

Note

A NUMERICAL PROCEDURE FOR CALCULATING CONSTANTS FOR THE ANTOINE EQUATION BY THE FIXED POINT METHOD

J.E. HOUSE, Jr.

Department of Chemistry, Illinois State University, Normal, IL 61761 (U.S.A.)

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The representation of the vapor pressure of a liquid as a function of temperature is a frequently encountered problem [1]. Although a number of different equations are used, the Antoine equation

$$\log p = A - \frac{B}{C + t} \quad (1)$$

where A , B , and C are constants, is perhaps the best of the three parameter equations for fitting vapor pressure data [1–5]. Furthermore, the constants A , B , and C are useful in calculating other thermodynamic quantities. However, determining the values for the constants is not a simple procedure and several methods have been discussed [3,4]. While the most accurate method is a nonlinear least squares approach, it is mathematically the most complex to perform. The least desirable method is to use the vapor pressures at three temperatures and solve the three equations for A , B , and C . Thompson [3] described a graphical method based on choosing one (p, t) point as being without error. While this might at first seem to be a gross approximation, it is not always so since the boiling point is usually accurately known. Thus, the method of Thompson can provide values for A , B , and C that are nearly as accurate as those from least squares methods. Furthermore, the graphical method of Thompson is easily implemented numerically using a programmable calculator of medium capacity. We describe here the procedure for this numerical evaluation of the Antoine constants.

METHOD

For simplicity, we begin by letting $y = \log p$. If it is assumed that the exact data pair is (y_0, t_0) , then

$$y_0 = A - \frac{B}{C + t_0} \quad (2)$$

All the other data are represented by

$$y_i = A - \frac{B}{C + t_i} \quad (3)$$

from which

$$B = -(y_i - A)(C + T_i)$$

Then

$$y_i - y_0 = (y_i - A)(C + t_i) \left[\frac{1}{C + t_i} - \frac{1}{C + t_0} \right] \quad (4)$$

or

$$y_i - y_0 = y_i - A - \frac{(y_i - A)(C + t_i)}{(C + t_0)} \quad (5)$$

Equation (5) simplifies to

$$y_i - y_0 = \frac{(t_0 - t_i)(y_i - A)}{(C + t_0)} \quad (6)$$

which is then placed in the form for computation as [3]

$$\frac{y_i - y_0}{t_i - t_0} = \frac{A}{C + t_0} - \frac{y_i}{C + t_0} \quad (7)$$

If we let $F_i = (y_i - y_0)/(t_i - t_0)$, a linear relationship exists between F_i and y_i with a slope of $-1/(C + t_0)$ and an intercept of $A/(C + t_0)$. Thus, A and C can be evaluated. However, a better way of calculating A and B is from eqn. (3) by using linear regression of y_i and t_i when the value of C has been obtained. Thus, a linear relationship exists between y_i and $1/(C + t_i)$ for which the slope is $-B$ and the intercept is A .

The program

In order to handle N data pairs, $2N$ registers are required. Additionally, most programmable calculators utilize six data registers for linear regression. Thus, $(2N + 6)$ registers are required in addition to some sort of loop control. The program described here for the Texas Instruments TI-59 calculator can load 20 (p_i, t_i) data pairs without repartitioning.

To program the analysis, it is necessary that linear regression be performed twice. The first case uses F_i and y_i . For N data pairs, there will be $(N - 1)$ values of F_i and y_i . In performing the second linear regression, all the data are used with y_i vs. $1/(C + t_i)$ so that there are N data pairs. Thus, the loop control must be changed during program execution. A data counter is built into the data storage routine and the loop control is changed as required during processing.

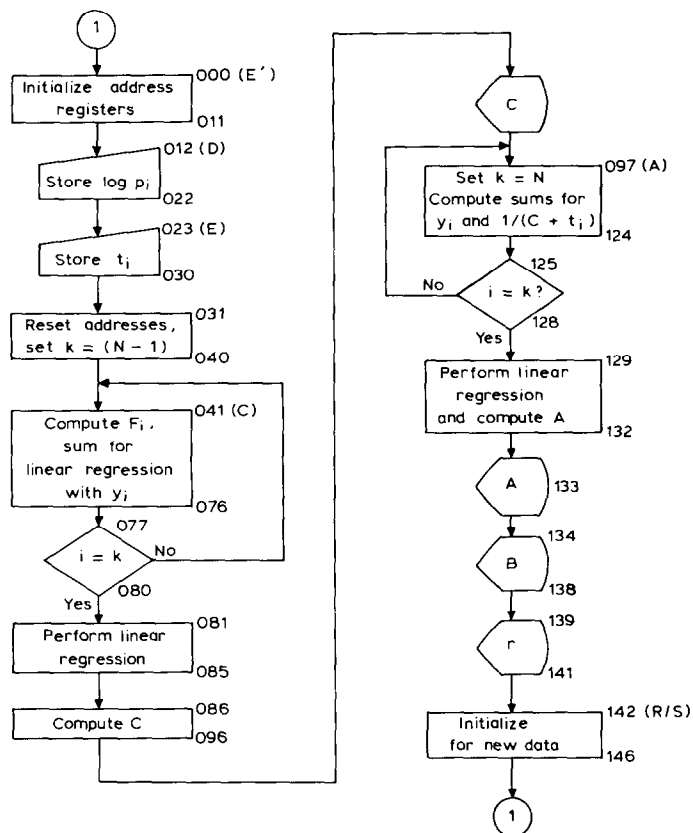


Fig. 1. Flowchart for computing Antoine constants. Numbers refer to steps in the program and letters refer to program labels.

The initialization consists of clearing memories (CMS) and pressing key E', a label to execute a subroutine to set initial address registers. Next, the p_i values are entered pressing key D after each. They are converted to $\log p_i$ and stored. Finally, the t_i values are entered pressing key E after each. Computation begins by pressing key C which results in the value of C being displayed. Pressing key A causes the computation of A and pressing key B causes the computation of B. Pressing key R/S computes the value of the correlation coefficient for the fit of the data to eqn. (1). Pressing the R/S key again clears the register holding the number of data pairs and initializes the address registers for a new data set. Figure 1 shows a flowchart of the computation and a complete program listing is given in the Appendix.

RESULTS

In order to determine the accuracy of the program, vapor pressure data for several types of compounds were studied [6-9]. In each case, the entire

TABLE I
Antoine constants for several compounds

Compound	Calculated using the program ^a			Published values			Ref.	
	A ^b	B	C	r ^c	A ^b	B		C
Cycloheptane	5.8584	1334.75	216.753	0.9(7)619	5.8568	1333.78	216.644	6
Cyclodecane	5.9879	1683.03	207.918	0.9(7)629	5.9813	1679.01	207.557	6
Cyclohexene	5.8663	1218.38	222.760	0.9(8)09	5.8724	1221.90	223.172	7
Propyl acetate	6.0196	1284.49	208.840	0.9(8)31	6.0199	1284.67	208.860	8
Methyl acetate	6.0304	1135.21	216.691	0.9(7)340	6.0876	1165.80	220.246	8
Butyronitrile	6.1461	1462.88	225.354	0.9(7)678	6.1291	1452.08	224.186	7

^a Using the vapor pressure data given in the refs. No data points were omitted.

^b Computed using vapor pressure in the same units as originally reported (kPa, mm Hg, cm Hg).

^c Number in parentheses is the number of nines following the decimal point. 0.9(7)619 is actually 0.9999999619, etc.

set of data was used as reported with no attempt being made to remove data which were less accurate. In all cases, the highest (p, t) pair listed was assigned as (p_0, t_0) . For other data sets for which the calculated results are not shown, it was frequently found that selecting a different pair as (p_0, t_0) improved the accuracy of the results. The results of the computations are shown in Table 1. For most purposes, the accuracy of ± 0.005 in A , $+0.5$ in C , and ± 1.0 in B is sufficient [3]. The results shown in Table 1 indicate that the calculator program implementing the numerical equivalent of Thompson's graphical method provides a sufficient level of accuracy. The computation is completed in a total computing time of 50 sec (for a set of 10 data pairs) and the computed constants enable data to be presented using the Antoine equation rather than the usual $\log p = A + B/T$ form.

REFERENCES

- 1 J.H. Weber, Chem. Eng., 86 (24) (1979) 111.
- 2 C. Antoine, C.R. Acad. Sci., 107 (1888) 681.
- 3 G.W. Thompson, Chem. Rev., 38 (1946) 1.
- 4 P.E. Liley and W.R. Gambill, in R.H. Perry and C.H. Chilton (Eds.), Chemical Engineers Handbook, McGraw-Hill, New York, 1969, Sec. 3, p. 246.
- 5 W.N. Trump, Comput. Chem., 4 (1980) 117.
- 6 E.F. Meyer and C.A. Hotz, J. Chem. Eng. Data, 21 (1976) 274.
- 7 E.F. Meyer and R.D. Hotz, J. Chem. Eng. Data, 18 (1973) 359.
- 8 E.F. Meyer, M.J. Awe and R.E. Wagner, J. Chem. Eng. Data, 25 (1980) 371.
- 9 E.F. Meyer, T.A. Renner and K.S. Stec, J. Phys. Chem., 75 (1971) 642.

APPENDIX

TI-59 Program for computing Antoine constants

000	76	LBL	037	01	1	074	54)	111	85	+
001	10	E'	038	95	=	075	54)	112	43	RCL
002	02	2	039	42	STO	076	78	$\Sigma+$	113	16	16
003	00	0	040	00	00	077	97	DSZ	114	54)
004	42	STO	041	36	PGM	078	00	00	115	35	1/X
005	10	10	042	01	01	079	00	00	116	32	$X \geq T$
006	04	4	043	71	SBR	080	45	45	117	73	RC*
007	00	0	044	25	CLR	081	60	OP	118	10	10
008	42	STO	045	01	1	082	12	12	119	78	$\Sigma+$
009	11	11	046	44	SUM	083	42	STO	120	01	1
010	92	RTN	047	10	10	084	14	14	121	44	SUM
011	91	R/S	048	44	SUM	085	32	$X \geq T$	122	10	10
012	76	LBL	049	11	11	086	42	STO	123	44	SUM
013	14	D	050	73	RC*	087	15	15	124	11	11
014	28	LOG	051	10	10	088	94	+/-	125	97	DSZ
015	72	ST*	052	42	STO	089	35	1/X	126	00	00
016	10	10	053	08	08	090	75	-	127	01	01

017	01	1	054	32	X ≥ T	091	43	RCL	128	08
018	44	SUM	055	73	RC*	092	40	40	129	69
019	10	10	056	11	11	093	95	=	130	12
020	69	OP	057	42	STO	094	42	STO	131	42
021	27	27	058	09	09	095	16	16	132	19
022	91	R/S	059	53	(096	91	R/S	133	91
023	76	LBL	060	53	(097	76	LBL	134	76
024	15	E	061	43	RCL	098	11	A	135	12
025	72	ST*	062	08	08	099	43	RCL	136	32
026	11	11	063	75	-	100	07	07	137	94
027	01	1	064	43	RCL	101	42	STO	138	91
028	44	SUM	065	20	20	102	00	00	139	69
029	11	11	066	54)	103	10	E'	140	13
030	91	R/S	067	55	÷	104	36	PGM	141	91
031	76	LBL	068	53	(105	01	01	142	00
032	13	C	069	43	RCL	106	71	SBR	143	42
033	10	E'	070	09	09	107	25	CLR	144	07
034	43	RCL	071	75	-	108	53	(145	61
035	07	07	072	43	RCL	109	73	RC*	146	10
036	75	-	073	40	40	110	11	11		