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# The 'E-ln(A)-f( $\alpha$ )' triplet in non-isothermal reaction kinetics analysis<sup>1,2</sup>

John P. Elder<sup>\*</sup>

Merck and Co. Inc., P.O. Box 4, WP78A-31 Sumneytown Pike, West Point, PA 19486, USA

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#### Abstract

In determining the kinetics of thermally stimulated solid-state transformations by non-isothermal techniques, there is, as yet, one unsurmountable barrier to establishing separate values of E, A, and  $f(\alpha)$ . By employing logarithmic plots of various expressions; reaction rate at one heating rate,  $\beta$ , i.e. Arrhenius analysis, isoconversional reaction rates at several heating rates, i.e. Friedman analysis, one easily obtains the value of  $E$ , from the plot slope. However, the plot intercept is the logarithm of the product of A and f( $\alpha$ ), and over the entire extent of a singular reaction should mirror the behavior of the f( $\alpha$ ) parameter defining the mechanism. In examining experimental non-isothermal data, either DSC or TG, obtained by either procedure, this behavioral pattern should assist in identifying the probable reaction mechanism. Rate and extent of reaction data for singular model systems, generated over a wide range of  $E_{in}$ ,  $A_{in}$ , and  $\beta$  for seven f( $\alpha$ ) mechanisms, have been employed as vehicles for isoconversional Friedman analysis in an attempt to verify the several model-dependent paradigms in the  $E_F\text{-}ln(A_F)\text{-}f(\alpha)$ relationships over the  $0.01 \le \alpha \le 0.99$  range. Isoconversational data resulting from analyses of simulated single solid-state reactions have also been employed to test the utility of proposed 'Kissinger-type' relationships for so-called 'model-free' kinetics. The isoconversional procedure has also been employed to investigate the behavioral patterns observed in the analyses of the three steps in a thermogravimetric study of the degradation of calcium oxalate monohydrate. © 1998 Elsevier Science B.V.

#### 1. Theoretical considerations

Eq. (1a) represents the generally accepted logarithmic expression for the rate of a non-isothermally stimulated solid-state reaction at a set extent of reaction,  $\alpha$ , at a temperature,  $T_{\alpha}$  [1].

$$
\ln(\mathrm{d}\alpha/\mathrm{d}t)_{\alpha} = \ln[A_{\alpha} \times T_{\alpha}^{m} \times \mathrm{f}(\alpha)] - E_{\alpha}/RT_{\alpha}
$$
\n(1a)

where  $f(\alpha)$  is the analytical function of the reaction

extent,  $\alpha$ , descriptive of the reaction mechanism.  $E_{\alpha}$ and  $A_{\alpha}$  are the so-called energy of activation and preexponential factor, respectively, and  $R$  is the general gas constant. In light of the remarks of Dollimore and Reading [2], it is considered reasonable to use the  $m=0$  approximation. This approach has been followed in this study.

$$
\ln(\mathrm{d}\alpha/\mathrm{d}t)_{\alpha} = \ln[A_{\alpha} \times \mathrm{f}(\alpha)] - E_{\alpha}/RT_{\alpha} \qquad (1b)
$$

Although recently heavily criticized by Zsako [3], the isoconversional procedure is highly popular. In this method, a set of actual or model-simulated experiments are performed at different linear heating rates. The rates and temperatures of reaction at a pre-defined extent of reaction are measured, and fitted to Eq. (1b).

<sup>\*</sup>Corresponding author. E-mail: john\_elder@merck.com

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The major premise underlying the use of the isoconversional method is that one is dealing with a single effective reaction with  $\alpha$ -invariant kinetics parameters,  $E_{\alpha}$  and  $A_{\alpha}$ , over the entire extent of reaction range,  $0 \le \alpha \le 1$ . Although it is possible to generate  $(d\alpha/dt, T_{\alpha})$  data sets for a simulated single reaction, which will yield a linear Eq. (1b) plot, it is highly unlikely that, in reality, experimentally measured data characterize a single reaction. Budrugeac et al. [4] have recently discussed some of the problems inherent in the application of the isoconversional technique.

In the early stages of the development of this analytical procedure, Friedman [5] employed the *n*th order model (2) to represent  $f(\alpha)$ .

Table 1 Solid-state reaction kinetics model parameters,  $f(\alpha)$  and  $g(\alpha)$ 



It has been demonstrated [6] that extent  $(\alpha)$  and rate  $(d\alpha/dt)$  of reaction data for various solid-state models can be analyzed using this empirical model. The resulting  $\alpha$ -T and d $\alpha$ /dt-T data for both the actual and the nth order equivalent model are identical. Furthermore, despite large discrepancies in the magnitude of the actual solid-state model kinetics  $(E, A)$ parameters compared with the nth order equivalent values  $(E_n, A_n)$ , extents of reaction as a function of temperature at any heating rate are superimposable with those generated using isoconversionally determined parameters with model-corrected pre-exponential factors [7].



 $\log(\alpha) = \int d\alpha/f(\alpha)$ .

 $T<sub>1</sub>$ 





Irrespective of the isoconversional procedure employed, either the differential or integral (vide infra) approach, the activation energy of the assumed single reaction is obtained from the slope of the logarithmic plots, Eq. (1b) in the differential method, at various degrees of conversion. However, one cannot calculate the pre-exponential factor without a knowl-

edge of the  $f(\alpha)$  function, which, as will be shown, varies significantly and differently with  $\alpha$  for all solidstate models. Budrugeac et al. [4] have discussed their views regarding how the investigator should proceed by suggesting criteria for establishing correct  $f(\alpha)$ functions when dealing with actual experimental data. Furthermore, they indicate their preference for using



Fig. 1. (a)  $f(\alpha)$  as a function of  $\alpha$  for the solid-state models A2, A3, R2, and R3. (b)  $f(\alpha)$  as a function of  $\alpha$  for the solid-state models D2, D3, and D4.

the differential rather than the integral approach, if one wishes to test quickly for constant  $E_{\alpha}$ . Zsako [3], who employs the empirical nth order model, Eq. (2), and, it appears, the integral approach, observes that much of the experimental data employed in support of his critique, show a systematic decrease in both calculated  $E_{\alpha}$  and  $A_{\alpha}$ , and an increase in the reaction order, 'n', with increasing heating rate. Furthermore, it is

stated that in performing model analyses, more realistic TG data, exhibiting such heating rate-dependent characteristics should be employed in investigating the utility of the `many curves' method. It is considered that such a selection protocol defeats the purpose of establishing the correctness of the isoconversional procedure. One should not be too surprised at his conclusions.



Fig. 2.  $g(\alpha)$  as a function of  $\alpha$  for the solid-state models A2, A3, R2, R3, D2, D3, and D4.

Table 3 ln  $[g(\alpha)]$  for seven solid-state reaction kinetics models

$\alpha$	A <sub>2</sub>	A <sub>3</sub>	R <sub>2</sub>	R <sub>3</sub>	D2	D <sub>3</sub>	D <sub>4</sub>
0.05	$-1.4851$	$-0.9901$	$-3.6761$	$-4.0773$	$-6.6677$	$-8.1547$	$-8.1661$
0.10	$-1.1252$	$-0.7501$	$-2.9697$	$-3.3665$	$-5.2638$	$-6.7330$	$-6.7563$
0.15	$-0.9085$	$-0.6057$	$-2.5505$	$-2.9425$	$-4.4347$	$-5.8851$	$-5.9209$
0.20	$-0.7500$	$-0.5000$	$-2.2484$	$-2.6355$	$-3.8404$	$-5.2710$	$-5.3200$
0.25	$-0.6229$	$-0.4153$	$-2.0101$	$-2.3921$	$-3.3744$	$-4.7842$	$-4.8470$
0.30	$-0.5155$	$-0.3436$	$-1.8119$	$-2.1884$	$-2.9892$	$-4.3768$	$-4.4545$
0.35	$-0.4211$	$-0.2807$	$-1.6411$	$-2.0117$	$-2.6594$	$-4.0234$	$-4.1168$
0.40	$-0.3359$	$-0.2239$	$-1.4899$	$-1.8543$	$-2.3697$	$-3.7085$	$-3.8188$
0.45	$-0.2572$	$-0.1715$	$-1.3533$	$-1.7110$	$-2.1104$	$-3.4221$	$-3.5504$
0.50	$-0.1833$	$-0.1222$	$-1.2279$	$-1.5784$	$-1.8745$	$-3.1569$	$-3.3048$
0.55	$-0.1125$	$-0.0750$	$-1.1112$	$-1.4538$	$-1.6572$	$-2.9075$	$-3.0769$
0.60	$-0.0437$	$-0.0291$	$-1.0009$	$-1.3349$	$-1.4546$	$-2.6697$	$-2.8627$
0.65	0.0243	0.0162	$-0.8955$	$-1.2199$	$-1.2639$	$-2.4397$	$-2.6589$
0.70	0.0928	0.0619	$-0.7935$	$-1.1069$	$-1.0823$	$-2.2139$	$-2.4628$
0.75	0.1633	0.1089	$-0.6931$	$-0.9941$	$-0.9078$	$-1.9883$	$-2.2716$
0.80	0.2379	0.1586	$-0.5928$	$-0.8790$	$-0.7379$	$-1.7580$	$-2.0821$
0.85	0.3202	0.2134	$-0.4899$	$-0.7579$	$-0.5702$	$-1.5157$	$-1.8903$
0.90	0.4170	0.2780	$-0.3801$	$-0.6239$	$-0.4009$	$-1.2478$	$-1.6898$
0.95	0.5486	0.3657	$-0.2531$	$-0.4595$	$-0.2229$	$-0.9190$	$-1.4656$

For single reactions with  $\alpha$ -invariant kinetics parameters, the intercepts of the Eq. (1b) plots should mirror the behavioral pattern of the  $f(\alpha)$  mechanistic functions over the entire  $\alpha$ -range. It is the purpose of this investigation to verify this fact by describing the findings of a set of isoconversional calculations of simulated singular non-isothermal reactions conforming to seven solid-state models A2, A3, R2, R3, D2, D3 and D4 [8]. A wide range of constant  $E$  and  $A$  input

parameters was used to generate extent and rate of reaction-temperature data at several linear heating rates from 1 to 100°C/min. Friedman isoconversional analyses were performed using a general purpose fortran program, KINMOD1, previously employed by the author to investigate several mutually independent multiple reaction systems [9]. All calculations used the  $m=0$  temperature exponent approximation. The original program has been modified by employing



Fig. 3. ln[f( $\alpha$ )] as a function of  $\alpha$  for the solid-state models A2, A3, R2, R3, D2, D3, and D4.



Fig. 4. ln[g( $\alpha$ )] as a function of  $\alpha$  for the solid-state models A2, A3, R2, R3, D2, D3, and D4.

the fourth degree rational approximation of Senum and Yang [10] to compute the  $p(E/RT)$  function used to calculate the  $f(\alpha)$  and the inverse integral function  $g(\alpha)$  employed at various stages in the data analyses. Recently, integral procedures have been described which purport to generate reaction kinetics parameters without specifying the  $f(\alpha)$  functional model, the socalled `model-free' kinetics [11], utilizing Vyazovkin's 'Kissinger-like' Eq. (3).

$$
\ln\{[\beta/(T_{\alpha})^2\} = \ln\{[A_{\alpha} \times R]/[E_{\alpha} \times g(\alpha)]\}
$$

$$
-E_{\alpha}/RT_{\alpha}
$$
 (3)

It appears that Vyazovkin employs the first two terms of the polynomial expression for the temperature integral  $p(E/RT)$  function [1,12] in developing this equation for  $E/2RT>>1$ . It is expected that the isoconversional logarithmic intercept of this equation will vary with  $\alpha$  in a manner similar to that expected for the comparable intercept in the more simple Eq. (1b). Calculations have been carried out in an attempt to verify this expectation.

#### 2. Model calculations

Table 1 lists the  $f(\alpha)$  and  $g(\alpha)$  i.e. [ $\int d\alpha/f(\alpha)$ ] functions for seven solid-state models [2]. Fig. 1(a) and (b), and Fig. 2, show the  $f(\alpha)$  and  $g(\alpha)$  functions, respectively, over the complete  $\alpha$ -range. The differences in the  $\alpha$ -dependencies of the several  $f(\alpha)$  and  $g(\alpha)$  functions are immediately obvious. Tables 2 and Table 3 list ln[f( $\alpha$ )] and ln[g( $\alpha$ )] as functions of  $\alpha$  for these seven models, and Figs. 3 and 4 show the dependencies. With  $E_{in}=220$  kJ/mol and  $A_{in} = 1.25 \times 10^{15} \text{ min}^{-1}$  as input parameters,  $\alpha - T$ and  $d\alpha/dt - T$  curves were generated, and from the Friedman isoconversional analyses,  $E_F$  and  $A_F$  values computed. Fig.  $5(a-c)$  show the logarithmic intercepts as functions of  $\alpha$  for the A2, R3 and D4 solid-state models. As can be seen, they almost exactly mirror the  $ln[f(\alpha)]$  variations depicted in Fig. 3 for these models. The results are summarised in Table 4. As can be seen, the activation energies are essentially constant at 99.9% of the correct value. Similarly, the logarithms of the pre-exponential factors are constant at  $>99.9\%$ of the correct value. Similar results were obtained with other input parameters. It is proposed that in dealing



Fig. 5. (a) Eq. (1b) isoconversional logarithmic intercepts as a function of  $\alpha$  for the A2 solid-state model with input parameters,  $E=220 \text{ kJ/mol}$  and  $A=1.25\times10^{15} \text{ min}^{-1}$ . (b) Eq. (1b) isoconversional logarithmic intercepts as a function of  $\alpha$  for the R3 solidstate model with input parameters,  $E=220$  kJ/mol and  $A=1.25\times10^{15}$  min<sup>-1</sup>. (c) Eq. (1b) isoconversional logarithmic intercepts as a function of  $\alpha$  for the D4 solid-state model with input parameters,  $E=220$  kJ/mol and  $A=1.25\times10^{15}$  min<sup>-1</sup>.

with experimental multi-heating rate TG/DSC data, one should use the characteristic  $\alpha$ -dependency of the intercept of the logarithmic isoconversional plot to establish by pattern recognition the probable  $f(\alpha)$ mechanistic parameter. If this is inconclusive, and





furthermore, if  $E_\alpha$  is not constant, then one is probably dealing with either a change in mechanism as the reaction proceeds or a more compex situation such as a mutually independent multiple, competitive or consecutive reaction system. Using these isoconversionally derived parameters, the logarithmic intercepts of the integral Eq. (3) were calculated and are listed in Table 5 for the several models and displayed in Fig. 6. As is seen, there is noticeable curvature at ca.  $\alpha$ <0.1 dependent upon the model. In general, as shown in Table 6, there is an approximate linear relationship between the logarithmic intercept and  $\alpha$ .



Table 6 Summary of 'model-free kinetics' equation  $\ln\left[\frac{A_{\alpha} \times R}{\right]}$  $[E_\alpha \times g(\alpha)]\} = K_1 + K_2 \times \alpha$ 

Model	$\alpha$ -range	# of pairs	$K_1$	$K_2$	$r^2$
A <sub>2</sub>	$0.05 - 0.95$	19	25.7264	$-1.8387$	0.9686
A <sub>3</sub>	$0.05 - 0.95$	19	25.3806	$-1.3098$	0.9608
R <sub>2</sub>	$0.05 - 0.95$	19	27.5032	$-3.0338$	0.9510
R <sub>3</sub>	$0.05 - 0.95$	19	27.9478	$-3.2670$	0.9376
D2	$0.05 - 0.95$	19	29.8017	$-5.8799$	0.9519
D <sub>3</sub>	$0.05 - 0.95$	19	31.3600	$-6.5453$	0.9594
D <sub>4</sub>	$0.05 - 0.95$	19	31.3385	$-6.0672$	0.9582

#### 3. Experimental data calculations

The application of the isoconversional analysis procedure to the three steps in the sequential degradation of calcium oxalate monohydrate, thermogravimetrically is presented [6]. The experimental TG data, previously described, are shown in Fig. 7. Table 7 summarizes the data at  $\Delta \alpha$  intervals from 0.05 to 0.95. As previously indicated [6], the input data were obtained by fitting the experimental TG data as a function of temperature, monitored over a 100% change in heating rates, to an *n*th order model using the instrumental system software. In agreement with





Fig. 6. Eq. (3) isoconversional logarithmic intercepts as a function of  $\alpha$  for the solid-state model A2, A3, R2, R3, D2, D3, and D4 with input parameters,  $E=220$  kJ/mol and  $A=1.25\times10^{15}$  min<sup>-1</sup>.



Fig. 7. Experimental non-isothermal TG curves of the three steps in the thermal degradation of calcium oxalate monohydrate at three heating rates, 0.25, 2.5 and  $25^{\circ}$ C/min.

Zsako [3], the parameters  $E_n$  and  $ln(A_n)$  decreased with increase in the heating rate but the reaction order, n, remained relatively constant. Since no assumption was made as to the reaction mechanism of each step in generating the input parameter set,  $[E_n, \ln(A_n), n]$ , it is

considered to be in order in this application to use such empirically generated extent and rate of reaction figures since it has been shown that, after sample weight normalization, they were essentially superimposable on the original weight change curves. For each

$\alpha$	Step 1	$-H2O$	$r^2 \ge 0.989$	Step 2	$-CO$	$r^2 > 0.998$	Step 3	$-CO2$	$r^2 > 0.998$
			Model D4		Model R3			Model D4	
	$E_{\alpha}$ (kJ/mol)	$ln[A_{\alpha} \times f(\alpha)]$	$A_{\alpha}$ ×10 <sup>-7</sup> $(min^{-1})$	$E_{\alpha}$ (kJ/mol)	$\ln[A_{\alpha} \times f(\alpha)]$	$A_{\alpha}{\times}10^{-17}$ $(min^{-1})$	$E_{\alpha}$ (kJ/mol)	$ln[A_{\alpha} \times f(\alpha)]$	$A_{\alpha}$ ×10 <sup>-8</sup> $(min^{-1})$
0.05	99.3	26.600	410.030	256.4	39.439	0.463	229.3	25.417	12.557
0.10	92.5	24.310	86.043	258.1	39.793	0.684	218.7	24.012	6.386
0.15	89.3	23.524	46.632	259.5	40.042	0.911	211.9	23.120	4.078
0.20	86.7	22.043	27.622	263.1	40.630	1.710	208.1	22.641	3.504
0.25	84.2	21.620	16.442	261.8	40.408	1.429	204.6	22.192	2.914
0.30	82.0	20.916	10.205	265.3	40.960	2.599	201.7	21.817	2.513
0.35	81.3	20.672	9.782	262.8	40.509	1.740	199.3	21.494	2.226
0.40	80.1	20.308	8.172	264.1	40.686	2.190	196.6	21.150	1.895
0.45	78.6	19.486	6.114	264.9	40.786	2.565	195.5	20.994	1.927
0.50	77.9	19.578	5.514	265.7	40.860	2.943	193.8	20.762	1.800
0.55	77.7	19.475	5.833	267.1	41.021	3.708	192.3	20.556	1.719
0.60	76.0	18.955	4.063	267.5	41.035	4.070	191.3	20.392	1.709
0.65	76.1	18.920	4.604	266.0	40.713	3.223	189.5	20.133	1.547
0.70	74.8	18.487	3.517	266.6	40.712	3.569	188.3	19.934	1.495
0.75	74.2	18.267	3.360	267.7	40.785	4.355	187.0	19.717	1.432
0.80	72.9	17.822	2.600	268.6	40.784	5.023	186.5	19.580	1.510
0.85	72.5	17.601	2.591	267.6	40.437	4.301	184.7	19.269	1.373
0.90	71.8	17.245	2.375	268.7	40.361	5.228	183.8	19.000	1.373
0.95	70.2	16.514	1.699	268.1	39.819	4.825	182.2	18.537	1.284

Table 7 Friedman analyses: Calcium oxalate monohydrate three-step degradation [6]

step, at each of the heating rates, the Friedman plot regression analyses showed excellent correlation coef ficients, as indicated at each  $\alpha$ . One observes in Fig. 8(a) that, for all three steps,  $E_{\alpha}$  is not constant but changes monotonically with  $\alpha$ , implying that one is monitoring an effective but not a singular reaction. The isoconversional logarithmic plot intercepts, shown in Fig. 8(b), indicate that the overall reactions conform best to the models:

- Step  $1 -$ loss of water  $-$  D4
- Step  $2 \text{loss of carbon monoxide} R3$
- Step  $3 \text{loss of carbon dioxide} D4$

as originally proposed, based upon `peak analysisdetermined' models [6]. Fig. 8(c) shows the computed logarithms of the pre-exponential factors.

## 4. Conclusions

 The procedure described should be considered as integral-differential, since its use necessitates integral methods in generating the Friedman data for the isoconversional analysis. Isoconversional analyses of modelled solid-state reactions demonstrate that the intercepts of logarithmic rate equations mirror exactly the characteristic  $f(\alpha)$  functions describing the reaction mechanism, confirming the utility of the procedure in assisting the investigator to ascribe a mechanism descriptive of monitored non-isothermally generated thermoanalytical behavior.

- The isoconversional logarithmic intercept of the integral Eq. (3) does exhibit  $\alpha$ -variant behavior in a manner similar to that shown for the comparable term in the more simple differential Eq. (1b) investigated in this study.
- Applications of the procedure in the analysis of non-isothermally generated experimental degradative data show that, in spite of the fact that the calculated effective reaction activation energy can vary slightly as the reaction proceeds to conclusion, the logarithmic plot intercept conforms well to realistic  $f(\alpha)$  functional behavior.



Fig. 8. (a) Isoconversional activation energy- $\alpha$  profiles for the three steps in the thermal degradation of calcium oxalate monohydrate. (b) Isoconversional logarithmic intercept- $\alpha$  profiles for the three steps in the thermal degradation of calcium oxalate monohydrate. (c) Isoconversional logarithm of the pre-exponential factor- $\alpha$  profiles for the three steps in the thermal degradation of calcium oxalate monohydrate.

- The effective first and third stages in the thermal degradation of calcium oxalate monohydrate appear to conform to the three-dimensional diffusion (Ginstling–Brounshtein D4) model, while the second stage appears to conform to the threedimensional phase boundary movement (R3) model. (See Elder [1] for detailed references.)
- It is considered unnecessary in modelling solidstate degradative reaction systems to employ as input parameters decreasing activation energies pre-exponential factors, and increasing reaction orders, rather than constant values, at different heating rates, as has been proposed [3], and it is considered that there is virtue in the use of the referred-to `many curves' methods.

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