Thermodynamics of liquid mixtures of acids. Vapourliquid equilibria and excess molar enthalpies of alkanoic acid mixtures with alcohols

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Abstract

Vapour-liquid equilibria and excess molar enthalpies of binary solutions of methanoic acid, ethanoic acid, propanoic acid, and butanoic acid with methanol, ethanol, propanols, and 1-butanol were calculated using the UNIQUAC associated-solution model, which takes into account the homo-association of like molecules and the hetero-association of unlike molecules. Ternary excess molar enthalpies of ethanoic acid + methanol + tetrachloro-methane were predicted from the same model with binary parameters. The calculated results agree well with the experimental values.

LIST OF SYMBOLS

A, A_1, A_2	acid, acid monomer and dimer
$a_{\rm JI}$	binary interaction parameter for J–I pair
AB_i	chemical complex involving an acid and alcohol <i>i</i> -mer
$\mathbf{B}, \mathbf{B}_1, \mathbf{B}_1$	alcohol, alcohol monomer and alcohol open <i>i</i> -mer
B_{II}^{F}	free contribution to second virial coefficient of component
-	Ι
$C_{\rm JI}, D_{\rm JI}$	coefficients of eqn. (16)
F	objective function as defined by eqn. (19)
H^{E}	excess molar enthalpy
hA	enthalpy of hydrogen bond formation for acid dimer
h _{AB}	enthalpy of complex formation between components A
	and B
h _B	enthalpy of a hydrogen bond formation for alcohol
κ _A	equilibrium constant of acid dimer formation
K _{AB}	solvation constant between components A and B
K _B	association constant of alcohol open-chain <i>i</i> -mer formation
Ν	number of experimental data points

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Р	total pressure
P_1^s	saturated vapour pressure of pure component I
q_{I}	molecular geometric area parameter of pure component I
R	universal gas constant
r _I	molecular geometric volume parameter of pure com- ponent I
Т	absolute temperature
V	true molar volume of acid $+$ alcohol mixture given by eqn.
	(10)
V^{0}_{A}, V^{0}_{B}	true molar volumes of pure acid and alcohol given by eqns.
	(11) and (12), respectively
$v_{\rm I}^{\rm L}$	pure liquid molar volume of component I
x _I	liquid-phase mole fraction of component I
y _I	vapour-phase mole fraction of component I
y_{I_1}	vapour-phase mole fraction of monomeric component I
Ζ	lattice coordination number, here set as 10

Greek letters

γ_1	activity coefficient of component I
θ_{I}	surface fraction of component I
$\sigma_P, \sigma_T, \sigma_x, \sigma_y$	standard deviations in pressure, temperature, liquid and
	vapour compositions, respectively
$ au_{ m JI}$	coefficient as defined by $\exp(-a_{\rm JI}/T)$
Φ_{I}	segment fraction of component I
Φ_{I_1}	monomer segment fraction of component I
$\Phi^{0}_{A_{1}}, \Phi^{0}_{B_{1}}$	monomer segment fractions of pure acid and alcohol given
	by eqns. (13) and (14), respectively
ϕ_1	fugacity coefficient of component I at P and T
$oldsymbol{\phi}_{\mathrm{I}}^{\mathrm{s}}$	fugacity coefficient of pure component I at P_1^s and T

Subscripts

A, B	acid and alcohol
A_1, A_2	acid monomer and acid dimer
AB	binary complex
B_1, B_i	alcohol monomer and alcohol open <i>i</i> -mer
chem	chemical
I, J, K	components I, J, and K
phys	physical

Superscripts

0	pure liquid reference state
*	323.15 K

INTRODUCTION

The dimerization constants and enthalpies of hydrogen-bond formation of alkanoic acids have been estimated [1] and vapour-liquid equilibria and excess molar enthalpies of mixtures of alkanoic acids with hydrocarbons have been well reproduced by using the UNIQUAC associated-solution model [2]. The UNIQUAC associated-solution model has shown its good performance in the correlation and prediction of vapour-liquid and liquid-liquid equilibria and excess molar enthalpies for alcohol solutions [3-6].

In this paper, we apply the UNIQUAC associated-solution model for the correlation of vapour-liquid equilibria and excess molar enthalpies of mixtures involving an alkanoic acid and an alcohol and for the prediction of ternary excess molar enthalpies of ethanoic acid + methanol + tetrachloromethane from binary information alone.

SOLUTION MODEL

According to the UNIQUAC associated-solution model, we assume that the alkanoic acid (A) and the alcohol (B) exist in the solution as dimers (A_2) and open chains (B_i) , and that acid monomers and alcohol open chains lead to chemical complexes AB_i. The equilibrium constants for these complex-forming reactions are defined by

$$K_{A} = \frac{\Phi_{A_{2}}}{\Phi_{A_{1}}^{2}} \frac{1}{2} \text{ for } A_{1} + A_{1} = A_{2}$$

$$= K_{A}^{*} \exp\left[-\frac{h_{A}}{R}\left(\frac{1}{T} - \frac{1}{T^{*}}\right)\right] \qquad (1)$$

$$K_{B} = \frac{\Phi_{B_{i+1}}}{\Phi_{B_{i}}\Phi_{B_{1}}} \frac{i}{i+1} \text{ for } B_{i} + B_{1} = B_{i+1}$$

$$= K_{B}^{*} \exp\left[-\frac{h_{B}}{R}\left(\frac{1}{T} - \frac{1}{T^{*}}\right)\right] \qquad (2)$$

$$K_{AB} = \frac{\Phi_{AB_i}}{\Phi_{A_i}\Phi_{B_i}} \frac{i}{r_A + ir_B} \quad \text{for } A_1 + B_i = AB_i$$
$$= K_{AB}^* \exp\left[-\frac{h_{AB}}{R}\left(\frac{1}{T} - \frac{1}{T^*}\right)\right] \tag{3}$$

Activity coefficient

The activity coefficient of associating component I is given by

$$\ln \gamma_{I} = \ln\left(\frac{\Phi_{I_{1}}}{\Phi_{I_{1}}^{0}x_{I}}\right) + r_{I}\left(\frac{1}{V_{I}^{0}} - \frac{1}{V}\right) - \left(\frac{Z}{2}\right)q_{I}\left[\ln\left(\frac{\Phi_{I}}{\theta_{I}}\right) + 1 - \frac{\Phi_{I}}{\theta_{I}}\right] + q_{I}\left[1 - \ln\left(\sum_{J} \theta_{J}\tau_{JI}\right) - \sum_{J}\frac{\theta_{J}\tau_{IJ}}{\sum_{K} \theta_{K}\tau_{KJ}}\right]$$

$$(4)$$

where Z is the lattice coordination number taken as 10 and the segment fraction Φ_{I} , the surface fraction θ_{I} , the binary coefficient τ_{JI} related to the binary interaction parameter a_{JI} are expressed by

$$\Phi_{\rm I} = r_{\rm I} x_{\rm I} \bigg/ \sum_{\rm J} r_{\rm J} x_{\rm J} \tag{5}$$

$$\theta_{\rm I} = q_{\rm I} x_{\rm I} \bigg/ \sum_{\rm J} q_{\rm J} x_{\rm J} \tag{6}$$

$$\tau_{\rm JI} = \exp(-a_{\rm JI}/T) \tag{7}$$

The monomer segment fractions of both associating components are solved simultaneously from eqns. (8) and (9)

$$\Phi_{\rm A} = \Phi_{\rm A_1} + 2K_{\rm A}\Phi_{\rm A_1^2} + \frac{K_{\rm AB}\Phi_{\rm A_1}\Phi_{\rm B_1}r_{\rm A}}{1 - K_{\rm B}\Phi_{\rm B_1}} \tag{8}$$

$$\Phi_{\rm B} = \frac{\Phi_{\rm B_1}}{(1 - K_{\rm B} \Phi_{\rm B_1})^2} (1 + K_{\rm AB} \Phi_{\rm A_1} r_{\rm B}) \tag{9}$$

The true molar volume of the mixture is given by

$$\frac{1}{V} = \frac{\Phi_{A_1}}{r_A} (1 + K_A \Phi_{A_1}) + \frac{\Phi_{B_1}}{r_B (1 - K_B \Phi_{B_1})} + \frac{K_{AB} \Phi_{A_1} \Phi_{B_1}}{1 - K_B \Phi_{B_1}}$$
(10)

At pure liquid states, eqn. (10) reduces to eqns. (11) and (12) and the monomer segment fractions $\Phi_{A_1}^0$ and $\Phi_{B_1}^0$ are expressed by eqns. (13) and (14)

$$\frac{1}{V_{\rm A}^0} = \frac{1 - K_{\rm A} \Phi_{\rm A_1}^{02}}{r_{\rm A}} \tag{11}$$

$$\frac{1}{V_{\rm B}^0} = \frac{1 - K_{\rm B} \Phi_{\rm B_1}^0}{r_{\rm B}}$$
(12)

$$\Phi_{A_1}^0 = [-1 + (1 + 8K_A)^{0.5}]/4K_A$$

$$\Phi_{B_1}^0 = [2K_B + 1 - (1 + 4K_B)^{0.5}]/2K_B^2$$
(13)
(14)

Excess molar enthalpy

The excess molar enthalpy of the acid-alcohol mixture is expressed as the sum of the chemical and physical contributions

$$H^{\rm E} = H_{\rm chem}^{\rm E} + H_{\rm phys}^{\rm E}$$

$$= h_{\rm A} x_{\rm A} K_{\rm A} \left(\frac{\Phi_{\rm A_1}^2}{\Phi_{\rm A}} - \Phi_{\rm A_1}^{02} \right) + h_{\rm B} x_{\rm B} K_{\rm B} (\Phi_{\rm B_1} - \Phi_{\rm B_1}^0)$$

$$+ h_{\rm AB} K_{\rm AB} r_{\rm B} \Phi_{\rm A_1} x_{\rm B} \frac{(1 - K_{\rm B} \Phi_{\rm B_1})}{(1 + K_{\rm AB} r_{\rm B} \Phi_{\rm A_1})}$$

$$- R \sum_{\rm I} q_{\rm I} x_{\rm I} \frac{\sum_{\rm J} \theta_{\rm J} \frac{\partial \tau_{\rm JI}}{\partial (1/T)}}{\sum_{\rm J} \theta_{\rm J} \tau_{\rm JI}}$$
(15)

The binary interaction parameters are assumed to be linearly dependent on temperature

$$a_{\rm JI} = C_{\rm JI} + D_{\rm JI}(T - 273.15) \tag{16}$$

CALCULATED RESULTS

Vapour-liquid equilibrium data were reduced using the thermodynamic relation

$$P\phi_{I}y_{I} = \gamma_{I}x_{I}\phi_{I}^{s}P_{I}^{s}\left[\frac{\nu_{I}^{L}(P-P_{I}^{s})}{RT}\right]$$
(17)

where P is the total pressure, y_1 the vapour-phase mole fraction of component I, P_1^s the pure component vapour pressure taken from the original vapour-liquid equilibrium data set, and v_1^L the pure-liquid molar volume estimated from the modified Rackett equation [7]. The fugacity coefficient ϕ_1 is calculated from the chemical theory of vapour imperfections [8]

$$\phi_{\rm I} = \frac{y_{\rm I_1}}{y_{\rm I}} \exp\left(\frac{PB_{\rm II}^{\rm F}}{RT}\right) \tag{18}$$

where y_{I_1} is the vapour-phase monomer mole fraction of component I and

Component A	<i>K</i> _A at 323.15 K	$-h_A/kJ \text{ mol}^{-1}$	r _A	q_{A}
Methanoic acid	15899.8	46.6	1.24	1.19
Ethanoic acid	14651.2	44.5	1.78	1.62
Propanoic acid	11948.5	47.9	2.32	2.05
Butanoic acid	11274.5	45.1	2.86	2.49
Methanol	173.9	23.2	1.15	1.12
Ethanol	110.4	23.2	1.69	1.55
1-Propanol	87.0	23.2	2.78	2.51
2-Propanol	49.1	23.2	2.78	2.51
1-Butanol	69.5	23.2	2.77	2.42
Tetrachloromethane			2.71	2.37

TABLE 1

Association parameters and molecular structural parameters for pure components

 B_{II}^{F} is the free contribution to the second virial coefficient [9]. Values of the critical constants and related parameters to evaluate v_{I}^{L} , B_{II}^{F} and the dimerization contribution to the second virial coefficient are available from Ambrose and Ghiassee [10] and Reid et al. [11].

The association parameters of the pure acids and alcohols were taken from Tamura and Nagata [1], Brandani [12] and Stokes and Burfitt [13], and the pure component molecular structural parameters, r and q, were calculated from the method of Vera et al. [14]. These values are given in Table 1. The solvation parameters are shown in Table 2.

TABLE 2

Solvation parameters for binary mixtures

System (A + B)	K_{AB} at	$-h_{AB}/$	
	323.15 K	$kJ mol^{-1}$	
Methanoic acid + 1-propanol	450	32	
Methanoic acid + 2-propanol	350	32	
Methanoic acid + 1-butanol	400	32	
Ethanoic acid + methanol	500	32	
Ethanoic acid + ethanol	400	32	
Ethanoic acid + 1-propanol	350	32	
Ethanoic acid + 2-propanol	300	32	
Ethanoic acid + 1-butanol	320	32	
Propanoic acid + methanol	400	32	
Propanoic acid + 2-propanol	280	32	
Propanoic acid + 1-butanol	280	32	
Butanoic acid + methanol	380	32	
Methanol + tetrachloromethane	1.5	5.5	

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The results of fitting the UNIQUAC associated-solution model to vapour-liquid equilibrium data and root-mean-square deviations δP , δT , δx and by for binary acid + alcohol systems

System (A + B)	Temp. v	Number of data	Energy par	ameters	Root-mean	-square devi	ations		Ref.
	4	points	a _{AB} /K	$a_{\rm BA}/{ m K}$	δ <i>P</i> /Torr	<i>δT</i> /K	$\delta x \times 10^3$	$\delta y imes 10^3$	
Methanoic acid + 1-butanol ^a	298.15	6	865.26	-364.45	3.18	0.00	1.1	10.0	16
Methanoic acid + 1-butanol ^a	308.15	6	929.54	-380.39	2.73	0.00	1.4	12.9	
Methanoic acid + 1-butanol ^a	318.15	6	903.56	-377.72	2.55	0.00	1.5	13.0	
Ethanoic acid + methanol ^a	308.15	6	- 347.68	-3.08	2.87	0.00	1.2	6.4	17
Ethanoic acid + methanol ^a	318.15	6	-363.64	20.83	1.83	0.00	1.1	7.0	
Ethanoic acid + ethanol ^a	308.15	13	-267.06	-7.86	2.00	0.00	1.4	7.9	17
Ethanoic acid + ethanol ^a	318.15	13	-262.74	-31.59	2.06	0.00	1.2	6.1	
Ethanoic acid + 1-butanol ^a	298.15	10	-92.06	-126.20	1.24	0.00	1.7	13.6	18
Ethanoic acid + 1-butanol ^a	308.15	10	-41.28	-164.23	1.84	0.00	1.4	12.4	
Ethanoic acid + 1-butanol ^a	318.15	10	-16.19	-174.96	2.85	0.00	1.3	11.6	
Propanoic acid + methanol ^a	298.18	16	-298.39	9.64	1.59	0.00	2.0	7.3	19
Propanoic acid + methanol ^b	300.07	32	-281.96	-9.17	1.04	0.00	0.5	4.0	
Propanoic acid + methanol ^b	308.21	31	-288.11	-9.00	1.46	0.00	0.9	6.5	
Propanoic acid + methanol ^b	318.16	32	-294.37	-6.36	1.91	0.00	1.3	7.2	
Butanoic acid + methanol ^b	301.75	16	-299.81	57.63	2.07	0.00	1.3	5.3	19
Butanoic acid + methanol ^b	308.26	16	-301.37	59.00	2.22	0.00	1.8	5.6	
Butanoic acid + methanol ^b	312.62	16	-295.39	48.35	2.32	0.00	3.1	5.7	
Butanoic acid + mcthanol ^b	318.20	16	-291.67	40.59	2.33	0.00	2.3	5.4	
^a Vapour mole fractions were ev	aluated by a	numerical pro	cedure given t	y Miksch et a	I. [20]. ^b Vapol	ur mole fract	tions were cal	culated by a 1	nethod

described by Fredenslund et al. [21].

An optimum set of the energy parameters was sought by minimizing the objective function [15]

$$F = \sum_{i=1}^{N} \left[\frac{(P_i - \hat{P}_i)^2}{\sigma_P^2} + \frac{(T_i - \hat{T}_i)^2}{\sigma_T^2} + \frac{(x_{1i} - \hat{x}_{1i})^2}{\sigma_x^2} + \frac{(y_{1i} - \hat{y}_{1i})^2}{\sigma_y^2} \right]$$
(19)

where a circumflex denotes the most probable calculated value corresponding to each measured variable. The standard deviations of the measured values are: for pressure, $\sigma_P = 1$ Torr; for temperature, $\sigma_T = 0.05$ K; for liquid-phase mole fraction, $\sigma_x = 0.001$; for vapour-phase mole fraction, $\sigma_y = 0.003$. Table 3 gives the results of vapour-liquid equilibrium data reduction and Figs. 1-4 show some typical examples.

Table 4 lists the binary results of the excess molar enthalpies. The coefficients, C_{II} and D_{II} , were obtained by minimizing the sum of the



Fig. 1. Vapour-liquid equilibria for the ethanoic acid + methanol system. Calculated: ——; experimental [17]: ▲, 308.15 K; ●, 318.5 K.



Fig. 2. Vapour–liquid equilibria for the ethanoic acid + ethanol system. Calculated: —; experimental [17]: ▲, 308.15 K; ●, 318.15 K.

squares of the deviations between the experimental and calculated values. The simplex method of Nelder and Mead [25] was used for this purpose. Figures 5–8 compare the calculated values with the observed results for selected systems.

The ternary excess molar enthalpies of the ethanoic acid +methanol + tetrachloromethane system at 298.15 K were predicted using the UNIQUAC associated-solution model with the binary parameters given in Table 4. The absolute arithmetic mean deviation between the predicted and experimental values for 35 data points [26] is 26.03 J mol^{-1} .

We may conclude that the UNIQUAC associated-solution model is useful in the correlation of vapour-liquid equilibrium and excess molar enthalpy data of binary acid + alcohol mixtures and could be used for the prediction of ternary excess molar enthalpy data of solutions including an acid, an alcohol and one non-associated component.





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Fig.

Activity coefficient

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Pressure/torr 33 29

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The results of fitting the UNIQUAC associated solution model to excess molar enthalpies of binary mixtures at 298.15 K

System (A + B)	Number of date	Parameters				Deviations	a	Ref.
	or uata points	C_{AB}/K	$C_{\rm BA}/{ m K}$	D_{AB}	$D_{\mathbf{BA}}$	AAM/ J mol ⁻¹	RMS/ J mol ⁻¹	
Methanoic acid + 1-propanol	19 ^b	700.45	225.20	4.1375	2.5392	5.41	6.00	22
Methanoic acid + 2-propanol	19	1123.69	121.10	28.8885	2.0340	9.65	10.91	22
Methanoic acid + 1-butanol	19	853.99	226.82	5.2450	2.4085	2.91	3.39	22
Ethanoic acid + methanol	19	991.98	31.68	6.9597	1.1175	0.87	1.36	22
Ethanoic acid + ethanol	19	940.97	197.78	6.2914	1.4682	0.82	1.20	22
Ethanoic acid + 1-propanol	19	962.92	431.02	5.9770	2.0505	5.17	5.88	22
Ethanoic acid + 2-propanol	19	801.37	122.51	4.7076	0.6691	8.65	10.20	22
Ethanoic acid + 1-propanol	19	1122.35	450.44	8.5736	2.0071	7.68	8.69	22
Propanoic acid + methanol	19	176.88	171.33	94.5072	1.7533	13.38	15.29	22
Propanoic acid + 2-propanol	19	1133.77	397.50	9.0842	2.0969	9.84	11.23	22
Propanoic acid + 1-butanol	19	1025.15	540.06	43.2225	2.8675	13.07	14.78	22
Butanoic acid + methanol	19	-138.10	-3954.65	0.0298	14.4560	14.77	16.45	22
Ethanoic acid + tetrachloromethane ^c	4	306.41	406.66	0.8252	1.3301	3.35	4.51	23
Methanol + tetrachloromethane	19	130.03	367.46	0.8833	2.1754	4.09	8.18	24
^a AAM, absolute arithmetic mean deviatio intervals from the smoothing equation give	m; RMS, roc n by Hasse a	t-mean-squa nd Lorenz (]	re deviation. 1985) [22]. ° 7	^b Experime $\Gamma = 293.15$ k	ental values	are interpola	ated at 0.05	mole fraction









Fig. 7. Excess molar enthalpies for two propanoic acid + alcohol systems at 298.15 K. Calculated: ---; experimental [22]: \blacksquare , propanoic acid + methanol; \blacktriangle , propanoic acid + nethanol.

propanol; •, propanoic acid + 2-propanol.

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