Thermodynamic properties of unsaturated vapour and liquid states from a cubic equation of state: supercritical and compressed liquid regions

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Abstract

The applicability of the Lielmezs-Merriman (LM) modification of the Peng-Robinson equation of state has been tested for supercritical and compressed liquid regions. The predicted thermodynamic properties (pressure, volume, temperature, departure functions: ΔH , ΔA , ΔS , ΔG , ΔU) and fugacity of the pure compounds have been compared with experimental data and with results calculated by the Soave-Redlich-Kwong (SRK) and Peng-Robinson (PR) equations of state.

LIST OF SYMBOLS

Greek symbols

 α temperature dependence of the parameter α in the cubic equation of state; see Lielmezs and Mak [1]

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Subscripts

 \mathbf{r} reduced property

Superscripts

 \circ reference state; ideal gas state

INTRODUCTION

Recently, Lielmezs and Mak [1] by means of a generalized L-H-C-type [2] temperature function, extended the work of Lielmezs and Merriman [3] over the saturated and subcritical regions of a *P-V-T* surface (Fig. 1). We now further explore Lielmezs and Mak's study [1] to include testing over the supercritical and compressed liquid regions. The thermodynamic properties tested for these two regions are pressure, volume, temperature, the departure functions (ΔH , ΔA , ΔS , ΔG , ΔU) and the fugacity, see Tables 1-3 and Figs. 2-7.

Fig. 1. Division of pressure-volume surface.

The results for a series of pure substances obtained here have been compared with those from the Soave-Redlich-Kwong (SRK) equation [14-16] and the Peng-Robinson [17] equation of state by means of RMS $\%$ error and AAD values, see Tables 2a, 2b, 3a, 3b and Figs. 2–7. RMS % error is defined as

RMS % error =
$$
\left[\frac{\Sigma(\% \text{ error})^2}{N}\right]^{1/2}
$$

Compound	MW	P_c (atm)	T_c (K)	$T_{\rm b}$ (K)	ω	\boldsymbol{p}	q
Methane	16.042	45.80	190.65	111.70	0.008	0.19584	0.78426
Ethane	30.068	48.20	305.42	184.47	0.098	0.25183	0.83742
Propane	44.094	42.01	369.96	231.10	0.152	0.27413	0.85176
n -Butane	58.123	37.47	425.16	272.67	0.193	0.28984	0.87067
<i>i</i> -Butane	58.123	36.00	408.13	261.32	0.176	0.27968	0.87124
n -Pentane	72.1498	33.25	469.65	309.19	0.251	0.30395	0.86468
<i>i</i> -Pentane	72.1498	33.37	460.39	301.025	0.227	0.29387	0.85979
Neopentane	72.1498	31.54	433.75	282.628	0.197	0.27709	0.87028
n -Hexane	86.170	29.91	507.87	341.87	0.296	0.30876	0.81677
n -Heptane	100.205	27.00	540.20	371.60	0.351	0.32020	0.82035
n -Octane	114.232	24.50	568.80	398.80	0.394	0.32632	0.81321
Benzene	78.108	48.70	562.65	353.25	0.212	0.30668	0.82281
Sulphur dioxide	64.066	77.79	430.65	263.00	0.251	0.36256	0.83570
Methanol	32.042	78.59	513.15	337.696	0.559	0.47075	0.80070
Ethanol	46.069	60.56	513.92	351.443	0.6436	0.47769	0.84658
1-Propanol	60.090	50.21	537.04	370.93	0.624	0.45585	0.91571
Acetylene	26.036	61.64	308.69	189.20	0.184	0.30632	0.80464
Propylene	42.078	45.60	364.91	225.45	0.148	0.27311	0.83694
1-Butene	56.104	39.67	419.59	266.90	0.187	0.29085	0.85773
Water	18.0152	218.3	647.30	373.15	0.344	0.44221	0.73237
Nitrogen	28.016	33.49	125.95	77.40	0.040	0.20477	0.81713
Ammonia	17.032	111.3	405.59	239.70	0.250	0.38595	0.85842
Neon	20.179	26.19	44.40	27.09	0.000	0.14466	0.76353
Argon	39.944	48.33	150.86	87.29	-0.004	0.18893	0.78649

TABLE 1 Summary of data and curve-fitted constants p , q used a

^a Physical properties that are not given in the original data source are taken from reference 4. Equations of state needed for calculations are given by Lielmezs and Mak [1].

 $Experimental - Calculated$ $%$ error = -Experimental $\times 100$

and AAD is defined as

I Deviation I AAD= N

where Deviation = Experimental – Calculated and $N =$ number of data points.

APPLICATION OF THE PROPOSED METHOD

The relations needed to calculate thermodynamic properties for this work over the supercritical and compressed regions of the *P-V-T* **surface (Fig. 1) have already been derived by Lielmezs and Mak [1]. Following that work [1], the saturated state temperature at 1 atm for carbon dioxide and acetylene has been taken as their "normal" boiling point temperature. The**

Comparison of prediction errors: Region III-supercritical

TABLE 2a

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TABLE 2b

Comparison of prediction errors: Region III-supercritical

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TABLE 2b (continued)

physical constants used as input data for all calculations have been listed in Table 1.

Region III-Supercritical

In the supercritical region predictions of $P-V-T$, five departure functions (ΔH , ΔS , ΔA , ΔG , ΔU) and the fugacity coefficient obtained from the SRK, PR and LM (this work) equations have been tested against the experimental data. The data sources for all the compounds are listed in

Fig. 2. Region III. Compressibility factors and reduced entropy departure functions of *n*-octane versus reduced temperature ($P_r = 4.03$) and reduced pressure ($T_r = 1.76$).

Comparison of prediction errors: Region IV-compressed liquid TABLE 3a

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TABLE 3b

Comparison of prediction errors: Region IV-compressed liquid

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TABLE 3b (continued)

Fig. 3. Region III. Reduced enthalpy and reduced Gibbs energy departure functions of *n*-octane versus reduced temperature $(P_r = 4.03)$ and reduced pressure $(T_r = 1.76)$.

Tables 2a and 2b. The results of the *P-V-T* **predictions of these equations are given in Tables 2a and 2b. For pressure prediction, the PR and LM equations are similar in accuracy. The overall average RMS % error from the SRK equation is about double that of the PR or LM equation. For the temperature prediction, all three equations are about the same in accuracy. For volume prediction, the PR and LM equations are again similar in accuracy for most compounds. However, in terms of the overall average RMS % error, the PR equation yields the lowest. In many cases, the error from the SRK equation is almost twice that of the PR or LM equation. Similar results were also obtained by Lielmezs and Mak [1] in the satura-**

Fig. 4. Region III. Reduced internal energy departure functions and fugacity coefficients of *n*-octane versus reduced temperature ($P_r = 4.03$) and reduced pressure ($T_r = 1.76$).

tion region. As a typical example, Fig. 2 presents plots of compressibility versus reduced temperature and reduced pressure for n -octane.

The results for the five departure functions $(\Delta H, \Delta S, \Delta A, \Delta G, \Delta U)$ are summarized in Table 2b. In terms of the overall deviation (AAD), the SRK and PR equations are similar for the ΔH , ΔS and ΔU departure functions. As already noted by Lielmezs and Mak [1], the LM equation is poor near the critical temperature because the derivative of the α -function (eqn. (3), Lielmezs and Mak [1]) with respect to temperature approaches infinity as the temperature approaches the critical value. However, above $T = 1.05$, the predictions from the LM equation are similar to those of the SRK and PR equations (Tables 2a and 2b, Figs. 2-4). For ΔA and ΔG departure

Fig. 5. Region IV. Compressibility factors and reduced entropy departure functions of *n*-heptane versus reduced temperature ($P_r = 0.73$) and reduced pressure ($T_r = 0.67$).

functions, error cancellation near the critical point results in similar accuracy for all three equations considered (Table 2b, Fig. 3). For the fugacity coefficient calculation, the SRK equation shows the best accuracy (Table 2b, Fig. 4). It is of interest to note that although the α -functions of the PR **and LM equations are so different (for a detailed discussion see Lielmezs and Mak [1]), the predictive accuracy of these two equations is similar for a number of compounds (Tables 2a and 2b).**

Region IV--compressed liquid

In the compressed liquid region predictions of *P-V-T,* **the five depar**ture functions $(\Delta H, \Delta S, \Delta A, \Delta G, \Delta U)$ and fugacity coefficient obtained

Fig. 6. Region IV. Reduced enthalpy and reduced Gibbs energy departure functions of *n*-heptane versus reduced temperature ($P_r = 0.73$) and reduced pressure ($T_r = 0.67$).

by means of the SRK, PR and LM (this work) equations have been compared with experimental data. The number of compounds studied, however, has been limited due to the lack of experimental data, especially data on departure functions. The experimental data sources are listed in Tables 3a and 3b. The RMS % errors of the *P-V-T* **prediction by the SRK, PR and LM equations are given in Tables 3a and 3b. For pressure prediction, all three equations yield very large errors. For temperature estimation, the magnitude of errors from the PR and LM equations is the same in all cases while the SRK equation gives substantially larger errors. In terms of an overall average RMS % error, the errors stemming from PR**

Fig. 7. Region IV. Reduced internal energy departure functions and fugacity coefficients of *n*-heptane versus reduced temperature ($P_r = 4.03$) and reduced pressure ($T_r = 1.76$).

and LM equations are about half that of the SRK equation (Table 3a). Similar conclusions are also drawn for the volume prediction (Table 3a). As a typical example of error distribution patterns, Fig. 5 presents the compressibility factor for n -heptane plotted against the reduced temperature and pressure.

The results of the five departure functions $(\Delta H, \Delta S, \Delta A, \Delta G, \Delta U)$ are given in Table 3b. As already noted, experimental data on departure functions in this region are limited. The TRC data book [5] contains enthalpy and entropy departure function data for seven of the compounds tested in this work (Table 3b).

The use of Lu et al.'s correlation [9] for estimating the enthalpy departure function for compressed liquids enabled the testing of the enthalpy and internal energy departure functions over a limited reduced temperature range [1] for an additional thirteen compounds (Table 3b). The results of the comparison for the five departure functions and fugacity coefficients are listed in Table 3b. Figures 5-7 give plots of the five departure functions and the fugacity coefficient against the reduced temperature and pressure for *n*-heptane. The overall results obtained (Tables 3a and 3b, Figs. $5-7$) show that for the compressed liquid region none of the three equations is adequate for predicting the considered thermodynamic properties. The accuracy of the PR and LM equations is similar for volume and temperature prediction. For pressure predictions all three equations are poor. For departure functions and fugacity coefficient prediction, all three equations are similar.

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