

Determination of thermal conductivities of solids by microcalorimetry

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Abstract

A method and a calculation formula for determining the thermal conductivity of solids under constant radial heat flow conditions by use of the Peltier and Joule effects is presented. The instrument constant and coefficient of thermal disequilibrium have been determined using four standard samples with known thermal conductivity: quartz glass, plexiglass, teflon and polyethylene. The reproducibilities of measurements for the heat of thermal disequilibrium in the investigated sample were less than 0.50% and the amount of heat flux through the wall of the investigated cylinder was within 0.07%. The standard deviation, relative error and accuracy of the thermal conductivity determined using this method were 1.8%, 0.22% and 5%, respectively. The thermal conductivities of quartz glass, plexiglass, polyethylene and teflon at 298 K were measured and it was found that the heat of thermal disequilibrium was the principal factor affecting the magnitude of the thermal conductivity of these samples.

INTRODUCTION

With the development of material engineering, material–heat transfer is of growing interest. As a basic transport property, determination of the thermal conductivity (λ) of materials becomes important. Many scientists devote their efforts to the measurement of the value of λ of solids. The basic principle and apparatus for measuring the values of λ of solids by microcalorimetry have been reported [1,2]. The aim of this work is to present a method and a calculation formula for determining the value of λ of solids by using microcalorimetry and to analyse the measuring error of the λ value using the experimental data obtained for quartz glass, plexiglass, polyethylene and teflon.

DERIVATION OF FORMULA OF THERMAL CONDUCTIVITY

If the cylinder shown in Fig. 1 is long enough, the heat conduction in the axial direction may be ignored. In this case, the temperature will change in

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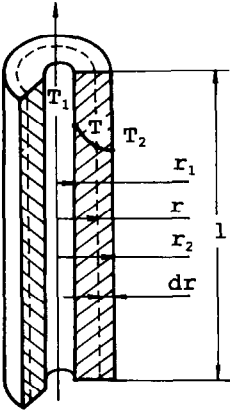


Fig. 1. Single wall cylinder where r_1 , r_2 are the internal and external radii, respectively, in m; T_1 , T_2 are the temperatures of the internal and external walls respectively in K; r is the arbitrary radius in m; dr is the thickness of the differential unit cylinder in m and l is the length in m.

the radial direction, and the temperature field will be one-dimensional when the cylinder coordinate is used.

For steady-state conditions the equation which describes one-dimensional radial heat conduction in the cylinder coordination is

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = 0 \quad (1)$$

By integrating eqn. (1) twice, the general solution can be obtained

$$T = c_1 \ln r + c_2 \quad (2)$$

where c_1 and c_2 are the integral constants which are determined by the following boundary conditions

$$\text{when } r = r_1, \quad T = T_1 \quad (3)$$

$$\text{when } r = r_2, \quad T = T_2 \quad (4)$$

Substituting eqns. (3) and (4) into eqn. (2), the values of c_1 and c_2 may be obtained. Substituting the values of c_1 and c_2 into eqn. (2), gives the following temperature distribution equation in the cylinder:

$$T = T_1 + \frac{T_2 - T_1}{\ln\left(\frac{r_2}{r_1}\right)} \ln\left(\frac{r}{r_1}\right) \quad (5)$$

Differentiation of eqn. (5) with respect to r gives

$$\frac{dT}{dr} = \frac{1}{r} \frac{T_2 - T_1}{\ln\left(\frac{r_2}{r_1}\right)} \quad (6)$$

For steady-state conditions the amount of heat flux passing through the test sample is a constant, and it can be described by Fourier's law

$$Q_s = -\lambda F \frac{dT}{dr} = -\lambda 2\pi r l \frac{dT}{dr} \quad (7)$$

where $F (= 2\pi r l)$ is the area of the cylinder in m^2 with a radius r , Q_s is the amount of heat flux passing through the wall of the cylinder in J s^{-1} ; and λ is the thermal conductivity of the investigated cylindrical sample in $\text{W m}^{-1} \text{K}^{-1}$ or $\text{J m}^{-1} \text{s}^{-1} \text{K}^{-1}$.

Substituting eqn. (6) into eqn. (7) the following equation may be obtained:

$$Q_s = \frac{2\pi\lambda l(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} \quad (8)$$

Similarly, the following equation which describes the amount of heat flux passing through the test sample from r to r_2 can be obtained

$$Q = \frac{2\pi\lambda l(T - T_2)}{\ln\left(\frac{r_2}{r}\right)} \quad (9)$$

For steady-state conditions, $Q = Q_s$, that is

$$Q_s = \frac{2\pi\lambda l(T - T_2)}{\ln\left(\frac{r_2}{r}\right)} \quad (10)$$

The accumulated heat in the differential unit cylinder shown in Fig. 1 is composed of dq_1 and dq_2 if the following experimental conditions are satisfied: (a) the length of the cylinder is much greater than its diameter, i.e. $l \gg 2r_2$ and its top and bottom are covered by thermal insulation; (b) the temperature of the test cylinder arrives at the equilibrium temperature T_c after simultaneously cutting the power produced in the hole of the cylinder by the Joule effect and the compensated power produced in the thermoelectric pile by the Peltier effect.

The value of dq_1 caused by $T > T_2$ is given by

$$dq_1 = (dV) \gamma (T - T_2) = 2\pi r l dr \gamma (T - T_2) \quad (11)$$

where γ is the heat capacity of unit volume of the cylindrical sample in $\text{J K}^{-1} \text{m}^{-3}$.

Combining eqns. (10) and (11), we have

$$dq_1 = \frac{\gamma}{\lambda} Q_s r \ln\left(\frac{r_2}{r}\right) dr = \frac{Q_s}{h} r \ln\left(\frac{r_2}{r}\right) dr \quad (12)$$

where

$$h = \frac{\lambda}{\gamma} = \frac{\lambda}{c\rho} \quad (13)$$

where h is the coefficient of thermal diffusion of the cylindrical sample in $\text{m}^2 \text{s}^{-1}$; c is the specific heat of the cylindrical sample in $\text{J kg}^{-1} \text{K}^{-1}$ and ρ is the density of the cylindrical sample in kg m^{-3} .

Integrating on both sides of eqn. (12)

$$\int_0^{q_1} dq_1 = \frac{Q_s}{h} \int_{r_1}^{r_2} r \ln\left(\frac{r_2}{r}\right) dr \quad (14)$$

the calculated formula of the heat of thermal disequilibrium (q_1) accumulated in the whole cylinder may be obtained as

$$q_1 = \frac{Q_s}{h} \left[\frac{r_2^2 - r_1^2}{4} - \frac{1}{2} r_1^2 \ln\left(\frac{r_2}{r_1}\right) \right] \quad (15)$$

The value of dq_2 caused by $T_2 > T_e$ is given by

$$dq_2 = 2\pi r l dr \gamma (T_2 - T_e) \quad (16)$$

where T_e is the equilibrium temperature of the cylinder.

Because the value of dq_2 is directly proportional to the amount of heat flux passing through the wall of the cylinder (Q_s) the temperature difference ($T_2 - T_e$) is also directly proportional to the value of Q_s , i.e.

$$T_2 - T_e = kQ_s \quad (17)$$

where k is a constant of the microcalorimeter in $\text{K J}^{-1} \text{s}$.

Substituting eqn. (17) into eqn. (16) gives

$$dq_2 = 2\pi r l dr \gamma k Q_s \quad (18)$$

Integrating on both sides of eqn. (18) gives

$$\int_0^{q_2} dq_2 = 2\pi l \gamma k Q_s \int_{r_1}^{r_2} r dr \quad (19)$$

and the following equation which describes heat of thermal disequilibrium

q_2 accumulated in the whole cylinder can be obtained

$$q_2 = \pi l \gamma k Q_s (r_2^2 - r_1^2) = V c \rho k Q_s = cm Q_s k \quad (20)$$

where m is mass of the cylindrical sample in kg.

Hence, the heat of thermal disequilibrium q_{12} accumulated in the whole cylinder is

$$q_{12} = q_1 + q_2 = \frac{Q_s}{h} \left[\frac{r_2^2 - r_1^2}{4} - \frac{1}{2} r_1^2 \ln \left(\frac{r_2}{r_1} \right) \right] + cm Q_s k \quad (21)$$

When the Joule current and the Peltier current are cut simultaneously, there is the heat of thermal disequilibrium q_3 within the all materials in the space around the cylinder. The value of q_3 is also directly proportional to the value of Q_s , i.e.

$$q_3 = D Q_s \quad (22)$$

Where D is the coefficient of thermal disequilibrium.

Thus, the total heat of thermal disequilibrium q is written as

$$q = q_{12} + q_3 = \frac{Q_s}{h} \left[\frac{r_2^2 - r_1^2}{4} - \frac{1}{2} r_1^2 \ln \left(\frac{r_2}{r_1} \right) \right] + cm Q_s k + D Q_s \quad (23)$$

Combining eqns. (13) and (23), we have

$$\lambda = \frac{c \rho}{\frac{q}{Q_s} - (cmk + D)} \left[\frac{r_2^2 - r_1^2}{4} + \frac{r_1^2}{2} \ln \left(\frac{r_1}{r_2} \right) \right] \quad (24)$$

Equation (24) is known as the calculated formula of thermal conductivity of the cylindrical sample.

By measuring the values of c , m , Q_s , q , ρ , r_1 and r_2 with four standard cylindrical samples of known thermal conductivity, the values of k and D can be obtained from eqn. (24).

By substituting the values of Q_s , q and c obtained by using microcalorimetry, the values of r_1 and r_2 obtained with a vernier callipers, the value of ρ obtained with densitometry, and the values of k , D and m into eqn. (24), the value of λ of the investigated cylindrical sample can be obtained.

To gain the value of specific heat of the sample in eqn. (24), the empty cell (system I), the cell containing sample with unknown specific heat c (system II), the cell containing first standard substance with known specific heat c_1 (system III), and the cell containing the second standard substance with known specific heat c_2 (system IV) are uniformly heated by the Peltier current of $10\sqrt{10}$ mA for 1 h. When thermal equilibrium of systems I–IV is established, it can be shown that, (a) the temperature is uniform within the cell, and (b) the equilibrium temperature θ is the same for a given Peltier

current, whatever the cell contents. When the Peltier current is cut off, the quantities of heat, Q_1 , Q_2 , Q_3 and Q_4 retained by the systems I–IV are given by the following equations.

$$\text{For system I} \quad Q_1 = KS_0 = a\theta \quad (25)$$

$$\text{For system II} \quad Q_2 = KS = (a + mc)\theta \quad (26)$$

$$\text{For system III} \quad Q_3 = KS_1 = (a + m_1c_1)\theta \quad (27)$$

$$\text{For system IV} \quad Q_4 = KS_2 = (a + m_2c_2)\theta \quad (28)$$

where K is a caloric constant of the microcalorimeter used under the experimental temperature in $\text{J} (\text{integral number})^{-1}$; a is the apparent heat capacity of the empty cell in JK^{-1} ; m , m_1 and m_2 are masses of the investigated sample, the first standard substance and the second standard substance, respectively, in kg; S is the area under the curve on its return to the experimental zero after cutting off the Peltier current, (integral number); and subscripts 0, 1 and 2 express the systems I, III and IV, respectively.

From eqns. (25) and (26), we have

$$mc\theta = K(S - S_0) \quad (29)$$

From eqns. (25) and (27), we have

$$m_1c_1\theta = K(S_1 - S_0) \quad (30)$$

From eqns. (29) and (30), we have

$$c = \frac{(S - S_0)m_1c_1}{(S_1 - S_0)m} \quad (31)$$

Similarly, we have

$$c = \frac{(S - S_0)m_2c_2}{(S_2 - S_0)m} \quad (32)$$

From eqns. (31) and (32), we can obtain

$$c = \frac{S - S_0}{2m} \left(\frac{m_1c_1}{S_1 - S_0} + \frac{m_2c_2}{S_2 - S_0} \right) \quad (33)$$

The value of c of the investigated sample can be obtained from eqn. (33) if the values of c_1 and c_2 of two standard substances are known.

EXPERIMENTAL

Sample

The cylindrical samples used to measure the thermal conductivity of quartz glass, plexiglass, polyethylene and teflon have same dimensions: 66.18 mm long with a outer diameter of 14.86 mm and an inner diameter of 2 mm (see Fig. 2).

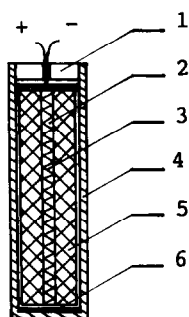


Fig. 2. Device used for measuring the thermal conductivity of the investigated cylindrical samples. 1, Teflon cover; 2, wooden rod; 3, enamel-insulated manganin wire; 4, calorimetric cell; 5, investigated cylindrical sample; 6, thermal insulation.

Electric heater

The electric heater is placed in the hole of the cylinder (inner diameter 2 mm) and is composed of a wooden rod of length 66.18 mm and diameter 1.40 mm and an enamel-insulated manganin wire ($85.4 \Omega \text{ m}^{-1}$) of 0.08 mm diameter which has a very low resistance–temperature coefficient. The wire is compactly wrapped in the wooden rod in the form of two layers. The leads of the heater are fixed onto the teflon cover. The two electric heaters used in the measurements have same resistance.

Experimental equipment

All measurements were made using a microcalorimeter, type RD496-II from Southwestern Electronic Engineering Institute, China, which has a sensitivity of $46.49 \mu \text{ V mW}^{-1}$ and is equipped with two 15 ml vessels. The microcalorimeter was calibrated by the Joule effect before use. The variation in enthalpy measurement is less than 1%.

Determination of the values of Q_s and q

The values of Q_s and q used to calculate the thermal conductivity of the sample are obtained by the testing method presented as a thermogram in Fig. 3. In Fig. 3, AH is the baseline; at time B, a constant calorific power is generated in the laboratory cell; at CD, the steady state is established; at point D, the Peltier effect begins; at EF, the calorific power produced by the Joule effect is totally compensated by the Peltier effect; at point F, the Joule and Peltier effects are simultaneously cut; FGH is the resulting curve, corresponding to a heat evolution; Q_s is the amount of heat flux passing through the wall of the test sample under the steady-state conditions; q is the heat of thermal disequilibrium accumulated in the test sample, and all materials in the space around the cylinder.

Determination of the values of c , r_1 , r_2 and ρ

The value c of the cylindrical sample was obtained using a microcalorimeter. The values of r_1 and r_2 were obtained with vernier callipers and ρ was obtained by densitometry.

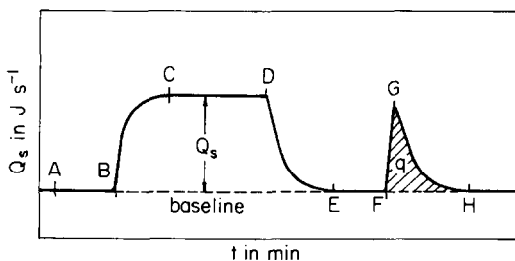


Fig. 3. Schematic thermogram of measurement of the thermal conductivity of solids.

EXPERIMENTAL RESULTS AND DISCUSSION

The values of Q_s , q , m , c , ρ , r_1 , r_2 , S_0 , S , S_1 , S_2 , m_1 , m_2 , c_1 and c_2 of quartz glass, plexiglass, teflon and polyethylene obtained using the above-mentioned methods are listed in Tables 1 and 2. It can be seen in Table 1 that the reproducibilities of the values of Q_s and q are within 0.07% and within less than 0.5% respectively.

By substituting the values of Q_s , q , c , m , ρ , r_1 , r_2 listed in Table 1 and the reported values of λ of $1.37 W m^{-1} K^{-1}$ for quartz glass, of $0.192 W m^{-1} K^{-1}$ for plexiglass, of $0.251 W m^{-1} K^{-1}$ for teflon, and of $0.460 W m^{-1} K^{-1}$ for polyethylene listed in Table 3 into eqn. (24), the corresponding values of k

TABLE 1
The original data of four standard samples

Sample	$c^a \times 10^{-3}$	$\rho^b \times 10^{-3}$	$m^c \times 10^3$	$r_1^d \times 10^3$	$r_2^e \times 10^3$	$q^{f,j}$	$\frac{\sigma_{g,k}}{q}$ in %	$Q_s^{h,i} \times 10^3$	$\frac{\sigma_{Q_s}}{Q_s}$ in %
Quartz glass	0.8409	2.2027	24.5948	1.01	7.42	0.6094	0.46	9.237	0.05 ^l
			24.8210	1.00	7.43	0.6143	0.49	9.236	0.04
Plexiglass	1.517	1.1900	12.6720	1.25	7.40	1.4498	0.27	9.200	0.05 ^l
			13.3863	1.00	7.41	1.5221	0.23	9.192	0.04
Teflon	1.122	2.2124	24.6217	1.06	7.42	1.7251	0.17	9.177	0.02 ^l
			24.5471	1.07	7.40	1.7142	0.12	9.191	0.06
Polyethylene	1.834	0.9707	10.7744	1.07	7.40	0.8677	0.25	9.204	0.04 ^l
			10.7762	1.07	7.44	0.8759	0.26	9.199	0.03

^a c , Specific heat of cylindrical sample in $J kg^{-1} K^{-1}$. ^b ρ , Density of cylindrical sample in $kg m^{-3}$. ^c m , Mass of cylindrical sample in kg . ^d r_1 , Internal radius of the cylindrical sample in m . ^e r_2 , External radius of the cylindrical sample in m . ^f q , Heat of thermal disequilibrium in $J s^{-1}$. ^g σ_q , Standard deviation of the heat of thermal disequilibrium in J . ^h Q_s , Amount of heat flux under the steady state in $J s^{-1}$. ⁱ σ_{Q_s} , Standard deviation of the amount of heat flux in $J s^{-1}$. ^j q and Q_s are the average values of nine measurements. ^k Values of σ_q and σ_{Q_s} are obtained with $\sigma = [\sum (x_i - \bar{x})^2 / (n - 1)]^{1/2}$, $x = q$, Q_s , $n = 9$. ^l Data used for error analysis.

TABLE 2

Original data used for calculating the values of c

	Empty cell	Benzoic acid	Distilled water	Quartz glass	Plexiglass	Teflon	Polyethylene
$S_0^{a,d}$	2994						
$S_1^{a,d}$		3246					
$S_2^{a,d}$			5116				
$S^{a,d}$				4064	3644	3977	3662
$m_1 \times 10^3{}^b$		3.0284					
$m_2 \times 10^3{}^b$			7.3857				
m^b				19.1310	6.4395	12.9555	5.4793
$c_1 \times 10^{-3}$		1.253 [3]					
$c_2 \times 10^{-3}$			4.180 [4]				
$c \times 10^{-3}$				0.8409	1.5173	1.1220	1.8337

^a S_0 , S_1 , S_2 and S , are the areas under the curve on its return to the experimental zero after cutting off the Peltier current for the empty cell, the cell containing first standard sample (benzoic acid), the cell containing second standard sample (distilled water), and the cell containing investigated sample, respectively, (integral number). ^b m_1 , m_2 and m are the masses of benzoic acid, distilled water, and the investigated sample, respectively in kg. ^c c_1 , c_2 and c are the specific heats of benzoic acid, distilled water, and the investigated sample, respectively in $\text{J kg}^{-1} \text{K}^{-1}$. ^d S_0 , S_1 , S_2 and S , Average value of six measurements.

of $2.3265 \text{ K J}^{-1} \text{ s}^{-1}$ and of D of 0.9668 s , and of the correlation coefficient of 0.9998 may be obtained. By substituting the values of Q_s , q , m , c , ρ , r_1 , r_2 listed in Table 1 and the values of k and D into eqn. (24), the corresponding value of λ can be obtained (data shown in Table 3).

To check the accuracy of measurements, the thermal conductivities of quartz glass, teflon, plexiglass and polyethylene were measured and compared with the reported values [5-8]. The comparison is shown in

TABLE 3

Comparison of the experimental values of λ with literature ones λ_1 of thermal conductivity of four standard solids at 298 K

Sample	$\lambda_1/\text{W m}^{-1} \text{K}^{-1}$	$\lambda/\text{W m}^{-1} \text{K}^{-1}$	$\frac{ \lambda - \lambda_1 }{\lambda_1}$ in %
Quartz glass	1.37 [5]	1.37	0
		1.37	
		Av. 1.37	
Plexiglass	0.192 [6]	0.192	0
		0.192	
		Av. 0.192	
Teflon	0.251 [7]	0.251	0
		0.251	
		Av. 0.251	
Polyethylene	0.460 [8]	0.462	0.22
		0.459	
		Av. 0.461	

Table 3. The results for quartz glass, plexiglass, teflon and polyethylene are in excellent accord with those reported in the literature. Because the reported methods have a relative error of 1% in measuring solids, it is reasonable to believe that the relative error of the microcalorimetry is less than 1.5%.

In order to gain the overall standard deviation of results obtained by microcalorimeter, we made the data treatment of indirect measuring quantities by using the functional eqns. (34) and (35), the overall standard deviation formulae (36) and (37), the propagation coefficient eqns. (38)–(53), the calculated formula, $\sigma = \Delta d/\sqrt{3}$ where Δd is the value of the minimal scale division of the measuring equipment), for the single measurement quantities, r_1, r_2, m, m_1, m_2 and ρ , and the standard deviation equation $\sigma = \sqrt{\{\sum[(x_i - x)^2]/(n - 1)\}}$, $x = q, Q_s, S_0, S_1$ and S_2 , for the repeatable measurement quantities, q, Q_s, S_0, S_1 and S_2 and the results are shown in Table 4.

TABLE 4

Values of the standard deviation, error propagation coefficient used for calculating the values of σ_λ , and the overall standard deviation σ_λ

	Quartz glass	Plexiglass	Teflon	Polyethylene
σ_{r_1}	5.77×10^{-6}	5.77×10^{-6}	5.77×10^{-6}	5.77×10^{-6}
σ_{r_2}	5.77×10^{-6}	5.77×10^{-6}	5.77×10^{-6}	5.77×10^{-6}
σ_q	2.76×10^{-3}	3.50×10^{-3}	2.90×10^{-3}	2.20×10^{-3}
σ_{Q_s}	5.00×10^{-6}	4.00×10^{-6}	2.00×10^{-6}	4.00×10^{-6}
σ_c	1.42×10^{-2}	1.42×10^{-2}	1.42×10^{-2}	1.42×10^{-2}
σ_m	5.77×10^{-8}	5.77×10^{-8}	5.77×10^{-8}	5.77×10^{-8}
σ_ρ	5.77×10^{-8}	5.77×10^{-8}	5.77×10^{-8}	5.77×10^{-8}
$\frac{\partial \lambda}{\partial r_1}$	-2.21×10^2	-3.59×10	-4.17×10	-7.78×10
$\frac{\partial \lambda}{\partial r_2}$	3.99×10^2	5.80×10	7.35×10	1.36×10^2
$\frac{\partial \lambda}{\partial q}$	-8.78	-1.87×10^{-1}	-2.22×10^{-1}	-1.06
$\frac{\partial \lambda}{\partial Q_s}$	5.79×10^2	2.94×10	4.18×10	1.00×10^2
$\frac{\partial \lambda}{\partial c}$	6.36×10^{-3}	1.78×10^{-4}	3.42×10^{-4}	5.02×10^{-4}
$\frac{\partial \lambda}{\partial m}$	1.59×10^2	6.06	5.33	4.17×10
$\frac{\partial \lambda}{\partial \rho}$	6.22×10^{-4}	1.61×10^{-4}	1.13×10^{-4}	4.76×10^{-4}
σ_λ	2.5×10^{-2}	7.7×10^{-4}	8.1×10^{-4}	2.5×10^{-3}

$$\lambda = f(r_1, r_2, q, Q_s, c, m, \rho, k, D) \quad (34)$$

$$c = f(S, m, S_0, m_1, c_1, S_1, m_2, c_2, S_2) \quad (35)$$

$$\sigma_\lambda = \left[\left(\frac{\partial \lambda}{\partial r_1} \right)^2 \sigma_{r_1}^2 + \left(\frac{\partial \lambda}{\partial r_2} \right)^2 \sigma_{r_2}^2 + \left(\frac{\partial \lambda}{\partial q} \right)^2 \sigma_q^2 + \left(\frac{\partial \lambda}{\partial Q_s} \right)^2 \sigma_{Q_s}^2 + \left(\frac{\partial \lambda}{\partial c} \right)^2 \sigma_c^2 + \left(\frac{\partial \lambda}{\partial m} \right)^2 \sigma_m^2 + \left(\frac{\partial \lambda}{\partial \rho} \right)^2 \sigma_\rho^2 + \left(\frac{\partial \lambda}{\partial k} \right)^2 \sigma_k^2 + \left(\frac{\partial \lambda}{\partial D} \right)^2 \sigma_D^2 \right]^{1/2} \quad (36)$$

$$\sigma_c = \left[\left(\frac{\partial c}{\partial S} \right)^2 \sigma_S^2 + \left(\frac{\partial c}{\partial m} \right)^2 \sigma_m^2 + \left(\frac{\partial c}{\partial S_0} \right)^2 \sigma_{S_0}^2 + \left(\frac{\partial c}{\partial m_1} \right)^2 \sigma_{m_1}^2 + \left(\frac{\partial c}{\partial c_1} \right)^2 \sigma_{c_1}^2 + \left(\frac{\partial c}{\partial S_1} \right)^2 \sigma_{S_1}^2 + \left(\frac{\partial c}{\partial m_2} \right)^2 \sigma_{m_2}^2 + \left(\frac{\partial c}{\partial c_2} \right)^2 \sigma_{c_2}^2 + \left(\frac{\partial c}{\partial S_2} \right)^2 \sigma_{S_2}^2 \right]^{1/2} \quad (37)$$

$$\frac{\partial c}{\partial S} = \frac{1}{2m} \left(\frac{m_1 c_1}{S_1 - S_0} + \frac{m_2 c_2}{S_2 - S_0} \right) \quad (38)$$

$$\frac{\partial c}{\partial m} = - \frac{S - S_0}{2m^2} \left(\frac{m_1 c_1}{S_1 - S_0} + \frac{m_2 c_2}{S_2 - S_0} \right) \quad (39)$$

$$\frac{\partial c}{\partial S_0} = \frac{1}{2m} \left[\frac{m_1 c_1 (S - S_1)}{(S_1 - S_0)^2} + \frac{m_2 c_2 (S - S_2)}{(S_2 - S_0)^2} \right] \quad (40)$$

$$\frac{\partial c}{\partial m_1} = \frac{c_1 (S - S_0)}{2m (S_1 - S_0)} \quad (41)$$

$$\frac{\partial c}{\partial c_1} = \frac{m_1 (S - S_0)}{2m (S_1 - S_0)} \quad (42)$$

$$\frac{\partial c}{\partial S_1} = - \frac{m_1 c_1 (S - S_0)}{2m (S_1 - S_0)^2} \quad (43)$$

$$\frac{\partial c}{\partial m_2} = \frac{c_2 (S - S_0)}{2m (S_2 - S_0)} \quad (44)$$

$$\frac{\partial c}{\partial c_2} = \frac{m_2 (S - S_0)}{2m (S_2 - S_0)} \quad (45)$$

$$\frac{\partial c}{\partial S_2} = - \frac{m_2 c_2 (S - S_0)}{2m (S_2 - S_0)^2} \quad (46)$$

$$\frac{\partial \lambda}{\partial r_1} = \frac{\rho c}{\frac{q}{Q_s} - cmk - D} r_1 \ln \left(\frac{r_1}{r_2} \right) \quad (47)$$

$$\frac{\partial \lambda}{\partial r_2} = \frac{\rho c}{\frac{q}{Q_s} - cmk - D} \frac{r_2^2 - r_1^2}{2r_2} \quad (48)$$

$$\frac{\partial \lambda}{\partial q} = -\frac{\rho c}{\left(\frac{q}{Q_s} - cmk - D\right)^2 Q_s} \left[\frac{r_2^2 - r_1^2}{4} + \frac{r_1^2}{2} \ln\left(\frac{r_1}{r_2}\right) \right] \quad (49)$$

$$\frac{\partial \lambda}{\partial Q_s} = \frac{\rho c q}{\left(\frac{q}{Q_s} - cmk - D\right)^2 Q_s^2} \left[\frac{r_2^2 - r_1^2}{4} + \frac{r_1^2}{2} \ln\left(\frac{r_1}{r_2}\right) \right] \quad (50)$$

$$\frac{\partial \lambda}{\partial c} = \frac{\rho q}{\left(\frac{q}{Q_s} - cmk - D\right)^2 Q_s} \left[\frac{r_2^2 - r_1^2}{4} + \frac{r_1^2}{2} \ln\left(\frac{r_1}{r_2}\right) \right] \quad (51)$$

$$\frac{\partial \lambda}{\partial m} = \frac{\rho c^2 k}{\left(\frac{q}{Q_s} - cmk - D\right)^2} \left[\frac{r_2^2 - r_1^2}{4} + \frac{r_1^2}{2} \ln\left(\frac{r_1}{r_2}\right) \right] \quad (52)$$

$$\frac{\partial \lambda}{\partial \rho} = \frac{c}{\left(\frac{q}{Q_s} - cmk - D\right)} \left[\frac{r_2^2 - r_1^2}{4} + \frac{r_1^2}{2} \ln\left(\frac{r_1}{r_2}\right) \right] \quad (53)$$

Because k and D are two constants, $\partial \lambda / \partial k = 0$ and $\partial \lambda / \partial D = 0$.

The percentages that the percentage error of eight direct measurement quantities accounts for of the overall error for quartz glass, plexiglass, teflon and polyethylene are shown in Table 5. It can be seen from Table 5 that the value of q in eight direct measurement quantities is the principal factor affecting the λ value of solids.

To check the effects of dimension of the cylinder and concentricity of the hole of the cylinder on the value of λ , we measured the values of the thermal conductivities of plexiglass with different dimensions and concentricity. It can be seen from Table 6 that the relative error and reproducibilities of the thermal conductivities of plexiglass determined by using this method are within than 1.1% and within 0.60% when the internal radius is changed from 1.00 mm to 1.25 mm, the external radius from 7.40 mm to 7.46 mm, and the concentricity from 0.02 mm to 0.12 mm.

TABLE 5

Percentage that the percentage error of eight direct measurement quantities accounts for of the overall error

	Quartz glass	Plexiglass	Teflon	Polyethylene
$\left(\frac{\partial \lambda}{\partial r_1} \frac{\sigma_{r_1}}{\sigma_\lambda}\right)^2$	0.26	7.19	8.79	3.15
$\left(\frac{\partial \lambda}{\partial r_2} \frac{\sigma_{r_2}}{\sigma_\lambda}\right)^2$	0.88	18.77	27.32	9.60
$\left(\frac{\partial \lambda}{\partial q} \frac{\sigma_q}{\sigma_\lambda}\right)^2$	97.50	71.74	62.83	84.75
$\left(\frac{\partial \lambda}{\partial Q_s} \frac{\sigma_{Q_s}}{\sigma_\lambda}\right)^2$	1.38	2.30	1.06	2.50
$\left(\frac{\partial \lambda}{\partial c} \frac{\sigma_c}{\sigma_\lambda}\right)^2$	1.34×10^{-3}	1.07×10^{-3}	3.58×10^{-3}	7.91×10^{-4}
$\left(\frac{\partial \lambda}{\partial m} \frac{\sigma_m}{\sigma_\lambda}\right)^2$	1.40×10^{-5}	2.04×10^{-4}	1.44×10^{-5}	9.02×10^{-5}
$\left(\frac{\partial \lambda}{\partial \rho} \frac{\sigma_\rho}{\sigma_\lambda}\right)^2$	2.14×10^{-16}	1.45×10^{-14}	6.45×10^{-15}	1.17×10^{-14}

TABLE 6

The effects of dimension of the cylinder and concentricity of the hole of the cylinder on the λ values of plexiglass^a

	N_1	N_2	N_3	N_4	N_5	N_6
r_1/mm	1.25	1.00	1.25	1.02	1.1	1.25
r_2/mm	7.40	7.41	7.44	7.45	7.45	7.44
Ct/mm	0.12	0.02	0.04	0.12	0.05	0.04
$\lambda/W \text{ m}^{-1} \text{ K}^{-1}$	0.192	0.192	0.191	0.190	0.190	0.192

^a N_{1-6} are experimental values and Ct is the concentricity of the hole of the cylinder.

CONCLUSIONS

The method of determining the thermal conductivity of solids under constant radial heat flux conditions by use of the Peltier and Joule effects is very satisfactory in accuracy, relative error and reproducibility. The results for quartz glass, plexiglass, teflon and polyethylene determined using this method agree with data available in the literature. The heat of thermal disequilibrium is the principal factor affecting the magnitude of the thermal conductivity of solids.

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