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A "UNIVERSAL" HUGONIOT FOR LIQUIDS

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ABSTRACT

The empirically derived universal Hugoniot curve for liquids was modified to satisfy boundary conditions in the initial state at 1 bar. The modification improves the description of liquids for pressures up to 20 kbar. The new form of the universal Hugoniot is $U/c_0 = 1.37 - 0.37\ \exp(-2u/c_0) + 1.62\, u/c_0$, where U is the shock velocity, u is the particle velocity, and c_0 is the sound speed at 1 bar.

INTRODUCTION

The universal Hugoniot curve[1–3] used previously to calculate shock temperature in liquid explosives[4,5] is not valid below 20 kilobars because it was constructed from shock wave data obtained at higher pressures. Moreover, it does not satisfy conditions defined in the initial state by the Rankine–Hugoniot jump conditions. The object of the present work was to extent the Hugoniot curve to the initial state and thus improve the thermodynamic description of liquids in the 1-bar to 20-kilobar region. The objective was attained by modifying the form of the Hugoniot curve to satisfy the initial conditions[6], calibrating it with static pressure data for water[7,8], and checking its validity with dynamic shock wave data for glycerin[9] and carbon tetrachloride[10].

The Hugoniot curve defines the locus of shocked states obtainable from a given initial condition. It is obtained experimentally[11] from a determination of the states produced by constant velocity shocks propagating at different velocities. Experimental determination of a Hugoniot is usually very expensive and time-consuming, and requires considerable amounts of materials which are destroyed in the process. There is therefore a need for estimating Hugoniots from easily measured physical properties.

RESULTS AND DISCUSSION

Following earlier work by Gibson *et al.*[1], Woolfolk and Amster[2], and Voskoboinikov, Afanasenkov and Bogomolov[3] have shown that the Hugoniots of liquids could be represented by a single normalized plot of the form

$$U/c_0 = a_1 + (a_2 u/c_0) \tag{1}$$

where U is the shock velocity, u is the particle velocity, c is the sound velocity, a_1 and a_2 are constants, and subscript zero denotes the initial state at a pressure (p) of 1 bar.

Equation (1) with $a_1 = 1.37$ and $a_2 = 1.62$ fitted[2] the experimental data which were in the range $u/c_0 = 0.5$ to 2.5, corresponding to shock pressures in the range of 20 to 150 kbar.

The main problem with Eqn. (1) is that it does not satisfy the boundary condition $U = c_0$ at $u = 0$. Equation (1) therefore cannot be used in the region from 1 bar to 20 kbar. This low-pressure region is of interest because of its importance in low-velocity detonations.

Jacobs[6] has suggested that an additional term $(1-a_1)\exp(-a_3u/c_0)$ with a_3 constant, be added to Eqn. (1) so that the boundary condition may be met. The form of the "universal" Hugoniot would then be

$$U/c_0 = 1.37 - 0.37 \exp(-a_3u/c_0) + 1.62\, u/c_0 \tag{2}$$

which reduces to $U/c = 1$ at $u = 0$. In Fig. 1 the experimental data are shown along with three calculated curves that correspond to $a_3 = \infty$, 5, and 1. Figure 1 indicates that the value of a_3 should be ≤ 5. An expression for a_3 was derived from thermodynamic identities relating the Hugoniot curve and the isentrope. The constant a_3 was evaluated using echo-sounding data for water.

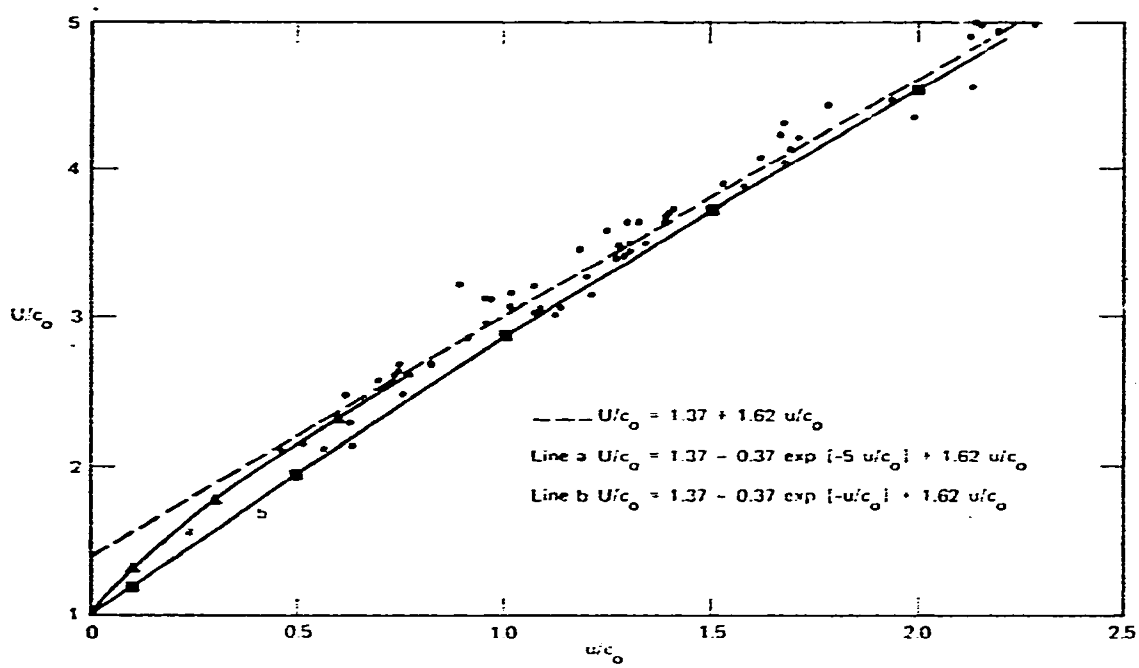

Fig. 1. Normalized U–u plot.

The identity for the initial slope of the Hugoniot in the (U–u) plane

$$\left(\frac{dU}{du}\right)_{p=1} = \frac{v_0^{\,3}}{4c_0^{\,2}}\left(\frac{\partial^2 p}{\partial v^2}\right)_{s0} \tag{3}$$

where v denotes specific volume and s denotes specific entropy, and the thermodynamic identity for an isentrope

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_s = \frac{2c}{v^2}\left[\frac{c}{v} - \left(\frac{\partial c}{\partial v}\right)_s\right] \tag{4}$$

are used to determine the constant a_3 in Eqn. (2). Differentiating Eqn. (2) with respect to u gives the slope of the Hugoniot as

$$\mathrm{d}U/\mathrm{d}u = 1.62 + 0.37\, a_3 \exp\left(-a_3 u/c_0\right) \tag{5}$$

and the relationship between the initial slope of the Hugoniot and a_3 is obtained as

$$0.37\, a_3 = \left((\mathrm{d}U/\mathrm{d}u)_{p=1} - 1.62\right) \tag{6}$$

by setting $u = 0$ in Eqn. (5). The calculation of $(\partial^2 p/\partial v^2)_s$ at $p = 1$ bar with sound velocity data is then sufficient to evaluate a_3. The identity

$$\left(\frac{\partial c}{\partial v}\right)_s = \left(\frac{\partial c}{\partial v}\right)_p\left[1 + \frac{c^2}{v^2}\left(\frac{\partial v}{\partial p}\right)_c\right] \tag{7}$$

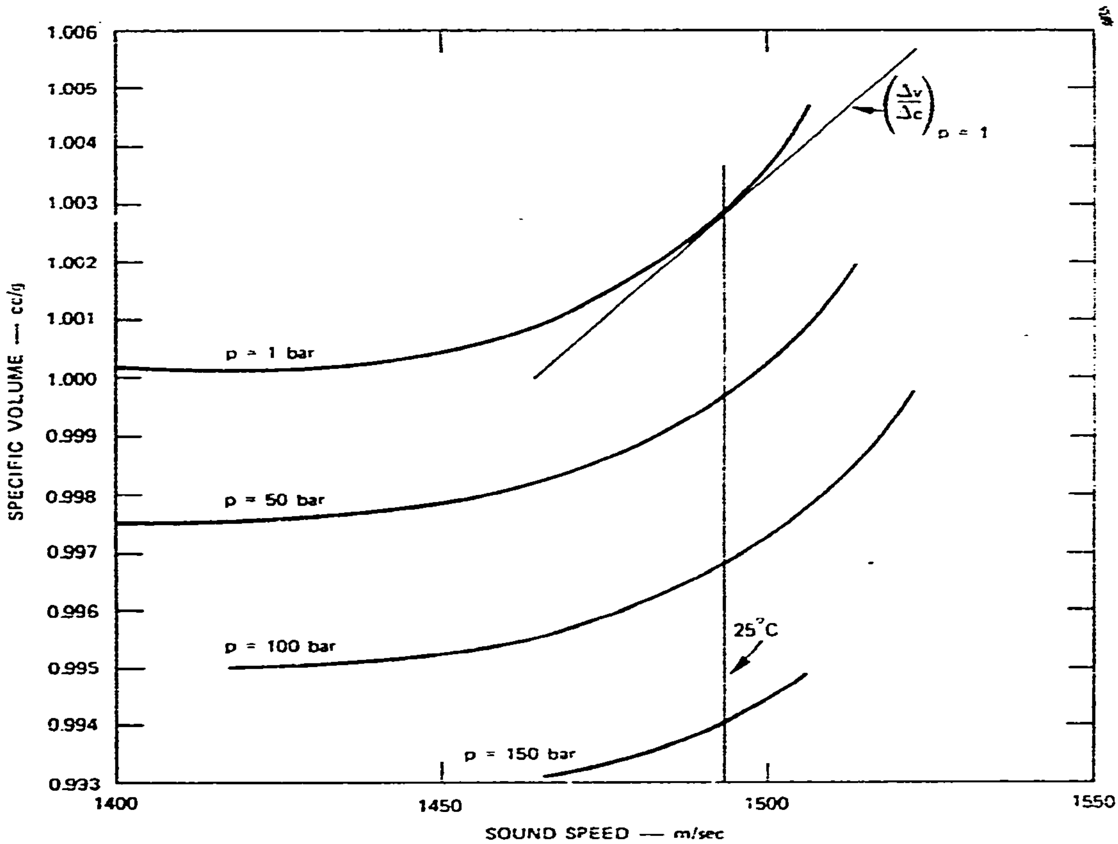

Fig. 2. Sound speed in pure water as a function of specific volume at pressures of 1, 50, 100, and 150 bar.

was used in the evaluation of a_3 because it is convenient to estimate the derivatives $(\partial c/\partial v)_p$ and $(\partial v/\partial p)_c$ with experimental data. Data from echo-sounding tables for pure water[7], together with p–v–T data[8], were used to construct a graph of the variation of sound speed with specific volume at pressures of 1, 50, 100, and 150 bar (see Fig. 2). At $p = 1$ bar, $T = 25°C$, and $c_0 = 1493$ m/sec, we estimate $(\partial c/\partial v)_{p=1}$ from the graph as

$$\left(\frac{\Delta c}{\Delta v}\right)_{p=1} = 1.027 \times 10^6 \text{ g/(sec cm}^2\text{)}$$

Similarly, from the graph, at constant $c = 1493$ m/sec, values of p and v were estimated as shown in Table I.

TABLE I

p (bar)	v (cc/g)	Δv (cc/g)
1	1.00295	
		0.00325
50	0.99970	
		0.00290
100	0.99680	
		0.00275
150	0.99405	

Extrapolating to $p = 1$, we obtain

$$\left(\frac{\partial v}{\partial p}\right)_{c_0} = \left(\frac{\Delta v}{\Delta p}\right)_{c_0} = -7 \times 10^{-11} \text{ cm}^5\text{/(dyne g)}$$

Substitution of $c_0 = 1.493 \times 10^5$ cm/sec, $v = 1.00295$ cm^3/g, $(\partial v/\partial p)_{c_0} = -7 \times 10^{-11}$ cm^5/(dyne g), and $(\partial c/\partial v)_{p=1} = 1.027 \times 10^6$ g (sec cm^2) into Eqn. (7) gives $(\partial c/\partial v)_s = 5.66 \times 10^5$ g/(sec cm^2). Evaluating $(\partial^2 p/\partial v^2)_s = 2.12 \times 10^{11}$ sec^2/(g^3 cm^{-7}) with Eqn. (4) and $(\mathrm{d}U/\mathrm{d}u)_{p=1} = 2.40$ with Eqn. (3) gives $a_3 = 2.1$ by substitution in Eqn. (5).

CONCLUSION

The region of Fig. 1 close to the origin has been replotted as Fig. 3, using Eqn. (2) and values of $a_3 = \infty$, 10, 5, 2, 1, and 0.1. Also plotted in Fig. 3 are experimental Hugoniot points for glycerin obtained by Erlich[9] and for carbon tetrachloride obtained by Lysne[10]. Until further data are available, we conclude that a value of $a_3 = 2$ is consistent with results of the calculations on water and with the experimental data on glycerin and carbon tetrachloride. In other words, Eqn. (2) becomes:

$$U/c_0 = 1.37 - 0.37 \exp(-2u/c_0) + 1.62\, u/c_0 \qquad (8)$$

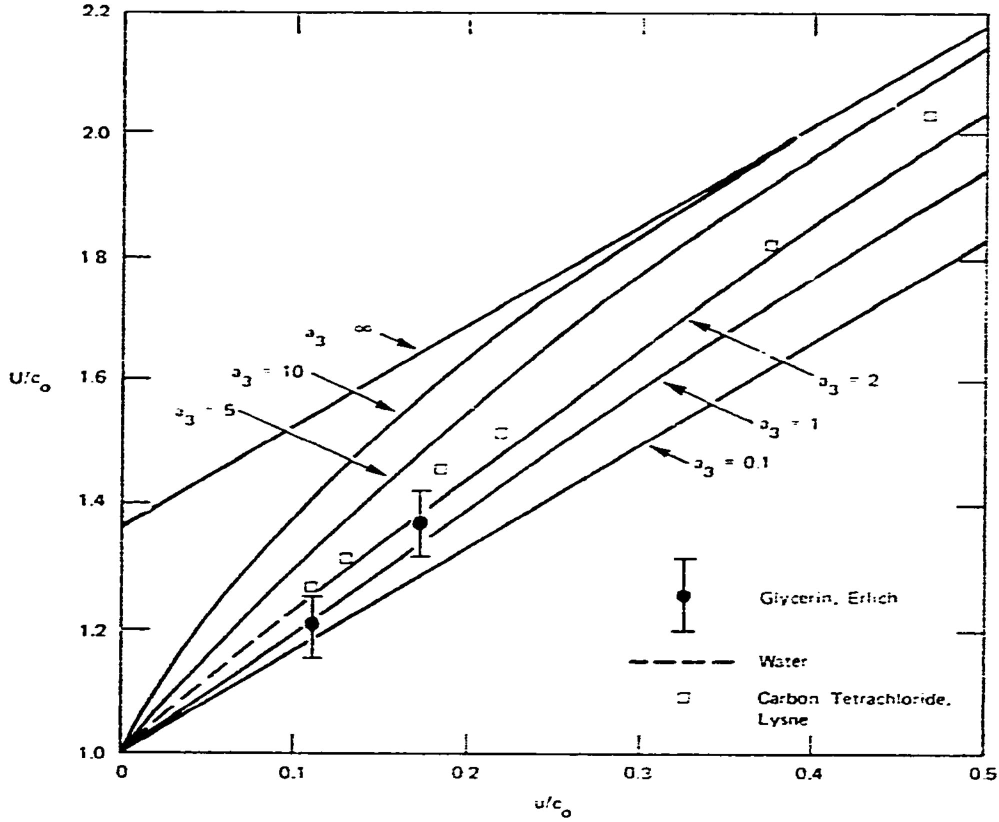

Fig. 3. Calculated normalized U–u plots for values of $a_3 = 0.1$, 1, 2, 5, 10, and ∞ in the Eqn. (8) $U/c_0 = 1.37 - 0.37\ \exp(-a_3 u/c_0) + 1.62u/c_0$ showing Erlich's experimental glycerin data, and Lysne's experimental carbon tetrachloride data.

It is concluded that the modified universal Hugoniot curve formulated in this note improves the thermodynamic description of liquids in the 1-bar to 20-kbar region.

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