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THE AUTOMOTIVE SYSTEM — A COMPLEX COUPLING OF MATTER AND ENERGY NETWORKS*

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ABSTRACT

Linear equations are shown with schematic diagrams which together describe the automotive dynamic model. However, the principal intent of the paper is not to study the dynamics of automotive operations. The schematic diagrams of each sub-system are presented to indicate the possible points of energy storage and dissipation and how they relate to the continuity of energy.

From the sub-system diagrams it is now possible to generate energy and matter balances for the automotive system. The final energie matrix expression (eqn 49), which relates to all components of the system, is developed.

I. INTRODUCTION

The automotive system has been analyzed in various ways in the past with regard to sub-system function. It is important to realize that this converter (as with other converter systems) is a network. A number of converter networks are shown in Figs. 1–3. In order to firmly set the converter system as a true network a number of conditions must be satisfied:

(1) A set of elements exist.

(2) The elements are active (they have inputs, outputs, storage).

(3) They are coupled together in some fashion.

(4) The coupled active elements perform some function (this may be nebulous in some selected systems).

*The work presented was done at Widener College.

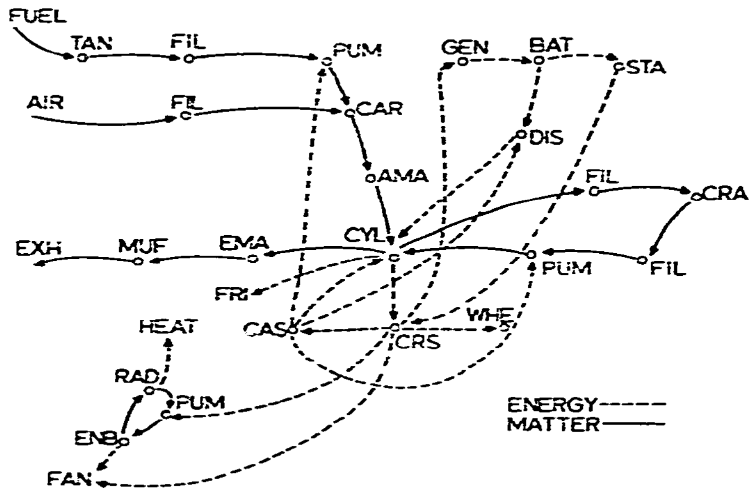
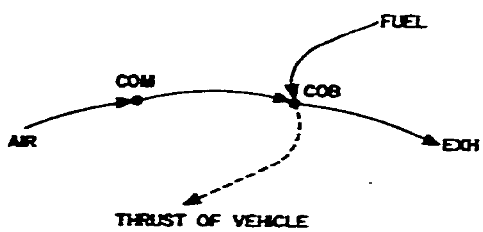


Fig. 1. Automotive network. EXH = Exhaust; TAN = tank; PUM = pump; CAR = carburetor; AMA = air induction manifold; CYL = cylinder; EMA = exhaust manifold; MUF = muffler; FIL = filter; CRA = crank case; FRI = friction; ENB = engine block; WHE = wheels; CAS = camshaft; CRS = crank shaft; DIS = distributor; GEN = generator; BAT = battery; STA = starter; RAD = radiator.



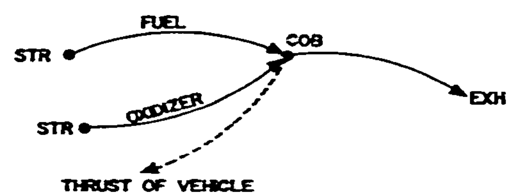


Fig. 2. Ram jet network. COM = compressor, COB = combustion.

Fig. 3. Rocket motor network. STR = storage.

The automotive system does fulfill all of the above conditions necessary to define a network (see Fig. 1). Most importantly the coupling which occurs between automotive elements (tank, filter pump, carburetor etc.) is of either matter or energy. If one decouples, in space, the matter and energy sub-networks, then configurations, as shown in Figs. 4 and 5, respectively, are produced. These decoupled primary sub-networks will now be discussed in some detail.

II. LINEAR SUB-SYSTEMS IN THE AUTOMOTIVE ENGINE

Using the matter and energy sub-networks or sub-systems, it is possible to write linear matrix equations and construct linear graphs which incorporate all resistive, capacitive and inductive (or inertance) effects for the sub-system. This type

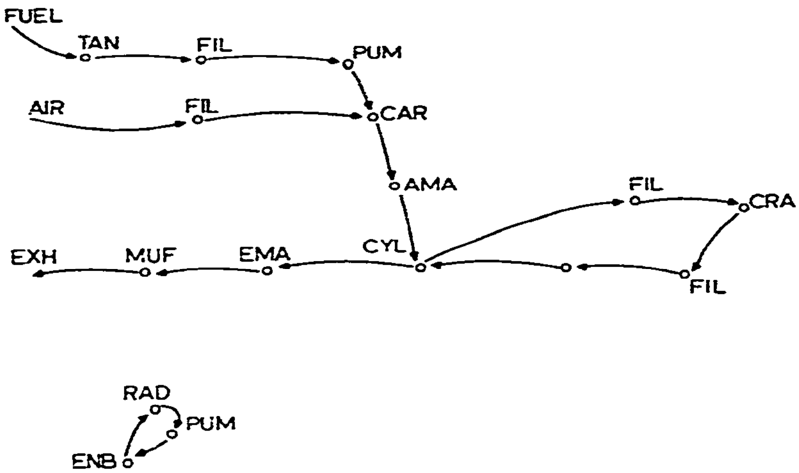


Fig. 4. Matter sub-network.

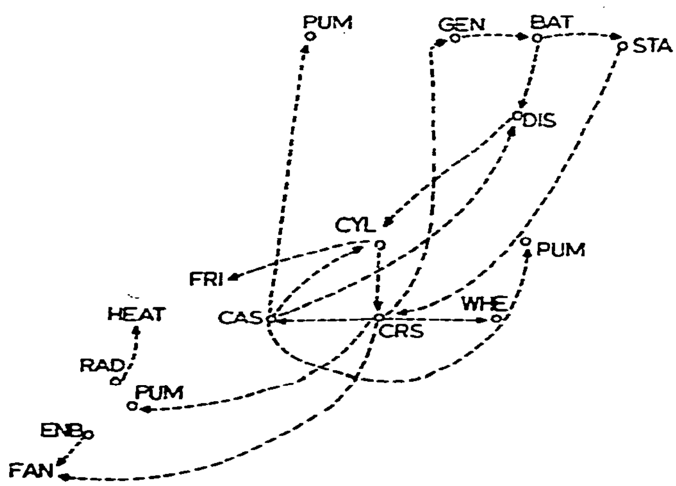


Fig. 5. Energy sub-network.

of general analysis is not new and has been used in linear systems analysis to define a multitude of diverse systems[1,2]. In the most generalized form the through and across variables within any network may be expressed as

$$\bar{F} = \bar{Y}\bar{V} \tag{1}$$

where F = through or flow variable matrix;

$\bar{V}$ = across or driving force variable matrix;

$\bar{Y}$ = admittance matrix.

Expanded into the usual matrix rotation (1) becomes

$$\begin{bmatrix} F_{1s} \\ F_{2s} \\ \vdots \\ F_{ns} \end{bmatrix} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} \dots -Y_{1j} \\ -Y_{21} & Y_{22} & -Y_{23} \dots -Y_{2j} \\ \vdots & & \\ -Y_{i1} & & Y_{ij} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \tag{2}$$

The matrices on the right-hand side of (2) must be conformable. Note two additional important parts: (1) the F matrix would be a source matrix; (2) $\bar{F}$, $\bar{Y}$ and $\bar{V}$ are matrices of complex amplitudes. This latter statement requires that exponential sources exist and that the systems be linear.

One may also define the sub-systems by

$$\bar{V} = \bar{Z}\bar{F} \tag{3}$$

where $\bar{V}$ is the source matrix and $\bar{Z}$ is defined as an impedance matrix. The relationship between $\bar{Y}$ and $\bar{Z}$ is that

$$\bar{Y} = \bar{Z}^{-1} \tag{4}$$

or

$$\bar{Z} = \bar{Y}^{-1} \tag{5}$$

$\bar{Z}^{-1}$ = the inverted matrix of $\bar{Z}$;

$\bar{Y}^{-1}$ = the inverted matrix of $\bar{Y}$.

A. The matter sub-network

In order to simplify the initial overall modelling of this sub-system, each loop or open mesh will be analyzed separately.

1. Induction air sub-system (pneumatic). The flow of air into the carburetor will be set as

$$\bar{F}_A = \bar{Y}_A \bar{V}_A \tag{6}$$

where $\bar{F}_A$ is the air-flow matrix for this part of the system. All resistive, inertance and capacitive effects in the air-flow system are incorporated with the admittance matrix, $\bar{Y}_A$. The potentials or driving forces (normally pressures) for the sub-systems are indicated as $\bar{V}_A$. See Figs. 6–8 for detailed schematic diagrams and linear graphs for a six-cylinder engine*.

2. Fuel flow sub-system. In a manner similar to the air-flow system, the fuel-flow matrix may be written:

$$\bar{F}_F = \bar{Y}_F \bar{V}_F \tag{7}$$

The detailed diagrams for this sub-system are shown in Figs. 9–11.

*All diagrams are for a Ford Falcon six engine.

3. *Lubrication sub-system.* The distribution of lubricating oil throughout the engine flow channels is a complicated network. However, the system may be written in matrix form as below with the appropriate graphs (Figs. 12–14) to describe the accompanying flows and potentials.

$$\bar{F}_L = \bar{Y}_L \bar{V}_L \tag{8}$$

4. *Hydraulic sub-system.* The most simple matter sub-system that exists within the total automotive network is the hydraulic sub-system. The linear matrix expression for the sub-system is

$$\bar{F}_H = \bar{Y}_H \bar{V}_H \tag{9}$$

Figures 15–17 show the sub-system in the three usual forms.

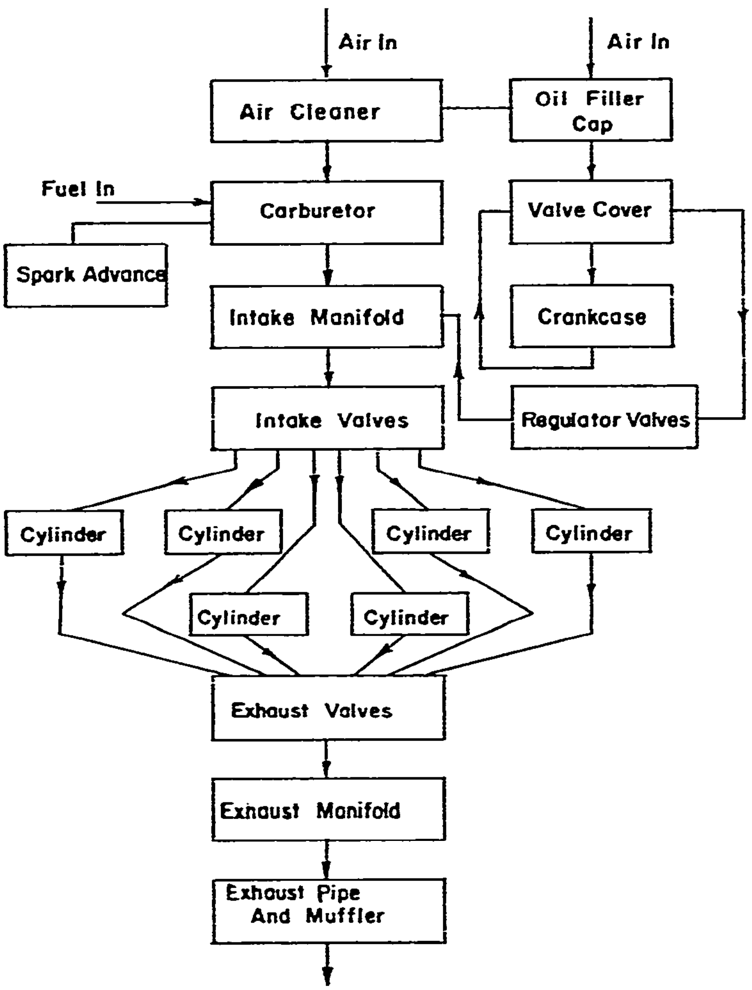
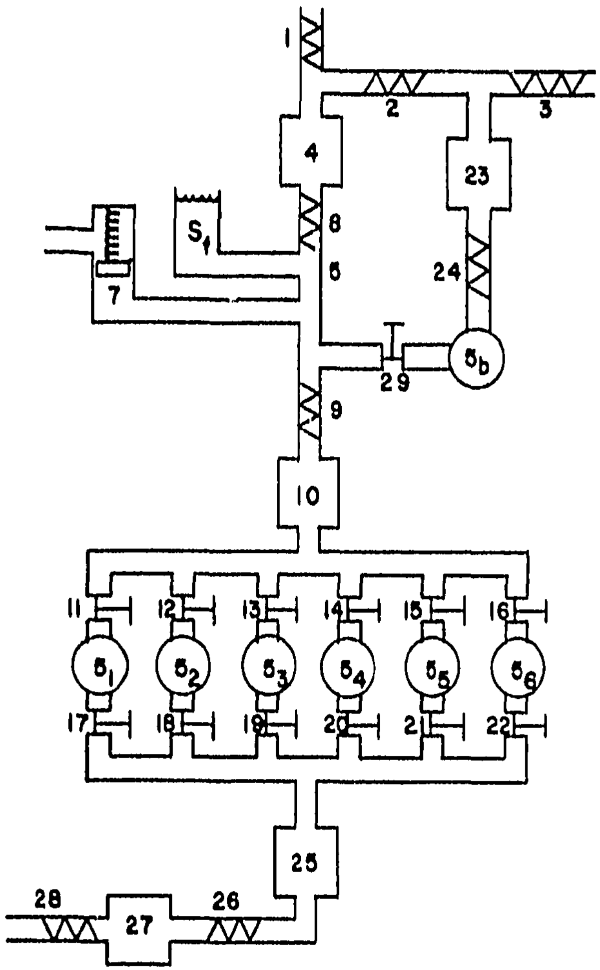
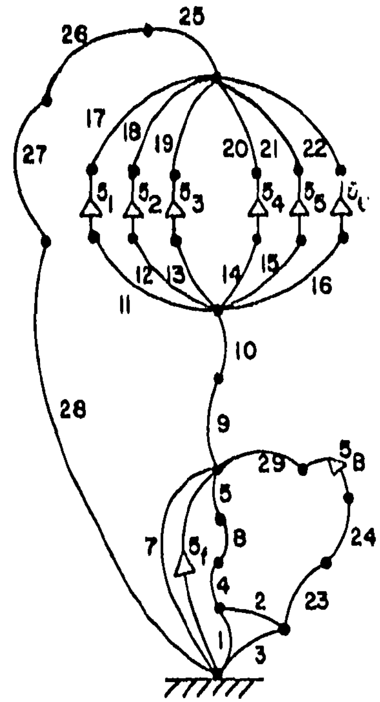


Fig. 6. Pneumatic sub-system flow diagram.





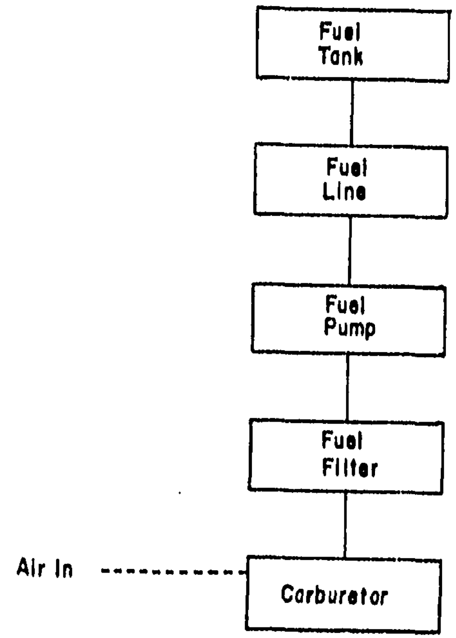
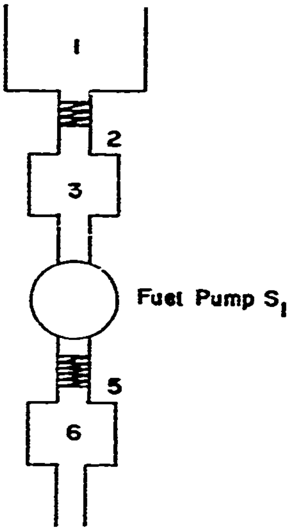


Fig. 7. Pneumatic sub-system idealized schematic. Numbers in system diagrams refer to elements in that system.

Fig. 8. Pneumatic sub-system linear graph. Numbers in system diagrams refer to elements in that system.

Fig. 9. Fuel sub-system flow diagram.



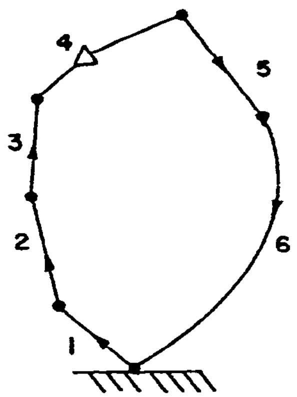


Fig. 10. Fuel sub-system idealized schematic.

Fig. 11. Fuel sub-system linear graph.

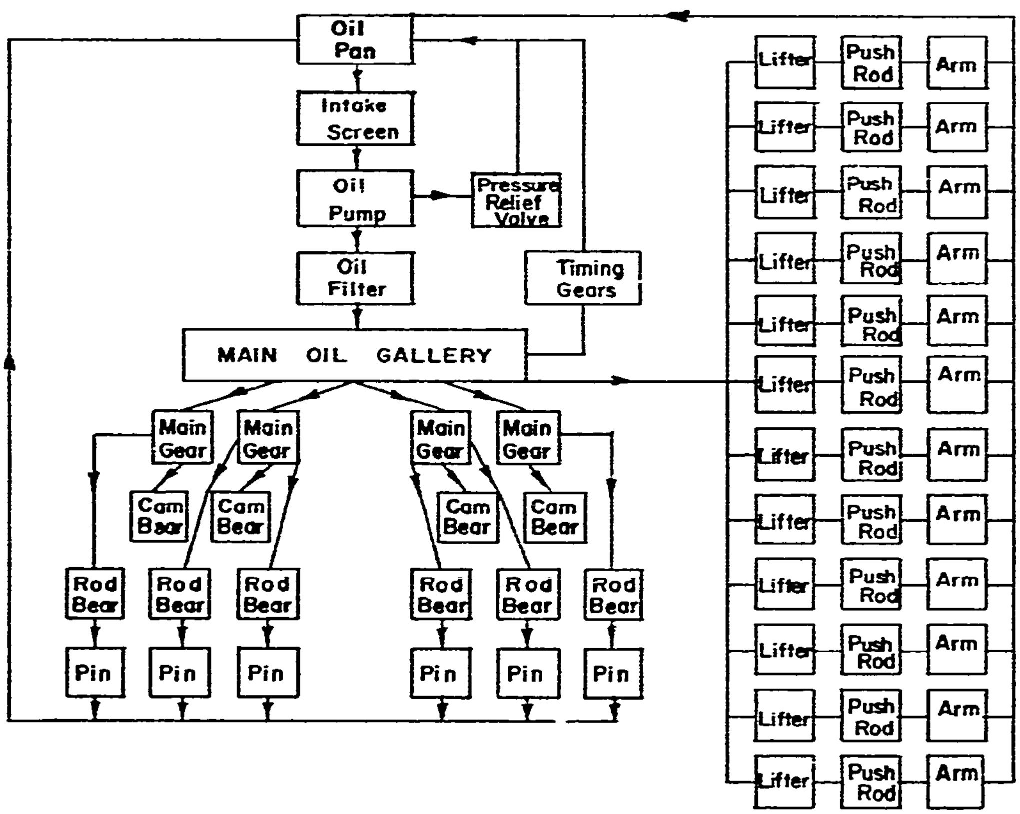


Fig. 12. Lubrication sub-system flow diagram.

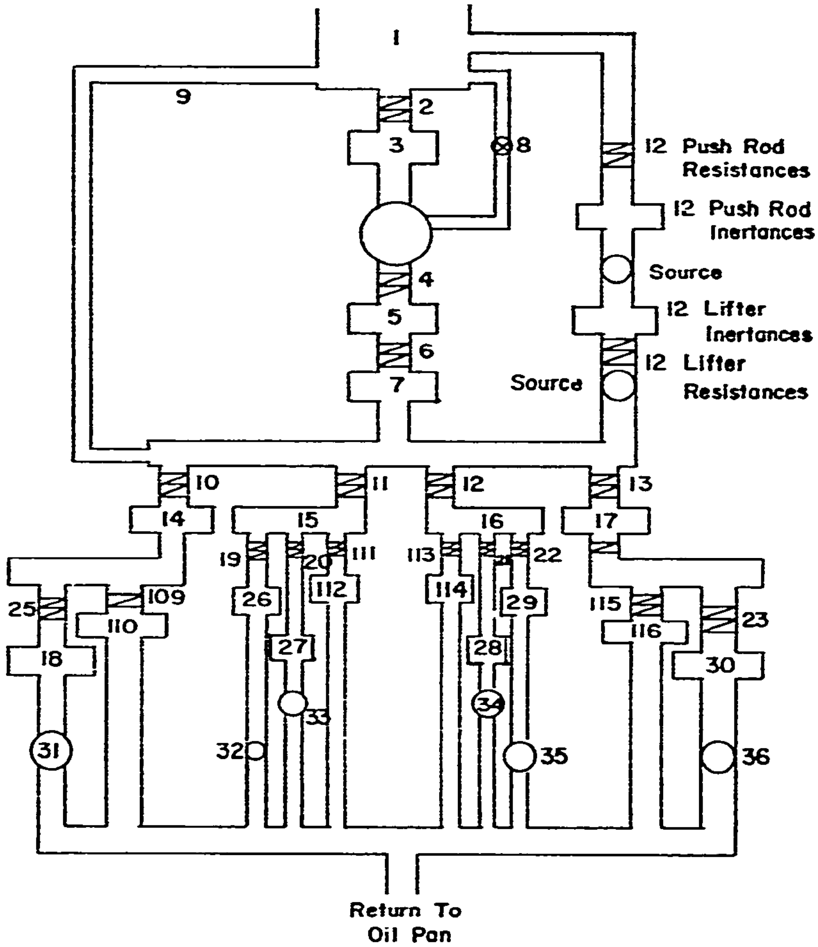


Fig. 13. Lubrication sub-system schematic.

B. The energy sub-network

In Fig. 5, the energy sub-network indicates the transmission of electrical and mechanical energies. The mechanical energy flows are power flows throughout the appropriate sub-system.

1. Electrical sub-system. The electrical sub-system in the automotive system is defined by the linear matrix equation

$$\bar{F}_E = \bar{Y}_E \bar{V}_E \tag{10}$$

The sub-system is shown in Figs. 18–20.

2. Mechanical sub-system (power flow). The mechanical sub-system is shown as

$$\bar{F}_M = \bar{Y}_M \bar{V}_M \tag{11}$$

Figures 21–23 are the pertinent graphs.

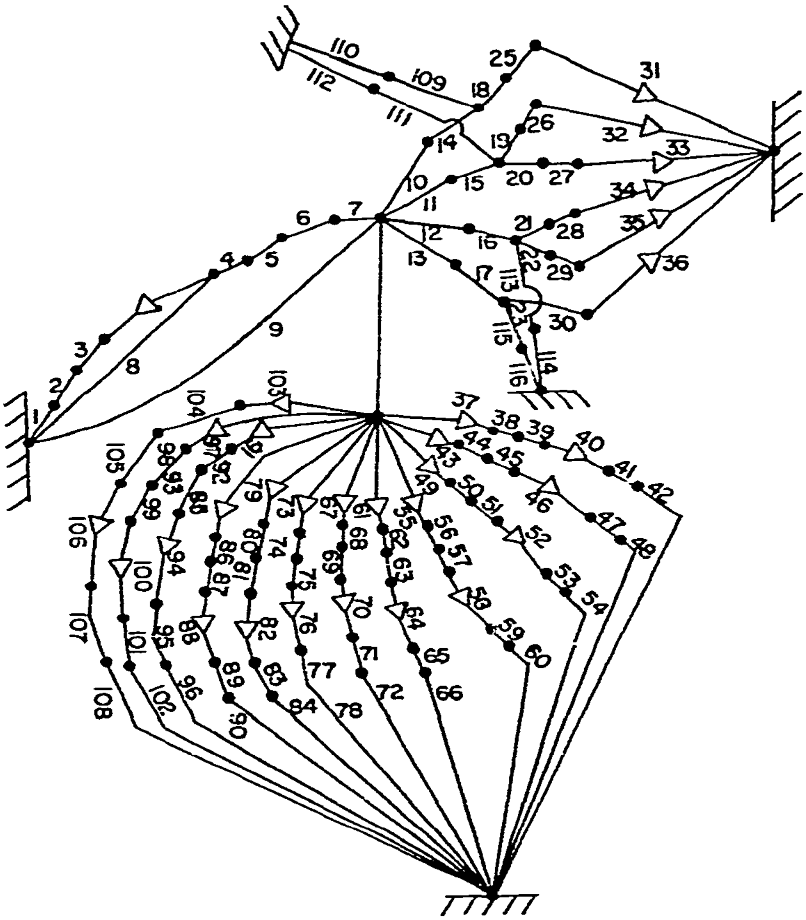


Fig. 14. Lubrication sub-system linear graph.

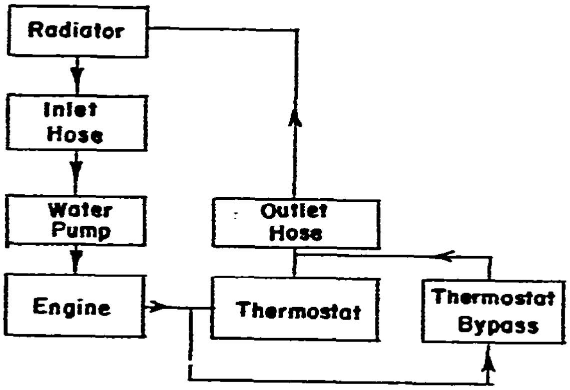


Fig. 15. Hydraulic sub-system flow diagram.

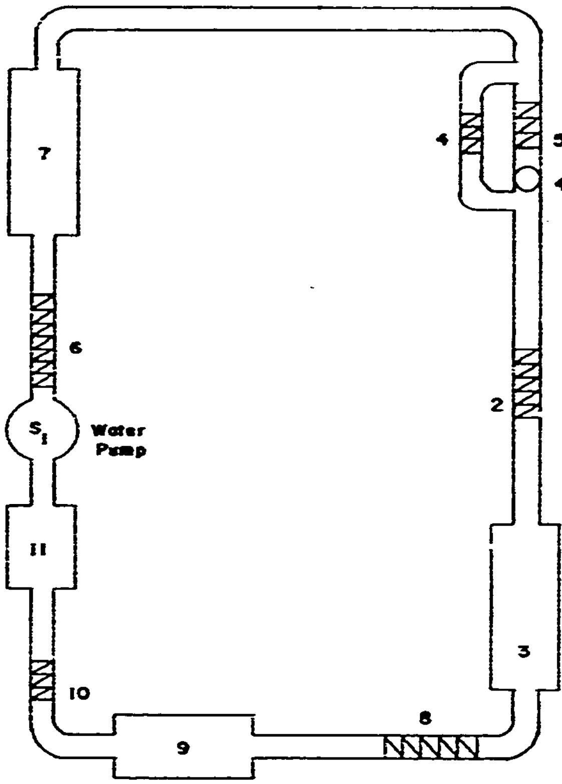


Fig. 16. Hydraulic sub-system idealized schematic.

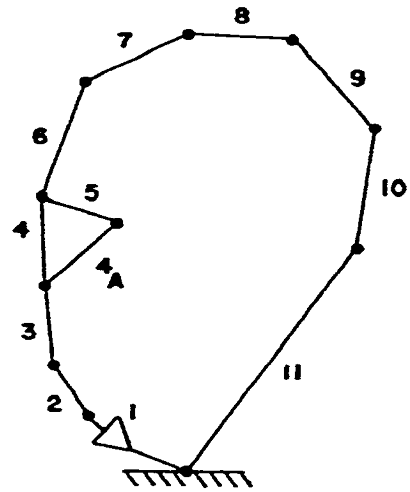


Fig. 17. Hydraulic sub-system linear graph.

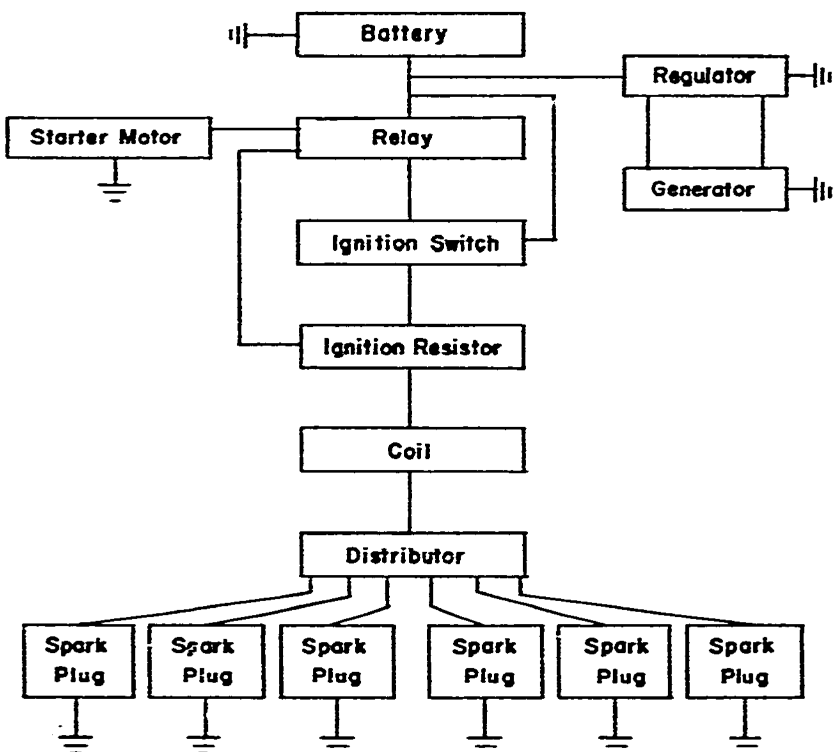


Fig. 18. Electrical flow sub-system flow diagram.

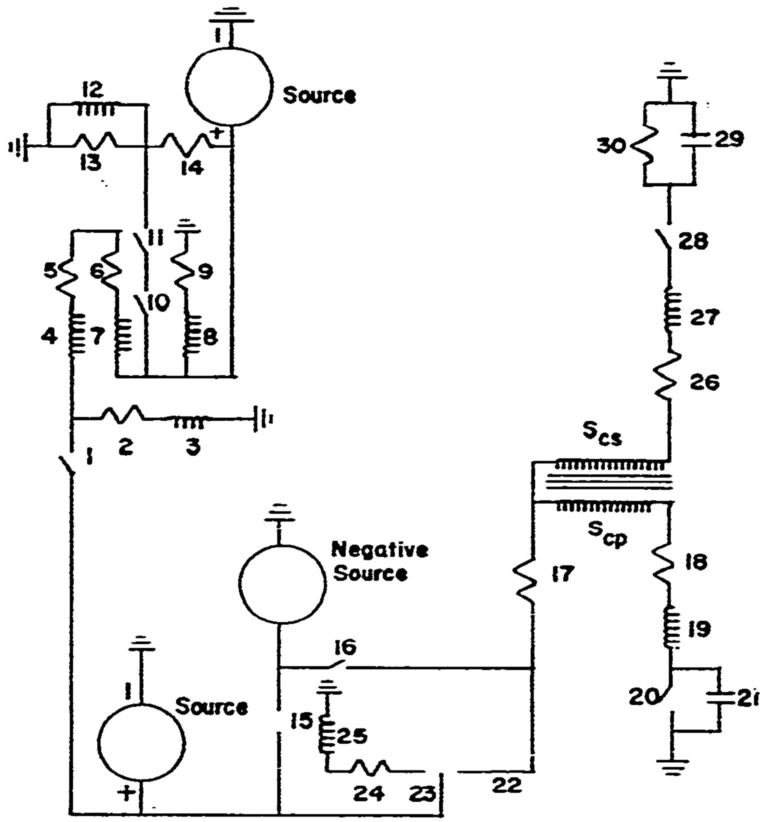


Fig. 19. Electrical flow sub-system schematic graph.

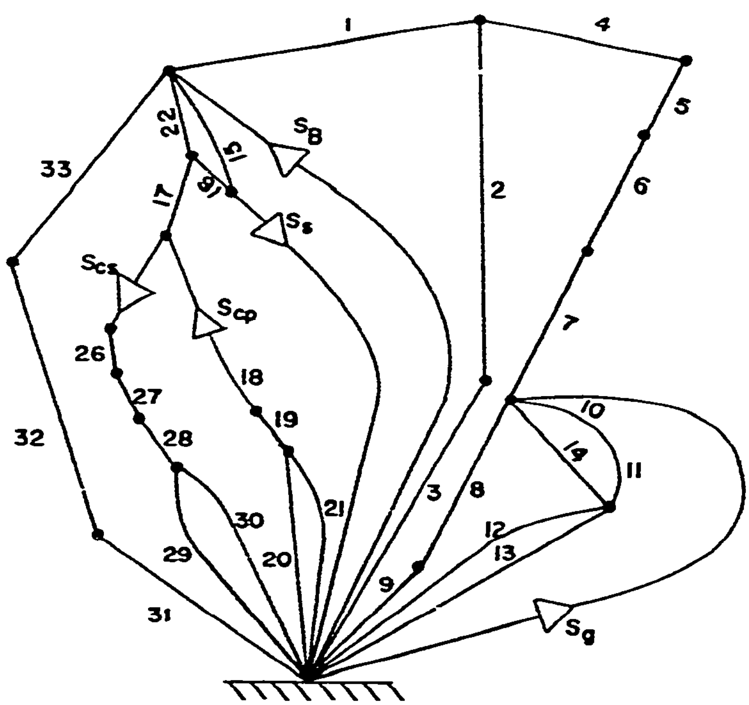


Fig. 20. Electrical flow sub-system linear graph.

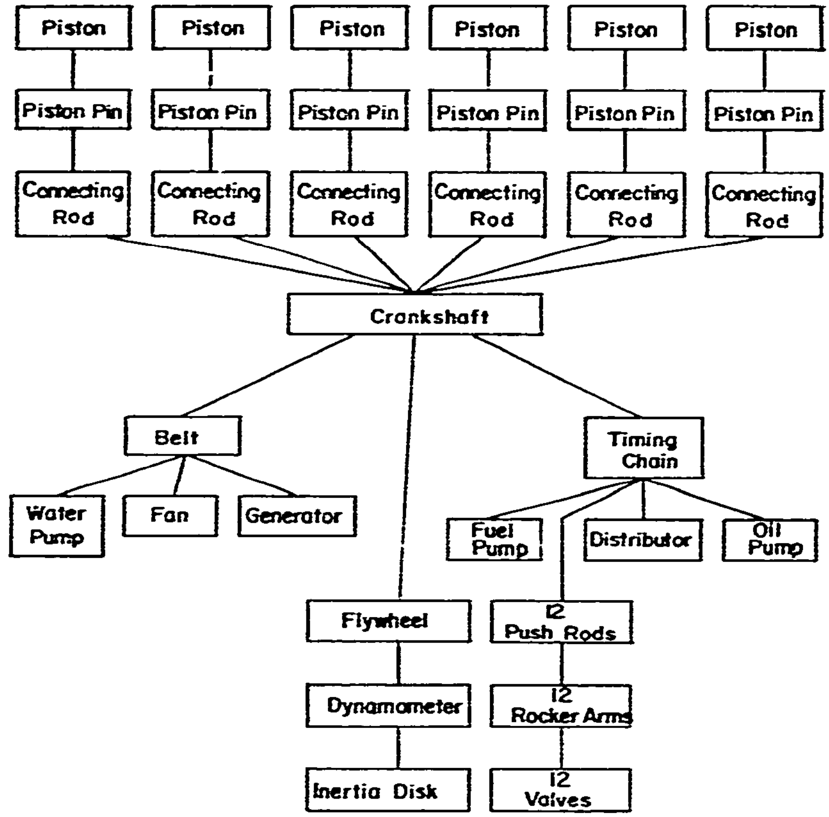


Fig. 21. Mechanical (power) flow sub-system flow diagram.

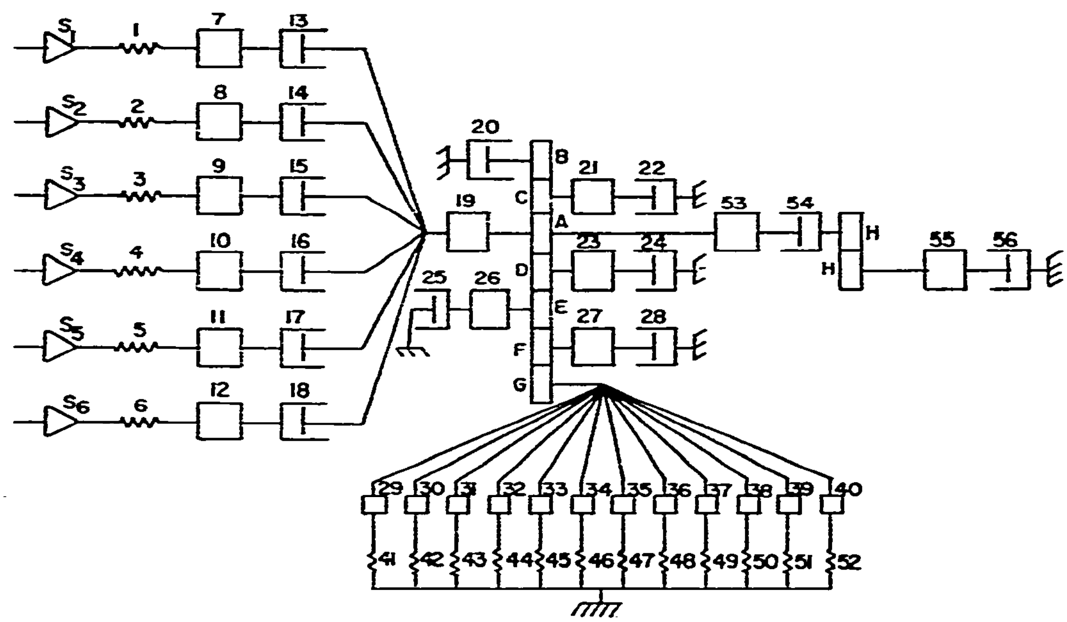


Fig. 22. Mechanical (power) flow sub-system schematic graph.

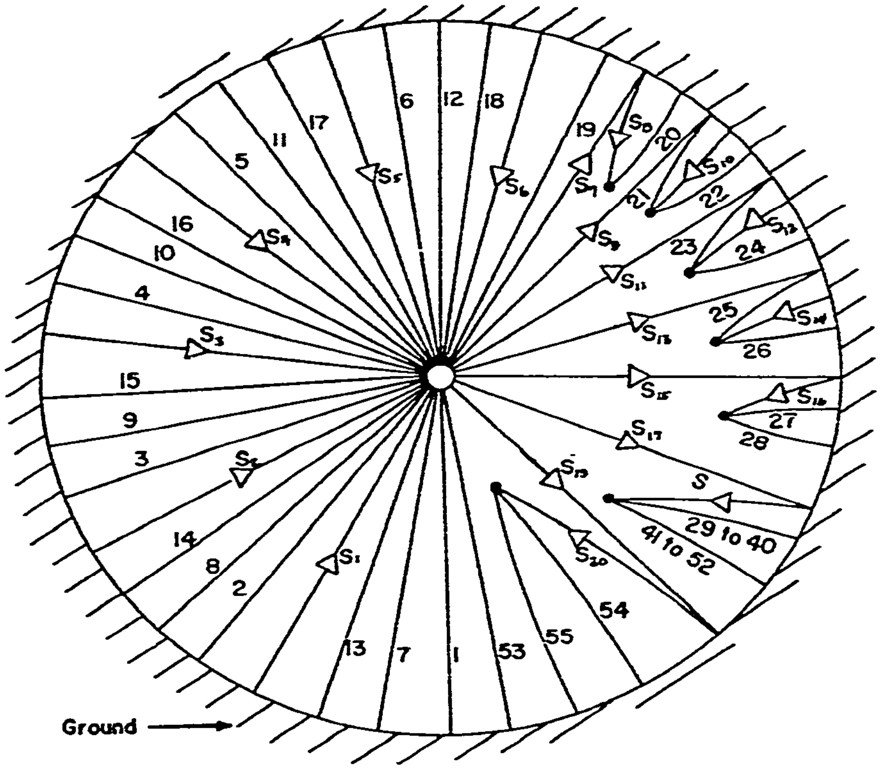


Fig. 23. Mechanical (power) flow sub-system linear graph.

3. Thermal sub-systems. Thermal energy losses occur throughout the automotive system due to the fact that chemical reactions and mechanical friction exist with the system's structure as heat sources.

A linear graph will not be shown for this sub-system since the flow of heat is diffusive throughout the entire automotive system. However, it still is possible to write a matrix thermal equation for the entire system.

$$\bar{F}_T = \bar{Y}_T \bar{V}_T \tag{12}$$

It should be expected that this latter expression should couple with those of other sub-systems in order to define a *coupled two-network space.*

All that has been shown above constitutes a simple linear model for a six cylinder engine. Of great importance is also the static continuity model for the engine. This latter development occupies the rest of this paper.

III. AUTOMOTIVE SYSTEM CONTINUITY BALANCES

A. Matter

The flow of matter into and out of the system may be expressed in matrix form as

$$\bar{F}_A + \bar{F}_F = \bar{F}_E \tag{13}$$

In terms of sub-systems* equation (13) would be

$$\bar{Y}_A \bar{V}_A + \bar{Y}_F \bar{V}_F = \bar{Y}_E \bar{V}_E \tag{14}$$

or

$$\bar{Y}_A \bar{V}_A + \bar{F}_F = \bar{Y}_E \bar{V}_E$$

$$\bar{F}_F = (\bar{Y}_E \bar{V}_E - \bar{Y}_A \bar{V}_A) \tag{15}$$

Expression (15) is a dynamic mathematical form which indicates that requisite fuel flows ($\bar{F}_F$) are not only a function of air potentials (pressures) and sub-system admittances but are also influenced by exhaust potentials (pressures) and sub-system admittances. Of course the time invariant information obtained from these equations is not new, but the dynamic information inherent in them is of prime importance.

Equation (13) may be modified in other ways to give air–fuel ratio matrix expressions. Pre-multiply eqn (13) by the inversion of $\bar{F}_{FD}$ or $\bar{F}_{FD}^{-1}$,

$$\bar{F}_{FD}^{-1} \bar{F}_{AD} + \bar{F}_{FD}^{-1} \bar{F}_{FD} = \bar{F}_{FD}^{-1} \bar{F}_{ED} \tag{16}$$

The matrix multiplication of square matrices, as $\bar{F}_{FD}^{-1} \bar{F}_{FD}$, results in a unit matrix, $\bar{I}$. Also it will be defined that $\bar{F}_{FD}^{-1} \bar{F}_{AD}$ be the air–fuel matrix, R. Then eqn (16) becomes

$$\bar{F}_{FD}^{-1} \bar{F}_{FD} = \bar{I} \tag{17}$$

$$\bar{R} + \bar{I} = \bar{F}_{FD}^{-1} \bar{F}_{ED} \tag{18}$$

$$\bar{R} = \bar{F}_{FD}^{-1} \bar{F}_{ED} - \bar{I} \tag{19}$$

Expression (19) relates in a generalized manner air-to-fuel ratios to fuel and exhaust flows.

B. Energy

1. Generation of energy through combustion

The only source of energy existing within the automotive system is the fuel used in the engine. The release of heat energy through exothermic combustion processes may be shown to be

$$\bar{E} = Q_C \bar{F}_F \tag{20}$$

Equation (20) expresses for all fuel streams the release of anergy through the oxidation of fuel. In most cases $\bar{E}$ and $\bar{F}_F$ matrices would be quite simple with only matrix element existing in each. But for multiple fuel injection systems the fuel through matrix would be more complex.

*Dynamic and static components of equations both used.

2. Storage

The stored energies in a moving automotive system may be viewed as matrix quantities by expressions developed below.

a. Electrical sub-system

For all electrical systems the battery must supply at least the power dissipated within the engine sub-systems.

$$\bar{e}_s = \begin{bmatrix} v_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & v_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & v_N \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \cdot \\ \cdot \\ i_N \end{bmatrix} \tag{21}$$

$$\bar{e}_s = \bar{v}_D \bar{i} \tag{22}$$

b. Mechanical sub-system

(1) The linear and rotational systems store the following kinetic energies:

Linear

$$\bar{e}_L = \tfrac{1}{2} \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & M_N \end{bmatrix} \left\{ \begin{bmatrix} V_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & V_N \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \cdot \\ \cdot \\ V_N \end{bmatrix} \right\} \tag{23}$$

$$\bar{e}_L = \tfrac{1}{2} \bar{M}_D [\bar{V}_D \bar{V}] \tag{24}$$

Equation (24) gives the linear mechanical energies stored (kinetic energy) in the various mechanical components of the automobile.

Rotational

Similarly for the rotational sub-systems the rotational mechanical energy stored is:

$$\bar{e}_R = \tfrac{1}{2} \bar{I}_{RD} [\bar{\omega}_D \bar{\omega}] \tag{25*}$$

(2) Linear and rotational systems store the following potential energy.

* $\bar{V}_D$, ϖ_D etc. are diagonal matrices.

Linear

$$\bar{e}_P = \tfrac{1}{2}\bar{k}_D[\bar{X}_D \bar{X}] \tag{26}$$

Rotational

$$\bar{e}_R = \tfrac{1}{2}\bar{K}_D[\bar{\theta}_D \bar{\theta}] \tag{27}$$

c. Thermal storage

The total amount of heat energy stored within the various engine sub-systems may also be represented by matrix expressions.

$$\begin{bmatrix} H_1 \\ H_2 \\ . \\ . \\ . \\ H_N \end{bmatrix} = \begin{bmatrix} C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 0 & C_N \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ . \\ . \\ . \\ T_N \end{bmatrix} \tag{28}$$

$$\bar{H} = \bar{C}_D \bar{T} \tag{28A}$$

H_1, H_2 etc. represent the amount of heat energy stored within the individual automotive sub-systems; C_1, C_2 etc. are the thermal capacitances existing within the various sub-systems. T_1, T_2, T_N are the temperatures of the sub-system masses above the ambient temperature. $T_1 = t_1 - t_0$, $T_2 = t_2 - t_0$ where t_0 = ambient temperature and t_1, t_2 are actual temperatures of masses storing thermal energy. Note also the $C_1 = C_{p1} m_1$, $C_2 = C_{p2} m_2$, etc.

d. Total stored energies

Equations (21)–(28) give the energies stored within the components of the automobile sub-systems. The $\bar{e}$ and $\bar{H}$ matrices do not give sums. In order to sum all individual component energies within sub-systems, the following is done.

Electrical

$$E_s = (v_1, v_2 \dots v_N) \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} \tag{29}$$

$$E_s = \bar{v}^* \bar{i} \; * \tag{30}$$

Note that E_s is a scalar matrix since energy is a scalar quantity.

*$\bar{v}^*$ is the transpose of $\bar{v}$ which is a column matrix; other matrices are similarly treated.

Linear kinetic energy

$$E_L = \tfrac{1}{2}(M_1, M_2 \ldots M_N)\left\{\begin{bmatrix} V_1 & 0 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & V_N \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_N \end{bmatrix}\right\} \tag{31}$$

$$E_L = \tfrac{1}{2}\overline{M}^*[\overline{V}_D \overline{V}] \tag{31}$$

Rotational kinetic energy

$$E_R = \tfrac{1}{2}\overline{I}_R^*[\overline{\omega}_D \overline{\omega}] \tag{32}$$

Linear potential energy

$$E_P = \tfrac{1}{2}\overline{k}^*[\overline{X}_D \overline{X}] \tag{33}$$

Rotational potential energy

$$E_R = \tfrac{1}{2}\overline{K}^*[\overline{\theta}_D \overline{\theta}] \tag{34}$$

Thermal energy

$$E_T = \overline{C}^* \overline{T} \tag{35}$$

3. Losses

Energy losses in the automotive system occur through friction and heat transfer in the various sub-systems. Energy or power losses may be indicated for multiple sub-system in a generalized fashion by a matrix of matrices.

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ . \\ . \\ . \\ P_N \end{bmatrix} = \begin{bmatrix} \overline{F}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{F}_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{F}_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \overline{F}_N \end{bmatrix}\begin{bmatrix} \overline{V}_1 \\ \overline{V}_2 \\ \overline{V}_3 \\ . \\ . \\ . \\ \overline{V}_N \end{bmatrix} \tag{36}$$

or $$\overline{P} = \overline{\overline{F}}_D \overline{\overline{V}} \tag{37}$$

It must be remembered that $\overline{\overline{F}}_D$ in eqn (37) is a diagonal matrix and $\overline{\overline{V}}$ a column

*Again it should be noted that $\overline{C}$, $\overline{K}$, $\overline{k}$, $\overline{M}$ and $\overline{v}$ are normally column matrices.

TABLE I

AUTOMOTIVE SUB-SYSTEM POWER LOSSES

Sub-system	*Power loss equation*
Induction	$P_A = F_A^* \bar{V}_A$
Fuel	$P_F = F_F^* \bar{V}_F$
Lubrication	$P_L = F_L^* \bar{V}_L$
Hydraulic	$P_H = F_H^* \bar{V}_H$
Electrical	$P_E = F_E^* \bar{V}_E$
Mechanical	$P_M = F_M^* \bar{V}_M$
Pneumatic	$P_P = F_P^* \bar{V}_P$

matrix. The power losses for the automotive sub-systems are listed in Table 1. Expression (21) could now be written with $P_1 = P_A$, $P_2 = P_F$ etc. so that

$$\begin{bmatrix} P_A \\ P_F \\ P_L \\ P_H \\ P_E \\ P_M \\ P_P \end{bmatrix} = \begin{bmatrix} \bar{F}_A^* & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{F}_F^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{F}_L^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{F}_H^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{F}_E^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{F}_M^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_P^* \end{bmatrix} \begin{bmatrix} \bar{V}_A \\ \bar{V}_F \\ \bar{V}_L \\ \bar{V}_H \\ \bar{V}_E \\ \bar{V}_M \\ \bar{V}_P \end{bmatrix} \tag{38}$$

The equivalent matrices for $\bar{F}_A$, $\bar{F}_F$ etc. expressed in $\bar{Y}_A \bar{V}_A$, $\bar{Y}_F \bar{V}_F$ could also be formed.

Thermal losses. Losses in the convective and radiative transport of heat from the engine is most important and can be written in simplified lumped form as

$$q_R = GA\sigma(T_E^4 - T_A^4) \tag{39}$$

$$q_C = hA(T_E - T_A) \tag{40}$$

or

$$\begin{bmatrix} q_R \\ q_C \end{bmatrix} = \begin{bmatrix} GA\sigma & 0 \\ 0 & hA \end{bmatrix} \begin{bmatrix} (T_E^4 - T_A^4) \\ (T_E - T_A) \end{bmatrix} \tag{41}$$

and

$$\bar{q} = \bar{k}_{TD} \bar{T}_L \tag{42}$$

Equation (42) is the loss of heat due to two mechanisms.

4. System energy balance

An overall system energy balance may be expressed as

$$\{\text{Energy out}\} - \{\text{Energy in}\} = \begin{Bmatrix} \text{Energy sources} \\ \text{Energy sinks} \end{Bmatrix} - \{\text{Energy storage}\} \tag{43}$$

Since no energy is coming into the vehicle system across its boundaries only three terms are pertinent in expression (43).

$$\{\text{Energy out}\} = \left\{\begin{matrix}\text{Energy sources}\\ \text{Energy sinks}\end{matrix}\right\} - \{\text{Energy storage}\} \tag{44}$$

Energy out = net work done by automotive system on surrounding and thermal losses; Energy source = stored chemical energy stored in gasoline; Energy sink = energy losses within automotive systems; Energy storage = all energy stored in the system or

$$\text{Total energy out} = J_0 W + (GA\sigma, hA)\begin{bmatrix} T_E^4 - T_A^4 \\ \\ T_E - T_A \end{bmatrix} + Q_{ex} \tag{45}$$

$$\text{or: Total energy out} = J_0 W + \bar{K}_T^* \bar{T}_L + Q_{ex} \tag{45A}$$

Q_{ex} = the heat lost in the exhaust gases.

$$\text{Energy source} = Q_C M_F \tag{46}$$

$$\text{Energy sink} = (\bar{F}_A^*, \bar{F}_F^* \ldots \bar{F}_M^*)\begin{bmatrix} \bar{V}_A \\ \bar{V}_F \\ \vdots \\ \bar{V}_M \end{bmatrix} \tag{47}$$

Equation (47) is the total power loss in system. In this equation appropriate factors must be used to convert systems resulting in work units to thermal energy units. This factor J_0 will be inserted in the overall balance (eqn 49).

Energy storage =

$$= \overbrace{(v_1, v_2 \ldots v_N)\begin{bmatrix} i_1 \\ i_2 \\ \cdot \\ \vdots \\ i_N \end{bmatrix}}^{\text{Electrical}} + \overbrace{\tfrac{1}{2}(M_1, M_2 \ldots M_N)\left\{\begin{bmatrix} V_1 & 0 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & V_N \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_N \end{bmatrix}\right\}}^{\text{Linear kinetic energy}} +$$

$$\overbrace{+\tfrac{1}{2}(I_{R1}, I_{R2} \ldots I_{RN})\left\{\begin{bmatrix} \omega_1 & 0 & 0 & 0 & 0 \\ 0 & \omega_2 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & \omega_N \end{bmatrix}\begin{bmatrix} \omega_1 \\ \omega_2 \\ \cdot \\ \cdot \\ \omega_N \end{bmatrix}\right\}}^{\text{Rotational kinetic energy}} +$$

$$+\overbrace{\tfrac{1}{2}(k_1, k_2 \dots k_N)\left\{\begin{bmatrix} X_1 & 0 & 0 & 0 & 0 \\ 0 & X_2 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & X_N \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_N \end{bmatrix}\right\}}^{\text{Linear potential}} +$$

$$+\overbrace{\tfrac{1}{2}(K_1, K_2 \dots K_N)\left\{\begin{bmatrix} \theta_1 & 0 & 0 & 0 & 0 \\ 0 & \theta_2 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & \theta_N \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \\ . \\ . \\ \theta_N \end{bmatrix}\right\}}^{\text{Rotational potential}} + \overbrace{(C_1, C_2, C_3 \dots C_N)\begin{bmatrix} T_1 \\ T_2 \\ . \\ . \\ T_N \end{bmatrix}}^{\text{Thermal storage}} \quad (48)$$

The energy balance for the system may now be indicated which will result in an overview model of the energy relationship existent in the automotive system.

$$\overbrace{J_0 w}^{\text{Work out}} + \overbrace{(GA\sigma, hA)\begin{bmatrix} (T_E^4 - T_A^4) \\ \\ (T_E - T_A) \end{bmatrix}}^{\text{Thermal energy out}} + \overbrace{Q_{ex}}^{\substack{\text{Chemical system} \\ \text{Energy release}}} = Q_C M_F - \overbrace{J_i(\bar{F}_A^*, \bar{F}_F^* \dots \bar{F}_M^*)\begin{bmatrix} \bar{V}_A \\ \bar{V}_F \\ \vdots \\ \bar{V}_M \end{bmatrix}}^{\text{System power losses}} -$$

$$-\left\{\overbrace{J_1(v_1, v_2 \dots v_N)\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}}^{\text{Electrical energy storage}} \quad + \overbrace{\tfrac{1}{2}J_0(M_1, M_2 \dots M_N)\left\{\begin{bmatrix} V_1 & 0 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & V_N \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_N \end{bmatrix}\right\}}^{\text{Linear kinetic energy storage}}\right.$$

$$+\overbrace{\tfrac{1}{2}J_0(I_{R1}, I_{R2} \dots I_{RN})\left\{\begin{bmatrix}\omega_1 & 0 & 0 & 0 & 0\\ 0 & \omega_2 & 0 & 0 & 0\\ 0 & 0 & . & 0 & 0\\ 0 & 0 & 0 & . & 0\\ 0 & 0 & 0 & 0 & \omega_N\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ .\\ .\\ \omega_N\end{bmatrix}\right\}}^{\text{Rotational kinetic energy storage}}+$$

$$+\overbrace{\tfrac{1}{2}J_0(k_1, k_2 \dots k_N)\left\{\begin{bmatrix}X_1 & 0 & 0 & 0 & 0\\ 0 & X_2 & 0 & 0 & 0\\ 0 & 0 & . & 0 & 0\\ 0 & 0 & 0 & . & 0\\ 0 & 0 & 0 & 0 & X_N\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ .\\ .\\ X_N\end{bmatrix}\right\}}^{\text{Linear potential energy storage}}+$$

$$+\overbrace{\tfrac{1}{2}J_0(K_1, K_2 \dots K_N)\left\{\begin{bmatrix}\theta_1 & 0 & 0 & 0 & 0\\ 0 & \theta_2 & 0 & 0 & 0\\ 0 & 0 & . & 0 & 0\\ 0 & 0 & 0 & . & 0\\ 0 & 0 & 0 & 0 & \theta_N\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ .\\ .\\ \theta_N\end{bmatrix}\right\}}^{\text{Rotational potential energy storage}}+\overbrace{(C_1, C_2 \dots C_N)\begin{bmatrix}T_1\\ T_2\\ .\\ .\\ T_N\end{bmatrix}}^{\text{Thermal storage}}\Bigg] \tag{49}$$

The system expression may be shown in a simpler form:

$$\overbrace{J_0 w + \bar{k}_T^* \bar{T}_L + Q_{ex}}^{\substack{\text{Net energy}\\ \text{release}}} = \overbrace{Q_C M_F}^{\substack{\text{Energy input}\\ \text{(source)}}} - \overbrace{J_i \bar{\bar{F}}^* \bar{V}}^{\substack{\text{Energy}\\ \text{sink}}}$$

$$-\overbrace{[J_1 \bar{v}^* \bar{i} + \tfrac{1}{2}J_0 \bar{M}^*(\bar{V}_D \bar{V}) + \tfrac{1}{2}J_0 \bar{I}_R^*(\bar{\omega}_D \bar{\omega}) + \tfrac{1}{2}J_0 \bar{k}^*(\bar{X}_D \bar{X}) + \tfrac{1}{2}J_0 \bar{K}^*(\bar{\theta}_D \bar{\theta}) + \bar{C}^* \bar{T}]}^{\text{Energy stored}} \tag{50}$$

Showing the term $J_i \bar{F}^* \bar{V}$ (all power losses) as a sink seems appropriate. It may be argued that $\bar{k}_T^* \bar{T}_L$ is also a sink term; however, "across total system boundary" energy transfer is quite apparent in this term.

If the stored energy in the system is assumed negligible then expression (50) becomes:

$$J_0 w + \bar{k}^* \bar{T}_L + Q_{ex} = Q_C M_F - J_i \bar{\bar{F}}^* \bar{\bar{V}} \tag{51}$$

In the latter equation only, thermal and power losses are indicated so that the network available for automobile motion is:

$$J_0 w = Q_C M_F - J_i \bar{F}^* \bar{V} - \bar{k}^* \bar{T}_L - Q_{ex} \tag{52}$$

This completes the overview analysis and modelling of the automotive system.

IV. CONCLUSIONS

A. The automotive system has been shown to exist as an assembly of sub-systems.

B. Each sub-system may be mathematically modelled using linear system theory.

C. Each important sub-system has been shown in linear graph form.

D. Matrix equations were shown to exist for various components of the automotive system.

E. The scalar matrix energy equation was constructed as an overview model for the system.

NOMENCLATURE

Latin symbols

$\bar{F}$ = generalized through variable matrix
$\bar{V}$ = generalized across variable matrix
$\bar{Y}$ = generalized admittance matrix
$\bar{Z}$ = generalized impedance matrix
$\bar{R}$ = air–fuel matrix
$\bar{I}$ = unit matrix
$\bar{e}$ = energy stored in system (a matrix)
Q = heat released in exothermic reactions, or heat in exhaust
i = current
$\bar{E}$ = energy matrix for fuel system
v = voltage
M, m = mass
V = linear velocity
N = number of sub-systems considered
A = area of heat transfer surface
R = resistance of electrical network
$\bar{P}$ = power loss (matrix of matrices)
$=$ = over a capital letter indicates a matrix of matrices

$\bar{k}$ = linear ideal spring constant matrix, conductivity matrix
$\bar{K}$ = rotary ideal spring constant matrix
C_p = specific heat of mass
G = radiation geometrical factor
h = film coefficient
T = temperature
q = heat flux
J_0 = conversion constant (work to thermal energy units)
J_1 = conversion constant (electrical to thermal energy units)
J_i = conversion coefficient for each system losing power (will be different for each system)

Subscripts
H = hydraulic
E = electrical, engine
D = diagonal matrix
M = mechanical
L = lubrication, losses
F, f = fuel
A = pneumatic (induction air and exhaust), ambient
T = thermal
L = linear
R = rotational, radiative
C = chemical, convective
D = signifies a diagonal matrix
i = when applied to conversion constants signifies that appropriate constant used where needed
ex = exhaust

Greek symbols
ω = rotational or angular velocity
θ = angular displacement
σ = Stefan–Boltzmann constant
Δ = difference

APPENDIX

Automotive system matrix equations

Pneumatic:	$\bar{F}_A = \bar{Y}_A \bar{V}_A$
Fuel:	$\bar{F}_F = \bar{Y}_F \bar{V}_F$
Lubrication:	$\bar{F}_L = \bar{Y}_L \bar{V}_L$
Hydraulic:	$\bar{F}_H = \bar{Y}_H \bar{V}_H$
Electrical:	$\bar{F}_e = \bar{Y}_e \bar{V}_e$
Mechanical:	$\bar{F}_M = \bar{Y}_M \bar{V}_M$
Heat (thermal):	$\bar{F}_T = \bar{Y}_T \bar{V}_T$

Network equipment nodes description

TAN	= tank	FRI	= friction
FIL	= filter	CAS	= camshaft
PUM	= pump	CRS	= crankshaft
GEN	= generator	MUF	= muffler
BAT	= battery	AMA	= air manifold
STA	= starter	EMA	= exhaust manifold
DIS	= distribution	RAD	= radiator
CRA	= crankcase	ENB	= engine block
CYL	= cylinder	FAN	= fan
WHE	= wheels	EXH	= Exhaust

Converter nodes

Generator	= mechanical to electrical
Battery	= chemical to electrical
Starter	= electrical to mechanical
Cylinder	= chemical to thermal to pressure
Pump (fuel)	= mechanical to pressure
Pump (water)	= mechanical to pressure
Pump (oil)	= mechanical to pressure
Mechanical contact (all zones)	= mechanical to friction (heat)
Fan	= mechanical to pressure
Hydraulic and pneumatic system	= fluid energy to friction (heat)

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