Thermochimica Acta, 218 (1993) 351–363
Elsevier Science Publishers B.V., Amsterdam

THERMAL TRANSPORT ACROSS DELAMINATIONS IN AEROSPACE COMPOSITE LAMINATE MATERIALS

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Keywords: aerospace, composite, testing, thermal diffusivity, optical measurements, infrared

ABSTRACT

A promising new technique for detecting aircraft structural cracks is called thermal diffusivity imaging. A heat flux is applied to one surface of a structural area while the opposite surface temperature field is scanned with an infrared television camera. In theory, layer delamination or other material non-uniformities will produce changes in the material thermal resistance between source and detector. These changes will produce regions of lower temperature than the surrounding material at the rear surface of the laminate. The technique has several advantages over the other methods available for detecting structural problems in aerospace applications: it is capable of scanning large areas in a short time; it avoids the use of potentially hazardous radiation, and it does not require forcing the material to the level of failure.

In the present paper, an effective thermal diffusivity model is used to provide estimates of the sensitivity of the technique. A solution is determined for the differential energy equation through the composite laminate which is subjected to flux type boundary conditions with a homogeneous initial condition. For a single material, this results in a compact mathematical solution which can easily be applied to the determination of both transient and steady-state thermal transport properties. For composites, the problem is more difficult but the application of the effective thermal diffusivity model greatly reduces the mathematical complexity. Thermal transport across a gap (delamination) is modeled as an optically-thin process and it is shown that thermal conduction is likely to be the dominant mode of heat transfer.

INTRODUCTION

As the United States commercial airline fleet ages, the question of structural integrity becomes extremely important {Derra, (1990}. A recent Wall Street Journal article indicated that the average age of the aircraft being flown by T.W.A. is 24+ years. Clearly this situation, which is typical of the industry, will not change substantially in the near future. New non-destructive test (NDT) techniques must, therefore, be developed for detecting material flaws before they become large enough to cause structural failure. One of the more promising new techniques for detecting structural cracks in composite laminates is called thermal diffusivity imaging. A uniform heat flux is directed onto one surface and the opposite surface is optically scanned to observe the temperature field. This method indicates regions of possible defects in much the same manner as the thermography method for detecting cancerous tumors. In this case, it is the body itself which acts as the heat source. In the present paper, the governing differential equations will be presented and applied to the case of a composite laminate which contains an air gap that simulates a delamination. This model will then be used to to estimate the thermography method sensitivity.

Good agreement results between prediction and experiment for transient thermal conduction problems if the analysis uses an experimentally determined value of thermal diffusivity. When the governing differential equation is examined, the reason for this is obvious--especially in the case of one-dimensional conduction. In terms of dimensionless quantities, the Fourier modulus is the only relevant parameter in the analysis at a particular position. Because the sample thickness can be measured and time is the independent variable, one can determine the thermal diffusivity directly from the temperature-time history at a given position in a sample. This position is usually the rear surface of the material sample.

The radiant pulsed method has become the standard experimental technique for determining thermal diffusivity for a wide range of materials {Taylor (1973), (1975), Lee and Taylor, (1976)}. A pulse of radiant energy is deposited in a very short time on the front surface of a thin specimen and the rear surface temperature excursion is monitored. A correlation between a one-dimensional thermal conduction model and the experimental record yields the thermal diffusivity. One estimate claims that "about 75% of the free world's thermal diffusivity values are currently being generated by this one technique" {Taylor, (1975)}.

In recent years, the method has been extended to the case of transient thermal conduction in composite materials. Both purely thermal problems {Wright, et.al., (1975), Tittle, (1965), Bulavin and Koscheev, (1965)} and coupled thermal-mechanical problems {Anderholm, (1968), Hartman, et.al., (1971)} have motivated this interest. A number of theoretical studies {Schimmel and Donaldson, (1975), Schimmel, et.al., (1975), Donaldson, et.al., (1977)} and experimental measurements {Taylor, (1975), Larson and Koyama, (1968), Brandt and Havranek, (1976), Lee, et.al., (1975)} have clearly established that some sort of "effective parameter" can be used to characterize the transient temperature response of composite materials.

The sensitivity of the composite effective thermal diffusivity to material delamination will be considered in the present paper. Both a step change in front surface temperature and a uniform heat flux applied at the front surface will be considered. The first case permits a simplified mathematical analysis but it is difficult to simulate in the measurement laboratory. Step changes are always difficult to produce and temperature step changes with out immersion are virtually impossible. The constant heat flux case can easily be simulated with electrical heating but it does not lead to a steady state condition for an adiabatic rear surface. This will be addressed by showing that the front and rear surfaces temperature increase linearly with time (ramp behavior).

THEORY - THERMAL MODELS

Three different mathematical models can provide a basis for experimentally determining the thermal diffusivity of a material sample. The first is the radiant pulsed technique, the second is a step change in front surface temperature and the third is the application of a constant heat flux at the front surface. The rear surface of all three is considered adiabatic and the initial temperature is zero in all cases. The thermal diffusivity, α, has its usual definition: the ratio of the thermal conductivity, λ, to the product of the density, ρ, and specific heat, C. These three cases are classical applications of the Sturm-Liouville system of linear partial differential equations. The first is an initial value problem while the last two are boundary value problems. Solutions can easily be obtained by separation of variables and expansion in Fourier sine/cosine series. To avoid repetition of the eigenvalues, a shorthand notation will be used in the last two models:

$$\mu_{n1}=\frac{n\pi}{L} \quad \& \quad \mu_{n2}=\left[\frac{(2n+1)\pi}{2L}\right] \tag{1}$$

The first idealized case to be discussed is that of the radiant pulsed technique for determining thermal diffusivity. The simplifying physical assumptions are that a planar, uniform pulse of energy is instantaneously deposited in a very thin region adjacent to the front surface of a thin, flat sample and the temperature excursion of the adiabatic rear surface is monitored. In practice, the effects of extended pulse duration {Donaldson and Faubian, (1976)}, pulse shape, and finite heat losses {Heckman, (1976)} must often be considered but they will be neglected here. It has been shown {Heckman (1976) and Taylor (1973)} that the omission of these mechanisms does not severely degrade the quality of the experimental results.

The deposition length, x_d is a region of unknown extent very near the surface in which the energy is instantaneously deposited by the incident radiant pulse. Although the actual deposition profile is exponential, it is assumed that it is uniform on $0 < x < x_d$. It is possible to determine T_s through the use of a radiation calorimeter or other device. For this analysis, however, the need for specifying this parameter is avoided by using the average sample temperature at long times, T_m It is clear from the application of the first law of thermodynamics that for zero losses:

$$T_m = \frac{x_d}{L} T_s \qquad (2)$$

With respect to these assumptions, the solution of the system of equations at the rear surface, $x = L$, is {Donaldson and Faubian, (1976)}:

$$\frac{T(L,t)}{T_m} = 1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2\alpha t}{L^2}} \qquad (3)$$

Rewriting slightly, the first few terms are:

$$\frac{T(L,t)}{T_m} = 1 - 2\left[e^{-\hat{a}\xi} - e^{-4\hat{a}\xi} + e^{-9\hat{a}\xi} - e^{-16\hat{a}\xi} + \dots \right] \qquad (4)$$

where:

$$\xi = \frac{t}{t_{1/2}} \quad \text{and} \quad \hat{a} = \frac{\pi^2 \alpha t_{1/2}}{L^2} = \pi^2 F_{o1/2} \qquad (5)$$

The quantity indicated by $F_{o1/2}$ is the Fourier number at the half-time, $t_{1/2}$. This is the time required for the rear temperature rise (above initial) to reach one-half the final value, T_m. It should be noted that the quotient of thermal diffusivity and length squared can be considered an inverse time constant. This observation will be used to advantage in later sections of this work. In terms of the experimental measurements, low temperature measurements can be made with a thermocouple and a recording oscilloscope or the more sophisticated digital data acquisition system {Taylor, (1985)}. At higher temperatures, optical techniques are used. In the case of the recording oscilloscope, the average temperature, T_m, is obtained from the difference of the base line and the scope retrace. If the above expression is solved for $T(L,t)/T_m = 1/2$, this corresponds to one half-time, or $\xi = 1.0$. Solving the above equation results in $F_{o1/2} = 0.139$ {Taylor, (1975)}.

For the second idealized case, the front surface of the slab is subjected to a unit step change in temperature at $t = 0$ to T_s and held there. The slab rear surface is adiabatic for all times of interest. Applying physical reasoning, it is clear that the rear surface temperature will eventually rise to the level of the front surface in the steady-state for the no-loss situation imposed by the boundary conditions. The total solution is:

$$\frac{T(x,t)}{T_s} = 1 - \frac{4}{\pi}\sum_{n=0}^{\infty} \frac{e^{-\mu_{n2}^2 \alpha t}}{(2n+1)} \sin(\mu_{n2} x) \qquad (6)$$

at the rear surface, this becomes:

$$\frac{T(L,t)}{T_s} = 1 - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n e^{-\mu_{n2}^2 \alpha t}}{(2n+1)} \tag{7}$$

This is given {Schimmel, et. al., (1975)} in a slightly different form with the definition of a modified Fourier modulus:

$$\frac{T(L,t)}{T_s} = 1 - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n e^{-(2n+1)^2 \hat{F}_o}}{(2n+1)} \tag{8}$$

where:

$$\hat{F}_o = \frac{\pi^2 \alpha t}{4L^2} = \frac{\hat{a}\xi}{4} \tag{9}$$

There is no physical reason for continuing to use this notation in terms of the half time, $t_{1/2}$ but it is instructive to compare the results from the various models in terms of a constant benchmark. The value of $\hat{a} = 0.139\,\pi^2$ should not change as a function of the particular model used for the analysis. If we let $T_s = T_m$, we can compare the results directly with the radiant pulsed method. The first few terms of the above series are the following:

$$\frac{T(L,t)}{T_m} = 1 - \frac{4}{\pi}\left[e^{-\frac{\hat{a}\xi}{4}} - \frac{e^{-9\frac{\hat{a}\xi}{4}}}{3} + \frac{e^{-25\frac{\hat{a}\xi}{4}}}{5} - \frac{e^{-49\frac{\hat{a}\xi}{4}}}{7} + \dots\right] \tag{10}$$

Note that the series clearly converges rapidly for values of ξ on the order of unity or larger. A comparison of the values obtained for a single term, two terms, etc, is given by {Schimmel, (1990)} and it is confirmed that the series is well represented by two terms except very near the $\xi = 0$ point, i.e., $t = 0$; in this region even the exact series solution requires a large number of terms.

Unfortunately, although the results of the present model are useful for understanding thermal response, there are several experimental difficulties inherent in attempting to produce a constant temperature at the sample front surface. A much easier situation to generate in the laboratory is a constant heat flux at the front surface (e.g., electrical heating). Ideally, a constant temperature rear surface (e.g., constant-temperature bath) would be specified but this would require measurement of the energy removed by the cooling device -- a measurement which is inherently imprecise. The alternative is to consider the rear surface adiabatic as has been done . It should be noted that the assumption of an adiabatic rear surface in the current case is only valid for the short time domain. In the steady-state, losses will occur under the best of conditions. In the following paragraph, an alternative to both the previous methods will be explored and shown that it can be used both to determine thermal diffusivity in the laboratory and material delaminations in the field. Hopefully, this technique will form the basis for a non-destructive technique that can be deployed in the aircraft maintenance field.

For the third model, the physical model once again consists of a constant property one-dimensional slab. A constant heat flux term is applied to the front surface while the rear surface is considered adiabatic. The solution for any position and time is given by:

$$\frac{T(x,t)}{-q_s L/\lambda} = \frac{\alpha t}{L^2} + \frac{2}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos(\mu_{n1}x)}{n^2} - \frac{2}{\pi^2}\sum_{n=0}^{\infty}\frac{e^{-\mu_{n1}^2\alpha t}}{n^2}\cos(\mu_{n1}x) \tag{11}$$

which converges to a transient plus a long term solution. There is a divergent component which increases with time so that steady-state does not appear to be a totally accurate term for the long term solution. It should be recalled, however, that a similar situation prevails whenever a surface flux boundary condition is applied to a one-dimensional solid.

Rewriting the expression in terms of these three components:

$$\frac{T(x,t)}{-q_s L/\Lambda} = \frac{\alpha t}{L^2} + \frac{x^2}{2L^2} - \frac{x}{L} + \frac{1}{3} - \frac{2}{\pi^2}\sum_{n=1}^{\infty} \frac{e^{-\mu_{n1}^2 \alpha t}}{n^2} \cos(\mu_{n1} x) \tag{12}$$

At the rear surface of the specimen, this is:

$$\frac{T(L,t)}{-q_s L/\Lambda} = \frac{\alpha t}{L^2} - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^n e^{-\mu_{n1}^2 \alpha t}}{n^2} \tag{13}$$

For comparison with the expressions given above, the first few terms can be expressed in terms of the previously defined quantities:

$$\frac{T(L,t)}{-q_s L/\Lambda} = \frac{\alpha t}{L^2} - \frac{1}{6} - \frac{2}{\pi^2}\left[e^{-\hat{a}\xi} - \frac{e^{-4\hat{a}\xi}}{4} + \frac{e^{-9\hat{a}\xi}}{9} - \frac{e^{-16\hat{a}\xi}}{16} + \cdots \right] \tag{14}$$

This model can be used to determine the thermal diffusivity of a thin material sample which is originally in thermal equilibrium with its surroundings. Note that the assumptions are consistent with those of the radiant pulsed method provided that the time and temperature levels are similar. The difference of temperature rises predicted by equation (12) when evaluated at the front and the rear surfaces is:

$$\frac{T(0,t) - T(L,t)}{-q_s L/\Lambda} = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=0}^{\infty} \frac{e^{-\mu_{n2}^2 \alpha t}}{(2n+1)^2} \tag{15}$$

Clearly the transient solution in terms of the first few terms is:

$$\frac{T(0,t) - T(L,t)}{-q_s L/\Lambda} = \frac{1}{2}\left\{ 1 - \frac{8}{\pi^2}\left[e^{-\hat{a}\xi} + \frac{e^{-9\hat{a}\xi}}{9} + \frac{e^{-25\hat{a}\xi}}{25} + \frac{e^{-49\hat{a}\xi}}{49} + \cdots \right] \right\} \tag{16}$$

and the long term component of the above equation slightly rewritten is:

$$\lim_{t \to \infty} \left[\frac{T(0,t) - T(L,t)}{-q_s L/2\Lambda} \right] = 1 \tag{17}$$

which permits direct comparison with the radiant pulsed method. The dimensionless temperature rise for the flux method also appraoches unity at long times as does the pulsed method. Recall that this reference is used in lieu of an accurate calorimetric measurement of the incident pulse.

The general expression for the nondimensional temperature difference is thus rewritten:

$$\frac{T(0,t)-T(L,t)}{-q_s L/2\Lambda} = 1 - \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{e^{-\mu_{n2}^2 \alpha t}}{(2n+1)^2} \tag{18}$$

This can be compared with equation (3) from the radiant pulsed method. The remarkable similarity between the two indicates the characteristic response of the sample to heating. This was to be expected because of the notion of input and output functions related by a transfer function

{Schimmel,(1990)}. Either expression can be used to determine α from the temperature-time history of the surfaces indicated. Recall that the requirement to explicitly measure the initial temperature rise in the deposition region is avoided by the use of an averaged temperature in the radiant pulsed method. It would appear at first glance that one requires additional information (and thus introduces additional experimental error) for the flux method. This is fortunately not the case as we now proceed to show.

Recall that the long term or "steady-state" temperature expression predicted by the constant heat flux method at the rear surface is:

$$\lim_{t\to\infty}\left[\frac{T(L,t)}{-q_s L/\Lambda}\right] = \frac{\alpha t}{L^2} - \frac{1}{6} \tag{19}$$

Thus, the rear surface temperature-time profile for long times is simply:

$$\lim_{t\to\infty} T(L,t) = \frac{-q_s t}{\rho C_p L} - \frac{-q_s L}{6\Lambda} = Bt + C_1 \tag{20}$$

while that of the front surface is:

$$\lim_{t\to\infty} T(0,t) = \frac{-q_s t}{\rho C_p L} + \frac{-q_s L}{3\Lambda} = Bt + C_2 \tag{21}$$

The difference between the two is clearly a constant so, the product of the parameter B and the reciprocal of this term is:

$$\left(\frac{-q_s}{\rho C_p L}\right)\left(\frac{2\Lambda}{-q_s L}\right) = 2\frac{\alpha}{L^2} \tag{22}$$

But, B is simply the slope of the steady-state portion of the temperature-time curvehis ; a constant as the above expression indicates (Figure 1). We can thus determine α/L^2, from the steady-state portion of the curve by evaluating the slope. The front and rear surface temperature excursions are presented in Figure 1 along with a straight line representing the slope, B. Here, we continue to use $t_{1/2}$ for comparison purposes, so the quantity ξ is indicated as the independent variable.

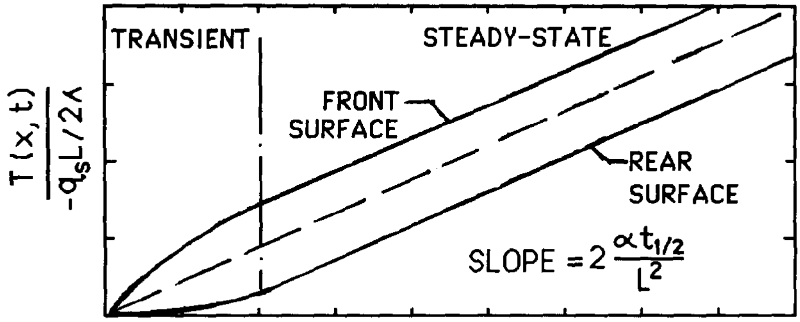

FIGURE 1

SURFACE TEMPERATURE RESPONSE

No scale is used on the figure because it is simply to identify the transient and steady-state time regimes and front/rear surface temperature-time profiles. In terms of the electrical analog, the front surface temperature profile corresponds to a "lead" circuit while that of the rear surface indicates a "lag" circuit The difference between the two is a lead-lag circuit which is a common element in automatic control systems analysis. The important feature to notice is the relation between input (heat flux) and output (temperature-time) is a constant transfer function. Thus homogeneous and multi-material laminate.transient thermal conduction problems can be analyzed by the use of simple transfer functions between input and output {Schimmel (1990)}.

EFFECTIVE THERMAL DIFFUSIVITY FOR A COMPOSITE LAMINATE

A general expression for the ratio of L^2_{eff} to α_{eff} can be determined {Schimmel, et al, 1975}:

$$\frac{L^2_{eff}}{\alpha_{eff}} = \left\{ \sum_{i=1}^{n} \frac{L_i^2}{\alpha_i} + 2 \sum_{i=1}^{n-1} \frac{L_i}{\alpha_i} \sum_{j=i+1}^{n} \frac{\rho_j C_j L_j}{\rho_i C_i} \right\} \tag{23}$$

For a three layer laminate, this reduces to:

$$\frac{L^2_{eff}}{\alpha_{eff}} = \frac{L_1^2}{\alpha_1} + \frac{L_2^2}{\alpha_2} + \frac{L_3^2}{\alpha_3} + 2\left[\left(\frac{L_1 L_2}{\alpha_1}\right)\left(\frac{\rho_2 C_2}{\rho_1 C_1}\right) + \left(\frac{L_1 L_3}{\alpha_1}\right)\left(\frac{\rho_3 C_3}{\rho_1 C_1}\right) + \left(\frac{L_2 L_3}{\alpha_2}\right)\left(\frac{\rho_3 C_3}{\rho_2 C_2}\right)\right] \tag{24}$$

If this expression, is now substituted into the single layer expression for the same thermal boundary and initial conditions, the effect of varying the gap thickness can be determined.

TECHNIQUE SENSITIVITY AND EXAMPLE CALCULATION

An example of the physical situation is given schematically in Figure 2. A panel joint is shown in its original condition and with a delamination as a result of a rivet which has failed in tension.

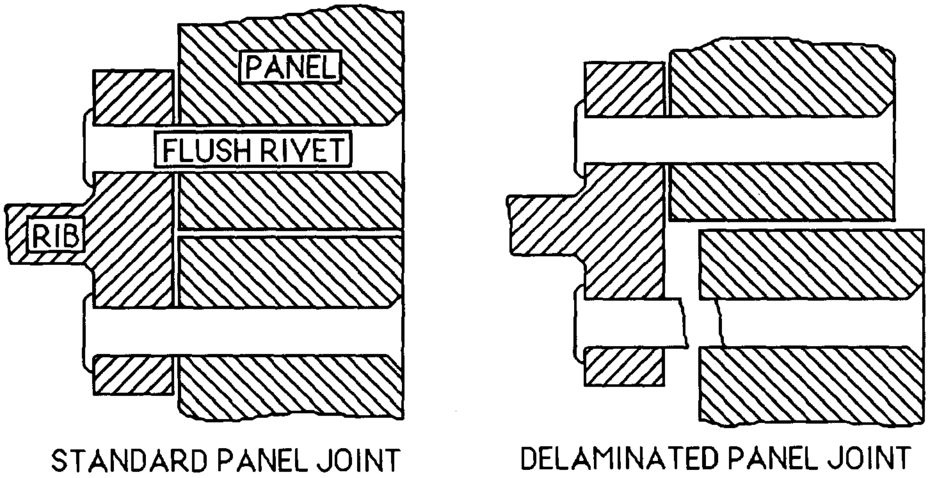

FIGURE 2

This is modelled as a thin air gap between two solid layers as indicated in Figure 3. The governing energy equation will now have an additional term because of the thermal radiation across the gap. Because the gap is assumed to be small, the process will be optically thin even if a radiatively participating gas is introduced into the gap. Were this not the case, our linear analysis would not be able to handle the nonlinear coupled radiation-conduction heat transfer process even under steady-state conditions {Schimmel, et al, (1970)}.

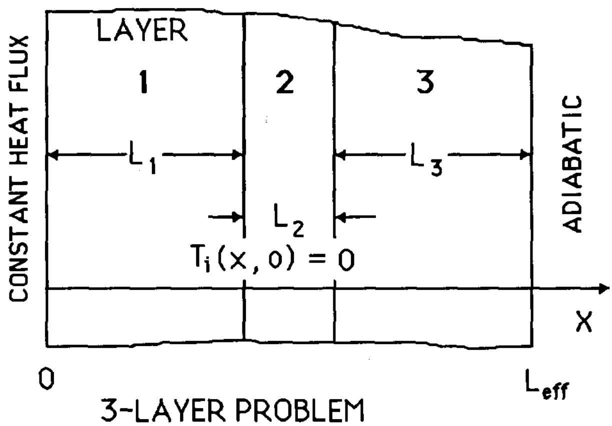

FIGURE 3

With the appropriate boundary and initial conditions, the general equation for a one-dimensional composite laminate consisting of n layers is:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} + q_R \qquad 0 < x < L, \ 0 < t$$

$$\frac{\partial T_1(0,t)}{\partial x} = -q_s \qquad 0 < t$$

$$\frac{\partial T_n(L,t)}{\partial x} = 0 \qquad 0 < t$$

$$T_i(x,0) = 0 \qquad 0 < x < L \quad i = 1,2,\cdots,n \tag{25}$$

Noting that the temperature and heat flux must be continuous at the interfaces indicates:

$$T_j(L_j,t) = T_{j+1}(L_{j+1},t)$$

$$\Lambda_j \frac{\partial T_j(L_j,t)}{\partial x} = \Lambda_{j+1}\frac{\partial T_{j+1}(L_{j+1},t)}{\partial x} \tag{26}$$

For the specific example being considered, the radiation flux term between layers 1 and 3, will be considered optically thin and we expect only solid radiative exchange between the surfaces. For this case, the radiative heat flux between gray diffuse surfaces is:

$$q_R = \frac{\sigma(T_3^4 - T_1^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} \tag{27}$$

where σ is the Stefan-Boltzmann constant and ϵ_i is the surface emittance of layer i.

If a Taylor series expansion is employed for T_3 in terms of T_1,

$$T_3^4 = T_1^4 + 4T_1^3(T_3 - T_1) + \cdots \tag{28}$$

The radiative heat flux can now be expressed in terms of an effective radiation conductivity

$$q_R = \Lambda_R \frac{\partial T(L_1,t)}{\partial x} \tag{29}$$

where

$$\Lambda_R = \frac{4\sigma T_1^3 L_2}{\epsilon_{13}} \tag{30}$$

where

$$\epsilon_{13} = \left[\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right] \tag{31}$$

Clearly this is not a thermophysical property in the usual sense but it permits us to make use of the notion of effective thermal properties for heat transfer through the three layer composite laminate. The total heat flux across the gap will consist of a sum of the conduction and a radiation components. Because the gap extent is considered small when compared with the gap length in the vertical direction, natural convection is not expected to be significant. Note that the presence of a radiatively participating gas in the gap will also not substantially change the total heat flux because of the optically thin assumption. An estimate of the magnitude of the radiative heat flux component can be obtained by comparing the radiative and molecular conductivities for typical values of T_1 and ϵ_{13} as L_2 is varied. This is presented in Table 1 for T_1 = 300 K AND $L_2)_{eq}$ indicates the value for which λ_R equals the molecular thermal conductivity (λ_2 = 0.026 W/m-K for air at 300 K).

TABLE 1

Gap required for equivalence between radiation and conduction heat transfer

$\epsilon_1 = \epsilon_3$	ϵ_{13}	Λ_R/L_2	$L_2)_{eq}$, cm
1.0	1	6.1236	0.042
0.5	3	2.0412	3.822
0.2	9	0.6804	34.39
0.1	19	0.3223	153.3
0.05	39	0.1570	645.9
0.01	199	0.0308	16815

Note that even if both surfaces are black (ϵ_i = 1), a gap of nearly 1/2 cm is required for the two conductivities to be of the same order of magnitude. Clearly a gap of this size would be detected visually. For the much lower emittances which are typical of aluminum, it is clear that the radiative component can be neglected. Increasing T_1 to 350 K would reduce the gap size for equivalent conductivities by about 30%. Again, this is not expected to be a significant factor so the radiative component will be dropped in our estimate of technique sensitivity.

The thicknesses and thermal properties of layers 1 and 3 will be taken to be the same. Layer 2 will be thin with respect to layers 1 and 3 and its thermal properties will be those of air at 300 K. Using the expression presented above for three layers, there results:

$$\frac{L_{eff}^2}{\alpha_{eff}} = \frac{4L_1^2}{\alpha_1} + \frac{L_2^2}{\alpha_2} + 2L_1L_2\left[\left(\frac{\Lambda_1}{\Lambda_2\alpha_1}\right) + \left(\frac{\Lambda_2}{\Lambda_1\alpha_2}\right)\right] \tag{32}$$

where the substitution has been made:

$$\rho_i C_i = \frac{\Lambda_i}{\alpha_i} \tag{33}$$

This can be expressed in a slightly more compact form:

$$\frac{L_{eff}^2}{\alpha_{eff}} = \frac{4L_1^2}{\alpha_1} + \frac{2L_1L_2}{\alpha_{12}} + \frac{L_2^2}{\alpha_2} \tag{34}$$

where:

$$\frac{1}{\alpha_{12}} = \left[\left(\frac{\Lambda_1}{\Lambda_2\alpha_1}\right) + \left(\frac{\Lambda_2}{\Lambda_1\alpha_2}\right)\right] \tag{35}$$

Note that the effective parameter is equal to the first term if L_2 = 0 because L_{eff} = 2 x L_1 Also note the continued use of the reciprocal of α/L^2 to simplify the algebra. Rewriting this slightly to express this parameter as a function of the ratio of L_1 to L_2 yields:

$$\frac{\alpha_1}{\alpha_{eff}}\left(\frac{L_{eff}}{2L_1}\right)^2 = 1 + \frac{2\alpha_1}{\alpha_{12}}\left(\frac{L_2}{L_1}\right) + \frac{\alpha_1}{\alpha_2}\left(\frac{L_2}{L_1}\right)^2 \tag{36}$$

Some typical values for aluminum (layers 1 & 3) are: $\lambda = 160.0$ W/m-K and $\alpha = 0.67$ cm^2/s and air (layer 2) are $\lambda = 0.026$ W/m-K and $\alpha = 0.24$ cm^2/s. The value of α_{12} for this situation is approximately 4.0×10^{-5} cm^2/s. Results of the calculation for this choice of parameters are given in Table 2.

TABLE 2

Temperature-time slope as a function of dimensionless gap width

L_2/L_1	0.005	0.001	0.005	0.01	0.05	0.1	1.0
$\frac{\alpha_1}{\alpha_{eff}}\left(\frac{L_{eff}}{2L_1}\right)^2$	1.007	1.022	1.447	2.73	43.1	169	16750

Note that a ratio of L_2/L_1 of only 0.001 is sufficient to produce an effective parameter change of more than 2%. Recalling that the reciprocal of this parameter is the slope of the temperature-time curve at the rear surface of the laminate indicates that the technique will have sufficient sensitivity to permit detection of very narrow gaps. This is not surprising when we recall the current biomedical uses of the technique.

RESULTS AND CONCLUSIONS

The NDT technique known as thermal diffusivity imaging or thermography appears to hold a great deal of promise for detecting structural failures in composite laminates of the type found in aircraft. The simplified model evaluated in the present paper considers both thermal conduction and optically thin radiation across an air gap between two lamina but shows that radiation is unlikely to be a significant transport mechanism at the temperatures of interest. The effective diffusivity model lends itself to an estimate of the technique sensitivity under typical operational conditions. It is shown that, within the validity of the model assumptions, the technique is capable of detecting very small delaminations. An experimental verification of the model is presently being carried out on a laboratory scale and field studies of the technique are planned for the near future.

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