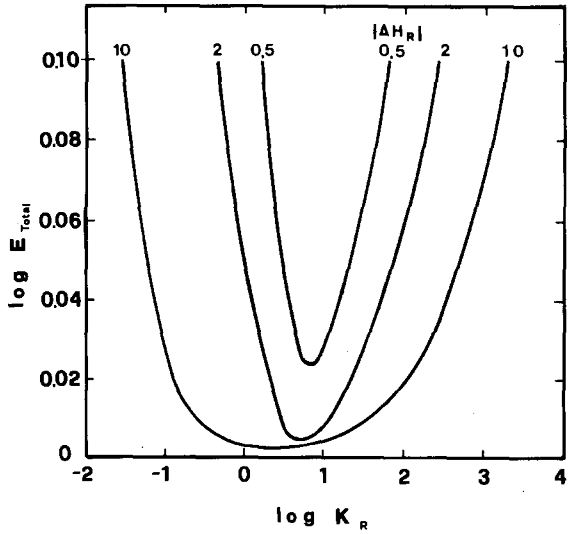

Fig. 8. Plot of log E_{TOTAL} versus log K_R. The $|\Delta H_R|$ value (kcal mol^{-1}) for the reaction is designated for each curve. $C_T = 0.2$. Errors: ±0.1% in C_T, ±0.1% in C_S and ±0.4% in Q_R. (Reprinted with permission from J.J. Christensen, D.P. Wrathall, J.O. Oscarson and R.M. Izatt, Anal. Chem., 40 (1968) 1713. Copyright 1968, American Chemical Society.)

(which cannot be obtained directly because of being outside the limits discussed above) from competing reactions of the type

$$HA + B^- = HB + A^- \qquad (31)$$

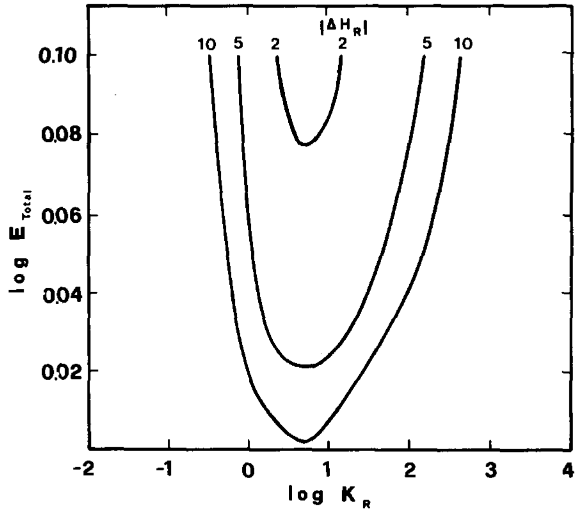

Fig. 9. Plot of log E_{TOTAL} versus log K_R. The $|\Delta H_R|$ value (kcal mol^{-1}) for the reaction is designated for each of the curves. $C_T = 0.05$. Errors: ±0.1% in C_T, ±0.1% in C_S and ±0.4% in Q_R. (Reprinted with permission from J.J. Christensen, D.P. Wrathall, J.O. Oscarson and R.M. Izatt, Anal. Chem., 40 (1968) 1713. Copyright 1968, American Chemical Society.)

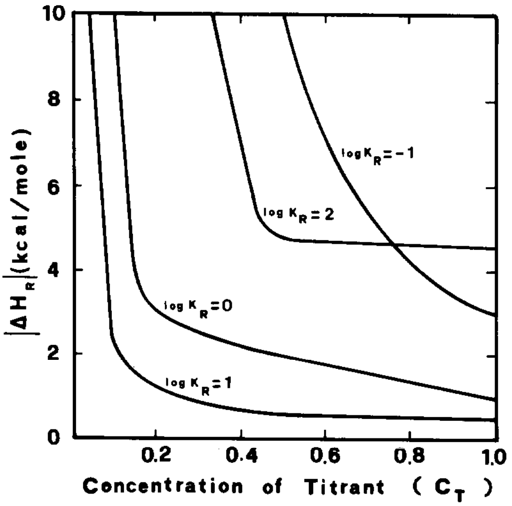

Fig. 10. Plot showing combinations of C_T and $|\Delta H_R|$ values which result in a log E_{TOTAL} value = 0.01. All combinations of C_T and $|\Delta H_R|$ values having coordinates above and to the right of the curve for a given log K_R value result in log E_{TOTAL} values less than 0.01. Errors: ±0.1% in C_T, ±0.1% in C_S and ±0.4% in Q_R. (Reprinted with permission from J.J. Christensen, D.P. Wrathall, J.O. Oscarson and R.M. Izatt, Anal. Chem., 40 (1968) 1713. Copyright 1968, American Chemical Society.)

provided that the equilibrium constant for the reaction

$$HA = A^- + H^+ \qquad (32)$$

is known.

For example (Table 3), if one wants to determine a p$K \cong 6$, pyridinium ion (p$K = 5.17$) could be titrated into a solution containing the ionized form of the compound with p$K \cong 6$, thus ending up with a reaction having a log $K_R = 1$. In such a case, at least in theory (see Fig. 10), a titrant with $C_T = 0.2$ could be used to determine the unknown pK with an accuracy of ± 0.01 log unit, provided $|\Delta H| > 1$ kcal mol^{-1}. Obviously, it is desirable that the reaction to be 'coupled' have a $|\Delta H|$ as large as possible; this is the reason why in Table 3, two titrants, having quite different $\Delta H^{\ominus}$ values, are reported for each pH range.

To take into account the propagation of errors under different experimental conditions, both the workers quoted in refs. 33 and 34 employ a weighting procedure. Although different, these procedures have as a common philosophy the attribution of a lower weight to those points with small thermal effects. In fact, despite being determined under correct experimental conditions, they may be significantly affected by random errors due to the smallness of their thermal effect. Different weighting procedures have also been suggested [28,36,37]. Other workers [30] suggest rejecting those measurements for which the 'error', obtained by minimizing the function U, is

TABLE 3

Suggested titrants ($HA = A^- + H^+$)

Effective pK range	Substance	pK	$\Delta H^\ominus$ (kcal mol^{-1})
< 2.5	Hydrogen ion	≪1	0
	Hydrazinium (+2) ion	−0.67	8.9
1– 4	Phosphoric acid	2.148	−1.88
3– 6	Anilinium ion	4.60	7.28
4– 7	Acetic acid	4.756	−0.01
	Pyridinium ion	5.17	4.98
6– 9	Dihydrogen phosphate ion	7.198	0.90
	Imidazolinium ion	6.99	8.78
8–11	Acetylacetone	9.02 (20 ° C)	2.8
	Tris(hydroxymethyl)aminomethane	8.069	11.33
9–12	Glycinium ion	9.780	10.57
11–14	Monohydrogen phosphate ion	12.39	4.20
	Hydroxide ion	13.998	13.335

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greater than three times the error calculated from the known precision of the calorimeter.

Systematic errors

Systematic errors can be of various types. Those more frequently encountered, although there are others, involve an erroneous evaluation of the thermal exchange between the calorimetric vessel and the surroundings (isoperibol calorimeters) and of the heat of dilution, a point which deserves consideration. The heat of dilution is usually so small as to be negligible. This is not the case, though, when dealing with concentrated reactants; in such a case, a non-negligible heat amount (due to solvation reactions, hydrolysis processes, etc.) will be involved. Then the ‘heat of dilution’ can be determined by performing ‘blank’ experiments. However, in the authors’ opinion, the use of dilution tables, such as those published by the National Bureau of Standards [38], should be avoided. In fact, experimentally determined heats of dilution also contain other heat effects, for example the heat of friction, which are obviously not included in the NBS heat of dilution.

For simplicity, we will suppose that systematic errors are due to an erroneous evaluation of the heat of dilution. A detailed study is reported in ref. 33. The authors of this last paper suppose they are dealing with a series of measurements with a constant error of 50 cal mol^{-1} in the heat of dilution of the titrant. The results thus obtained are listed in Table 4. The

TABLE 4

Effect of a systematic error on the evaluation of K and ΔH of dissociation of a compound AB ($K = 2 \times 10^{-2}$ mol l^{-1}; $\Delta H = 5000$ cal mol^{-1}) from calorimetric data

$10^3\ m_{\mathrm{A}}$[a]	$10^3\ m_{\mathrm{B}}$[a]	α[b]	q_{r}[c]	q[d]	$10^2\ K$[e]	q[f]	$10^2\ K$[e]	ΔH[e] (cal mol^{-1})
0.5	2.5	0.1056	1.3195	0.0250	2.181	−0.0015	2.176±0.000	5300± 0.8
1.0		0.2000	2.5000	0.0500	2.177	−0.0008		
1.5		0.2835	3.5435	0.0750	2.175	+0.0004		
2.0		0.3566	4.4575	0.1000	2.174	+0.0012		
2.5		0.4202	5.2525	0.1250	2.175	+0.0013		
3.0		0.4753	5.9512	0.1500	2.175	+0.0008		
3.5		0.5230	6.5370	0.1750	2.176	+0.0002		
4.0		0.5642	7.0527	0.2000	2.176	−0.0003		
4.5		0.6000	7.5000	0.2250	2.176	−0.0002		
5.0		0.6311	7.8892	0.2500	2.175	+0.0006		
0.5	1	0.1492	0.7460	0.0250	2.254	+0.0041	2.271±0.003	5575± 5
1.0		0.2680	1.3400	0.0500	2.260	+0.0044		
1.5		0.3625	1.8125	0.0750	2.269	+0.0014		
2.0		0.4385	2.1925	0.1000	2.275	−0.0014		
2.5		0.5000	2.5000	0.1250	2.280	−0.0045		
3.0		0.5505	2.7525	0.1500	2.283	−0.0060		
3.5		0.5925	2.9625	0.1750	2.282	−0.0054		
4.0		0.6277	3.1385	0.2000	2.278	−0.0035		
4.5		0.6577	3.2885	0.2250	2.271	+0.0009		
5.0		0.6834	3.4170	0.2500	2.259	+0.0067		
0.5	0.25	0.1850	0.2312	0.0250	2.798	+0.0086	2.919±0.018	7200±33
1.0		0.3154	0.3942	0.0500	2.864	+0.0061		
1.5		0.4114	0.5142	0.0750	2.914	+0.0006		
2.0		0.4844	0.6055	0.1000	2.951	−0.0045		
2.5		0.5418	0.6772	0.1250	2.971	−0.0076		
3.0		0.5880	0.7350	0.1500	2.975	−0.0083		
3.5		0.6258	0.7822	0.1750	2.965	−0.0063		
4.0		0.6574	0.8217	0.2000	2.939	−0.0030		
4.5		0.6840	0.8550	0.2250	2.900	+0.0028		
5.0		0.7070	0.8837	0.2500	2.846	+0.0105		

[a] Moles of reagent in 100 ml of a solution of unit density.

[b] Degree of formation of AB.

[c] Reaction heats calculated with $K = 2 \times 10^{-2}$ mol l^{-1} and $\Delta H = 5000$ cal mol^{-1}.

[d] Systematic error calculated as $\delta q = 50\ m_{\mathrm{A}}$.

[e] Values obtained with the procedure described in the text by introducing into eqn. (22) reaction heats: $q_{\mathrm{r}}' = q_{\mathrm{r}} + \delta q$.

[f] Difference between q_{r}' value and reaction heats calculated with $\overline{K}$ and ΔH reported in the last columns of the table.

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authors assume a (dissociation) reaction of known K (2×10^{-2} mol l^{-1}) and ΔH (5000 cal mol^{-1}) and titrate the same amounts of A (see column 1) into solutions containing 2.5×10^{-3}, 1×10^{-3} and 0.25×10^{-3} moles of B (see column 2), respectively. The degree of formation of AB is calculated from the known true value of K and is reported in column 3. From these data a true value of q_r is obtained (column 4). The error due to the evaluation of the heat of dilution of the titrant is listed in column 5. The K value is now determined (see column 6) by means of eqn. (22), by introducing the quantity $q_r' = q_r + \delta q$, i.e. by introducing the heat of reaction which contains an error (δq) in the evaluation of the heat of dilution of A. By averaging the K values reported in column 6, $\overline{K}$ and $\Delta\overline{H}$ values are obtained (see last two columns). In all three cases 'good' minima are obtained even if these good values of K and ΔH differ not only from the true value ($\Delta H = 5000$ cal mol^{-1} and $K = 2 \times 10^{-2}$), but also between one case and another. Despite this, the Δq values (column 7), obtained in all three cases as the difference between the q_r' values having a constant error of 50 cal mol^{-1} (in the heat of dilution) and the q values calculated with the 'best' data, reported in the last column of Table 4, are quite satisfactory.

TABLE 5

Effect of a systematic error on the evaluation of K and ΔH of dissociation of AB

K [a]	ΔH [a]	$10^2\,C_A$ [b]	$10^2\,C_B$ [b]	a [c]	$\overline{K}$ [d]	$\Delta\overline{H}$ [d]
0.02	3000	0.5–5	2.50	0.10–0.63	$(2.115 \pm 0.000)10^{-2}$	3118 ± 0.3
			1.00	0.15–0.68	$(2.181 \pm 0.002)10^{-2}$	3228 ± 2
			0.25	0.18–0.71	$(2.622 \pm 0.013)10^{-2}$	3862 ± 13
	5000	0.5–5	2.50	0.10–0.63	$(2.073 \pm 0.000)10^{-2}$	5122 ± 0.3
			1.00	0.15–0.68	$(2.116 \pm 0.002)10^{-2}$	5235 ± 2
			0.25	0.18–0.71	$(2.355 \pm 0.008)10^{-2}$	5822 ± 12
	8000	0.5–5	2.50	0.10–0.63	$(2.053 \pm 0.000)10^{-2}$	8133 ± 0.5
			1.00	0.15–0.68	$(2.074 \pm 0.001)10^{-2}$	8237 ± 2
			0.25	0.18–0.71	$(2.216 \pm 0.005)10^{-2}$	8802 ± 11
0.30	500	5 –150	25	0.09–0.81	0.480 ± 0.005	712 ± 3
			5	0.13–0.83	1.038 ± 0.032	1691 ± 36
			2.5	0.13–0.83	1.803 ± 0.054	3507 ± 87
	1500	5 –150	25	0.09–0.81	0.358 ± 0.002	1700 ± 3
			5	0.13–0.83	0.528 ± 0.013	2459 ± 32
			2.5	0.13–0.83	0.762 ± 0.024	3614 ± 76
	3000	5 –150	25	0.09–0.81	0.329 ± 0.001	3196 ± 3
			5	0.13–0.83	0.408 ± 0.007	3890 ± 29
			2.5	0.13–0.83	0.518 ± 0.013	4872 ± 69

[a] K and ΔH for the reaction AB = A + B.

[b] Reagent concentration; for each C_B value, ten different C_A values included in the range reported in the table were used for the calculations.

[c] Range of variation of degree of formation of AB; α values are calculated with K of the first column.

[d] Values calculated by introducing into eqn. (22) reaction heats: $q_r' = q_r + \delta q$, q_r being the correct reaction heat and $\delta q = 20\ m_A$.

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TABLE 6

Comparison of pK and ΔH values with and without systematic errors included in their calculation [a]

Run	pK	pK_e	ΔpK	ΔH [b]	ΔH_e [b]	$\Delta(\Delta H)$ [b]
Error = 1.0%						
1	1.638	1.627	0.011	3.62	3.71	0.09
2	1.777	1.767	0.010	3.13	3.20	0.07
3	1.865	1.858	0.007	3.01	3.07	0.06
4	2.217	2.209	0.008	2.26	2.30	0.04
Error = 0.4%						
1	1.638	1.633	0.005	3.62	3.66	0.04
2	1.777	1.773	0.004	3.13	3.16	0.03
3	1.865	1.863	0.002	3.01	3.03	0.02
4	2.217	2.214	0.003	2.26	2.28	0.02

[a] pK = pK value determined without systematic error added; pK_e = pK value determined with the systematic error added.

[b] ΔH in kcal mol^{-1}.

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This indicates that having a good fit does not necessarily imply having good final K and ΔH values. In order for a good fit to be satisfactory, a few precautions should be taken when designing an experiment (see below). The same authors [33] have also evaluated the effect of a smaller error (20 cal mol^{-1}) when varying the dissociation constant and the ΔH values (Table 5). At constant K, the error is larger the smaller the ΔH. Also, at constant ΔH, the error is larger the larger the dissociation constant and becomes smaller at higher B (titrate) concentrations. From these observations one might deduce that it would be better to run a larger number of measurements at higher B concentrations. However, it must always be kept in mind that working at high concentrations may lead to a stoichiometry of reaction different from that observed at relatively lower concentrations.

The effect of systematic errors has also been evaluated by Christensen et al. [39]. These authors have considered the effect of 1% and 0.4% errors on the parameters to be determined (Table 6). For example, errors of 0.01 log unit in log K are obtained when assuming a systematic error of 1%. The errors in log K are obviously lower (0.005 log unit) when assuming errors of 0.4%. These errors in log K (0.005–0.01), arising from systematic errors of this magnitude (0.4–1%), are far smaller than those calculated by Cabani and Gianni [33]. This is not surprising, however, if one looks at the data correctly. In fact, the seemingly low systematic error in the heat of dilution (50 cal mol^{-1}), assumed by Cabani and Gianni [33], turns out to be of the order of 2–3, 3–7 and 11–28% (Table 4) in the three experiments respectively, if calculated with respect to the total heat generated in the reaction.

CONCLUDING REMARKS

We can conclude that whereas owing to random errors, K_a values $> 10^3$ are difficult to determine, K_a values significantly lower than unity cannot be determined because of the influence of systematic errors.

In any case, to minimize both systematic and random errors a careful worker should: (a) appropriately weigh the experimental points; (b) vary the order of addition of reactants; (c) make sure that K and ΔH values obtained give a good fit when the concentration ratios as well as the concentrations themselves are varied; and (d) run as high a number of experiments as possible, for both continuous and incremental titrations, varying the concentrations as much as possible. Only by taking these precautions can a good fit also be regarded as a reliable fit.

Point (d) cannot easily be achieved. In fact, it is not always possible to vary the concentrations of reactants over a wide range without significantly altering the ionic strength of the solution and, consequently, the activity coefficients. In such cases it is undoubtedly more correct to work at constant ionic strength and obtain the parameters by extrapolation to zero ionic strength. This procedure, though unexceptional from a theoretical point of view, implies an enormous amount of experimental work. For diluted solutions ($\mu < 0.1$), it is preferable to make use of some iterative method to find the correct activity coefficients. One might for instance use an equation of the type

$$\log \gamma_{\pm} = \frac{-z^2 A\sqrt{\mu}}{1 + \beta \mathring{a}\sqrt{\mu}} \tag{33}$$

For the values of the parameters in eqn. (33) see references 15, 36 and 40. A correct procedure involves assuming an ionic strength, calculating the activity coefficients by means of eqn. (33), transforming the equilibrium constant into concentration constant, calculating the concentration of the various species and comparing assumed and calculated ionic strengths. Of course, this procedure is repeated until estimated and calculated ionic strength values agree to a desired degree. It must be noted that improper use of equations that do not describe the activity coefficients correctly may lead to discrepancies, as demonstrated by Cabani and Gianni [40].

For ionic strengths significantly higher than 0.1, it is desirable to determine the activity coefficients experimentally or, if this is not possible, to calculate concentration constants only.

The problem of the simultaneous determination of K and ΔH by means of calorimetry becomes more complicated when more than one species is present in the solution under investigation [15,30,32,41,42,43]. As pointed out by those authors who have tackled this problem [4], the choice of a system such as silver–pyridine, as an example of the simultaneous determination of K and ΔH, is not fortuitous. In fact in this system only two species are present, competing reactions are negligible and the equilibrium constants are expected to be independent of ionic strength. Nevertheless, even in simple cases, such as that just mentioned, significant differences are observed among the values obtained by different authors [30,41]. Even those who have extensively used the method of simultaneous determination of K and ΔH by calorimetry, point out that "the results become less quantitative as the number of reactions considered increases, and very favorable ΔH and log K values are necessary when multiple equilibria are considered" [39]. In the best of cases, equilibrium reactions involving two species are to be considered a border-line case if quantitative results are desired.

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REFERENCES

1 J.H. Sturtevant, in H.A. Skinner (Ed.), Experimental Thermochemistry, Wiley, New York, vol. II, 1962, p. 427.
2 T.H. Benzinger and C. Kitzinger, in I.D. Hardy (Ed.), Temperature—Its Measurement and Control in Science and Industry, Reinhold, New York, vol. 3, 1963.
3 T.F. Bolles and R.S. Drago, J. Am. Chem. Soc., 87 (1965) 5015.
4 J.J. Christensen, R.M. Izatt, L.D. Hansen and J.A. Partridge, J. Phys. Chem., 70 (1966) 2003.
5 B. Nelander, Acta Chem. Scand., 20 (1966) 2189.
6 S. Ahrland, Helv. Chim. Acta, 50 (1967) 306.
7 S. Ahrland, Struct. Bonding, 5 (1968) 118.
8 Y. Nakatsuji, J.S. Bradshaw, P.K. Tse, G. Arena, B.E. Wilson, N.K. Dalley and R.M. Izatt, J. Chem. Soc., Chem. Comm., (1985) 749 and references therein.
9 J.S. Bradshaw, D.A. Chamberlin, P.E. Harrison, B.E. Wilson, G. Arena, N.K. Dalley, J.D. Lamb, R.M. Izatt, F.G. Morin and D.M. Grant, J. Org. Chem., 50 (1985) 3065 and references therein.
10 J.S. Bradshaw, M.L. Colter, Y. Nakatsuji, N.O. Spencer, M.F. Brown, G. Arena, P.K. Tse, B.E. Wilson, J.D. Lamb, N.K. Dalley, R.M. Izatt, F.G. Morin and D.M. Grant, J. Org. Chem., 50 (1985) 4865 and references therein.
11 J.S. Bradshaw, R.B. Nielsen, P.K. Tse, G. Arena, B.E. Wilson, N.K. Dalley, J.D. Lamb, J.J. Christensen and R.M. Izatt, J. Heterocyclic Chem., 23 (1986) 361 and references therein.
12 L. Lamberts, Ind. Chim. Belg., 36 (1971) 347.
13 D.V. Fenby and L.G. Hepler, Chem. Soc. Rev., 3 (1974) 193.
14 J.J. Christensen, J. Ruckman, D.J. Eatough and R.M. Izatt, Thermochim. Acta, 3 (1972) 203.

15 D.J. Eatough, J.J. Christensen and R.M. Izatt, Thermochim. Acta, 3 (1972) 219.
16 D.J. Eatough, R.M. Izatt and J.J. Christensen, Thermochim. Acta, 3 (1972) 233.
17 J.H. Sturtevant, J. Phys. Chem., 45 (1948) 127.
18 J. Oscarson and J.J. Christensen, Proceedings of the 1st Eurasia Conference on Chemistry of Solution, Bangkok, January 3–8, 1988, p. 125.
19 L. Lamberts and F. Zeeger-Huyskens, J. Chim. Phys., 20 (1963) 435.
20 L. Abello and G. Pannetier, Bull. Soc. Chim. Fr., (1967) 3752.
21 T.D. Epley and R.S. Drago, J. Am. Chem. Soc., 89 (1967) 5770.
22 N.J. Rose and R.S. Drago, J. Am. Chem. Soc., 81 (1959) 6138.
23 D. Neerink, A. Audenhaege, L. Lamberts and P. Huyskens, Nature, 218 (1968) 461.
24 E.J. King, Acid–Base Equilibria, Pergamon Press, Oxford, 1965.
25 E.M. Woolley, J.G. Travers, B.P. Erno and L.G. Hepler, J. Phys. Chem., 75 (1971) 3591.
26 J.J. Christensen, D.P. Wrathall, R.M. Izatt and D.O. Tolman, J. Phys. Chem., 71 (1967) 3001.
27 J.J. Christensen, D.P. Wrathall and R.M. Izatt, Anal. Chem., 40 (1968) 175.
28 J. Mullens, J. Yperman, J.P. Francois and L.C. Van Poucke, J. Phys. Chem., 89 (1985) 2937.
29 L.G. Sillèn, Acta Chem. Scand., 16 (1962) 159.
30 P. Paoletti, A. Vacca and D. Arenare, J. Phys. Chem., 70 (1966) 193.
31 W.C. Davidon, Argonne National Laboratory, Report ANL 5990, Rev. 2, Argonne, IL, 1966.
32 R.M. Izatt, D. Eatough, J.J. Christensen and R.L. Snow, J. Phys. Chem., 72 (1968) 1208.
33 S. Cabani and P. Gianni, J. Chem. Soc. (A), (1968) 547.
34 J.J. Christensen, D.P. Wrathall, J.O. Oscarson and R.M. Izatt, Anal. Chem., 40 (1968) 1713 and references therein.
35 J.J. Christensen, R.M. Izatt and L.D. Hansen, Rev. Sci. Instr., 36 (1965) 779.
36 C. Rigano, E. Rizzarelli and S. Sammartano, Thermochim. Acta, 33 (1979) 211.
37 A. Jerzewski, T.M. Pietrzak and P. Goralski, Pol. J. Chem., 61 (1987) 861.
38 D.D. Wagman, S.M. Bailey and R.H. Schumm, Selected Values of Chemical Thermodynamic Properties, NBS., Technical Note 270, Jan. 1968.
39 J.J. Christensen, J.H. Rytting and R.M. Izatt, J. Chem. Soc. (A), (1969) 861.
40 S. Cabani and P. Gianni, Anal. Chem., 44 (1972) 253 and references therein.
41 F. Becker, N.G. Schmahl and H.D. Pflug, Z. Phys. Chem. (Frankfurt), 39 (1963) 306 and references therein.
42 P. Paoletti, A. Dei and A. Vacca, J. Chem. Soc. (A), (1971) 2656.
43 M. Bjorkman and L.G. Sillèn, Trans. Roy. Inst. Technol. Stockholm, No. 199 (1963).