Molecular symmetry depresses the entropy of fusion of organic molecules with regard to their expected values when comparing structural isomers

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Abstract

Using published data, the symmetry effect on the entropy of fusion, ΔS_f, of rigid organic molecules has been investigated by comparing structural isomers. Because ΔS_f is dependent on the enthalpy of fusion, ΔH_f, [A.S. Gilbert, Thermochim. Acta 339 (1999) 131–142] this dependence must be allowed for as ΔH_f values can vary quite widely. When this is done, it is found that more symmetric isomers usually have lower than expected values of ΔS_f, in relation to ΔH_f, than their less symmetric counterparts. This is the reason that more symmetric isomers generally melt at higher temperatures, a fact noted by many workers over the years.
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1. Introduction

Many workers over the years have drawn attention to the fact that the more symmetric examples of structural isomers of organic compounds almost always possess higher melting points than their less symmetric counterparts. The subject continues to generate interest and speculation [1–4] (and see references therein).

Molecular symmetry here is defined by the rotational symmetry number, σ, for reasons that will become evident below. This parameter is given by the number of indistinguishable orientations of the molecule under rotation. Thus $\sigma = 1$ for 1,2-bromochlorobenzene, but is equal to 2 for the 1,4-isomer and 4 for 1,4-dichlorobenzene.

The melting point, T_{mp}, is related to the entropy of fusion, ΔS_f, and the enthalpy of fusion, ΔH_f, according to the simple thermodynamic relation

$$T_{mp} = \frac{\Delta H_f}{\Delta S_f}. \tag{1}$$

It is usually rearranged such that the entropy is given in terms of ΔH_f and T_{mp} which are the experimentally measured quantities

and this allows ΔS_f to be readily obtained. It is obvious that, in isolation, either ΔH_f must increase or ΔS_f must decrease for T_{mp} to rise.

1.1. Possible ways for symmetry to play a role

The entropy of fusion of an organic compound is of general interest in providing insight into the various molecular factors that determine the melting point and a number of methods have been developed to predict it. A well known scheme [5–7] takes explicit account of symmetry considering that the more (indistinguishable) ways that the molecule can be fitted into the crystal the greater is its entropy and thus the smaller the change on fusion compared to the situation in the melt where all orientations are allowed. Thus for the case of rigid molecules (i.e. those that do not possess elements of structure that can take up multiple conformations in the melt) ΔS_f is given by the simple formula

$$\Delta S_f = C - R \ln \sigma \tag{2}$$

where C is a constant put to $56.5 \, \text{J K}^{-1} \, \text{mol}^{-1}$ (after Walden's Rule [8]) and R is the gas constant [5–7].

An alternative suggestion is that symmetry acts instead to lower the entropy of the melt [9] thereby reducing ΔS_f by different means. This assumes some degree of free rotation in the liquid, the rotational entropy of the symmetric molecules being

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then reduced by an amount $R \ln \sigma$ due to certain rotational levels being forbidden.

However despite such theories indicating that ΔS_f should be lower for more symmetric isomers there are many instances where this is not so.

Unfortunately there is no simple theory relating symmetry to enthalpy of fusion. The hypothesis that more symmetric compounds should in general fit more compactly into the crystal lattice to give greater cohesion has been considered [2,4] and seems to have some validity. Yet there are many exceptions to this where the more symmetric isomers possess lower values of ΔH_f but still melt at higher temperatures. This inconsistency in ΔH_f is evidently a reflection of the subtle interplay of many factors when complex molecules pack together in crystals [2,4].

1.2. Entropy–enthalpy compensation

It has recently been discovered [10,11] that ΔS_f varies in a linear fashion with ΔH_f (entropy–enthalpy compensation) for many different types of rigid organic compounds that are not electrolytes and that Walden's Rule [8], which states that ΔS_f for such compounds is constant, is somewhat of an illusion. The relationship can be simply described as

$$\Delta S_f = a + b \Delta H_f \tag{3}$$

where a and b are positive constants. The apparent constancy of ΔS_f is a consequence of the fact that collections of compounds tend to be clustered in terms of ΔH_f which in turn is due to clustering of the molecular weights [10].

This dependency of entropy on enthalpy means that it is only appropriate to ask whether symmetry significantly affects ΔS_f in isolation as any effect on ΔH_f will necessarily influence the former. To do this requires that the entropy–enthalpy dependence must somehow be allowed for when comparing isomers.

2. Examination of literature data

Fortunately there exists a considerable body of published information and this has been utilised in this work to investigate whether symmetry exerts any systematic effects by examining one to one comparisons of structural isomers. Data was taken from two large databases of melting points, enthalpies of fusion and temperatures and enthalpies of first order solid–solid, s–s, transitions of organic compounds [12,13].

In order to reduce complications, only rigid molecules were considered, i.e. those not possessing, for example, flexible methylene chains that would be expected to take up multiple conformations in the liquid state that are not seen in the solid.

In addition, compounds with groups that might engage in H-bonding such as hydroxyls, combinations of amino with nitro, etc. were excluded. This was done to avoid any disruption to the symmetry effect that might occur from intermolecular interactions in either the solid or liquid states. Also nitro compounds

with two or more substituents on a planar ring were excluded due to uncertainties in ascertaining the correct symmetry numbers. This is because of the variable out of plane distortions of the nitro groups from steric hindrance which can be observed in both the crystalline state [14] and often to a different degree in the gas phase [15], the latter being presumably relevant to the case of the melt.

For purposes of calculating σ, hydrogens on methyl and amino substituents were considered to be invisible, therefore assuming free internal rotation in both solid and melt. Biphenyls were assumed to adopt a conformation with out of plane rings. Finally no data was used from compounds with listed s–s transitions.

Supplementary Table 1 shows ΔS_f, ΔH_f and T_{mp} data along with σ for 87 suitable compounds which were almost all aromatic. These made up 30 sets of isomers and allowed 60 one to one comparisons of a less symmetric with a more symmetric structure. The change in symmetry number in all but one of the comparisons was either 1 to 2 or 2 to 4, the difference in the logarithms of σ in these instances therefore being the same.

The more symmetric isomer was found to possess a higher melting point in 51 cases out of the 60, but a lower ΔS_f in only 29, and a higher ΔH_f in only 36 cases. The average for $\Delta\Delta S_f$ (more symmetric isomer minus the less symmetric) was just $-1.08 \, \text{J K}^{-1} \, \text{mol}^{-1}$, as against the average for the absolute differences (i.e. ignoring the signs) of $8.06 \, \text{J K}^{-1} \, \text{mol}^{-1}$ and an average ΔS_f of $50.28 \, \text{J K}^{-1} \, \text{mol}^{-1}$. Figures for the corresponding enthalpy changes ($\Delta\Delta H_f$, defined in the same way as for $\Delta\Delta S_f$, and ΔH_f) were 1558, 4550 and $16{,}110 \, \text{J mol}^{-1}$, respectively.

As the figures above indicate, both $\Delta\Delta S_f$ and $\Delta\Delta H_f$ show quite a scatter of values. There are also significant differences in ΔS_f and ΔH_f between isomers of the same σ (averages for the absolute differences of $7.04 \, \text{J K}^{-1} \, \text{mol}^{-1}$ and $3490 \, \text{J mol}^{-1}$, respectively) which points to other influences that presumably help to mask any systematic effects of symmetry.

2.1. Offsetting the effect of entropy–enthalpy compensation

The immediate conclusions to be drawn from the above is that symmetry has no or very little net overall effect on ΔS_f whereas it does appear to cause a small net increase for ΔH_f although, as already mentioned, there are many cases where $\Delta\Delta H_f$ is negative.

One feature of the data is that, whatever the effects of symmetry, it is evident that ΔS_f is correlated with ΔH_f as would be expected [10,11]. For a collection of 187 quite varied (both aromatic and non-aromatic) asymmetric compounds with no listed s–s transitions, the values of a and b have been found to be $29.91 \, \text{J K}^{-1} \, \text{mol}^{-1}$ and $0.0013 \, \text{K}^{-1}$, respectively by linear regression [11].

Fig. 1 shows a plot of ΔS_f versus ΔH_f for the compounds listed in Supplementary Table 1. Linear regression yields

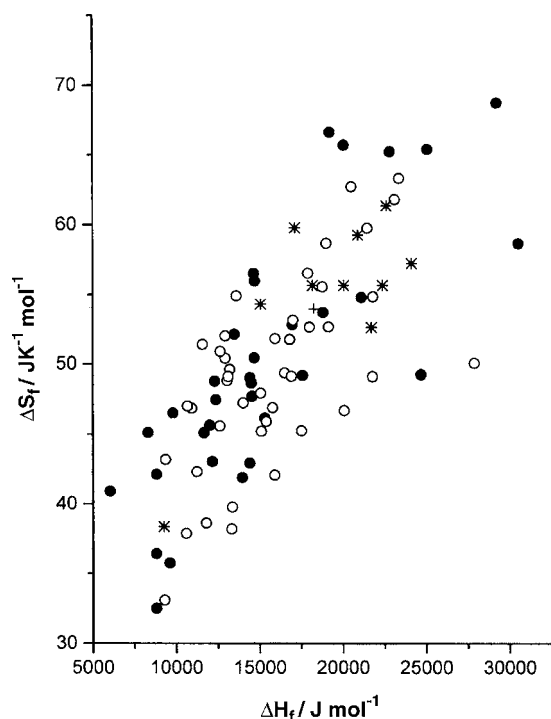$$\Delta S_f = 31.66(1.89) + 0.00156 \Delta H_f(0.000112) \tag{4}$$

Fig. 1. Plot of ΔS_f versus ΔH_f for the compounds listed in Supplementary Table 1. Compounds are distinguished by symmetry number as follows: (●) $\sigma = 1$; (○) $\sigma = 2$; (*) $\sigma = 4$; (+) $\sigma = 6$.

the figures in brackets being the standard errors. The coefficient of determination (R^2) is 0.587 with $P = 1.1 \times 10^{-16}$. This set of compounds therefore seems fairly ordinary.

If the symmetry number is included then multiple regression gives

$$\Delta S_f = 31.66(1.92) + 0.00157\Delta H_f(0.000114) + 0.03 \ln \sigma(1.21)$$
(5)

with $R^2 = 0.557$, no significant improvement which suggests that symmetry plays a very minor part if any. This is seemingly backed up by the plot itself where it can be seen that there is little if any differentiation between compounds of differing symmetry numbers. Such an exercise however throws all the compounds together regardless of structure.

Fig. 2 shows a plot of $\Delta\Delta S_f$ versus $\Delta\Delta H_f$ for the 34 one to one comparisons between isomers of the same value of σ. This is a measure of the symmetry independent effects and acts as a sort of 'control'. It indicates that comparisons only between isomers show a different and steeper dependence of entropy on enthalpy as linear regression yields

$$\Delta\Delta S_f = 0.43(1.1) + 0.001892\Delta\Delta H_f(0.000239)$$
(6)

R^2 being 0.662 and $P = 4.9 \times 10^{-9}$.

Fig. 3 shows two sets of the one to one comparisons between isomers of different symmetry. The larger set is the 36 examples where $\Delta\Delta H_f$ is positive, the other are the 24 examples where $\Delta\Delta H_f$ is negative, but for the latter $\Delta\Delta S_f$ and $\Delta\Delta H_f$ have been reversed in sign so that both sets can be plotted in the same quadrant. It is evident that the entropy–enthalpy dependence differs
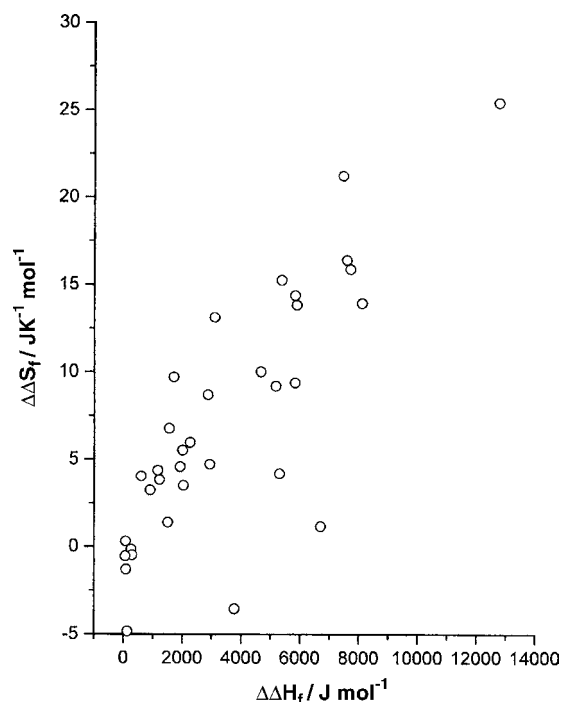
being steeper for the set where $\Delta\Delta H_f$ values are all negative therefore clearly showing that symmetry does have an effect in reducing ΔS_f.

If the symmetry independent values of the entropy change $\Delta\Delta S_f'$, i.e. the differences in ΔS_f assuming both compounds

Fig. 2. Plot of $\Delta\Delta S_f$ versus $\Delta\Delta H_f$ for one to one comparisons between isomers of the same value of the symmetry number.
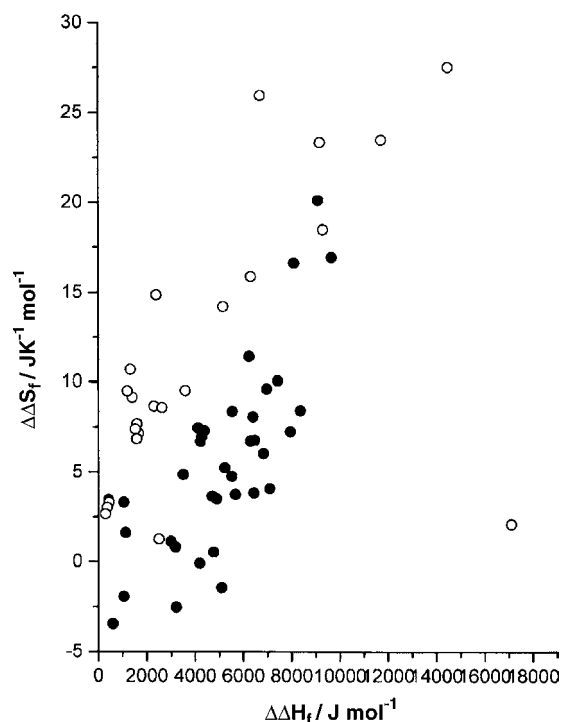

Fig. 3. Plots of $\Delta\Delta S_f$ versus $\Delta\Delta H_f$ for one to one comparisons between isomers of different symmetry number as follows: (●) compounds where $\Delta\Delta H_f$ is positive and (○) compounds where $\Delta\Delta H_f$ is negative but the signs of both $\Delta\Delta S_f$ and $\Delta\Delta H_f$ have been reversed.

in the comparison possess the same value of σ, are now calculated as

$$\Delta\Delta S_f' = b'\Delta\Delta H_f \qquad (7)$$

where $b' = 0.001892\,K^{-1}$ (the 'control' value of b) the overall symmetry independent enthalpy dependence can then be offset to give

$$\Delta S_f^{sym} = \Delta\Delta S_f - \Delta\Delta S_f' \qquad (8)$$

which should be the effect of symmetry acting in isolation from ΔH_f on the entropy change at fusion.

When this was done the vast majority of the values of ΔS_f^{sym} were found to be negative, in fact 54 out of the 60 cases. The results of the calculations are shown in Supplementary Table 2. It would be expected that ΔS_f^{sym} should show no significant correlation with $\Delta\Delta H_f$ and none was in fact found ($R^2 = 0.008$, $P = 0.51$).

3. Discussion

3.1. The gradient of the $\Delta\Delta S_f$ versus $\Delta\Delta H_f$ plot

The value of b' is about 50% larger than for b, i.e. when compounds of all types are considered [11]. An explanation for this apparent anomaly however can be offered by recourse to the proposed mechanism for entropy–enthalpy compensation [11] where theory and observation show that ΔS_f is dependent on the ratio of ΔH_f to ΔH_v the latter being defined as the enthalpy of vapourisation of the melt at T_{mp}. ΔH_v is found to increase in line with ΔH_f [11]. But structural isomers possess very similar values of enthalpy of vapourisation at the boiling point [16], which, even given slightly different melting points still show very similar values of ΔH_v. The corresponding values of ΔH_f of course differ widely. Thus the entropy of fusion can be expected to vary more rapidly with the enthalpy within a series of structural isomers, compared to when all compounds are lumped together.

3.2. Anomalies

In three of the 60 comparisons a higher T_{mp} for the symmetric isomer is associated with a positive calculated value of ΔS_f^{sym} while in another six cases negative values of ΔS_f^{sym} correspond to lower values of T_{mp}. The positive values of ΔS_f^{sym} are associated with some of the amino compounds, the methylacenaphthenes and the methylnaphthalenes.

There may be a number of explanations. The values of ΔS_f^{sym} are for whatever reasons scattered so that aberrant values could just be outliers in the distribution of ΔS_f^{sym}. Alternatively assumptions about structure could be wrong for certain of these compounds, e.g. amino groups may be symmetry destroying if there is no free rotation as gas phase data [15] implies that the NH2 group is not flat. Finally ΔS_f^{sym} does not of course have to be negative to give a higher T_{mp} if $\Delta\Delta H_f$ is sufficiently large and positive.

3.3. How does symmetry reduce ΔS_f?

The reductions in ΔS_f in relation to ΔH_f are evidently sufficiently negative to cause T_{mp} to be raised in the majority of cases of higher symmetry. The average ΔS_f^{sym} was found to be $-4.02\,J\,K^{-1}\,mol^{-1}$ and the average value of $\Delta S_f^{sym}/(\ln\sigma_1 - \ln\sigma_2)$ where σ_1 and σ_2 refer to the higher and lower symmetry numbers, respectively was calculated to be $-5.76\,J\,K^{-1}\,mol^{-1}$. This is not as high as the coefficient R (the gas constant, equal to $8.31\,J\,K^{-1}\,mol^{-1}$) on symmetry given by Eq. (2) but is a significant fraction of it.

The hypothesis that symmetry increases the entropy of the crystalline state has been rationalised [5–7] on the basis of the symmetry number. This view has been accepted by others, e.g. [17], but it is difficult to see the theoretical justification for it.

It is a central thesis of statistical mechanics that indistinguishable entities are counted only once. An interesting discussion on this point with relevance here took place a few years ago [18–20]. Therefore the rationalisation above [5–7], which effectively states that the partition function for determination of entropy in the crystal is equal to σ, cannot be correct. Another objection is that the proposed increase in entropy of the crystal $R\ln\sigma$ must, according to the above theory [5–7] and as pointed out elsewhere [17], persist at absolute zero. This would of course be a violation of the Third Law. Such violations are known, for instance the oft described example of carbon monoxide [21], but in such cases the residual entropy is small, about $5\,J\,K^{-1}\,mol^{-1}$ or so, consistent with some simple disorder in the crystal. Consider however the case of benzene, a molecule with a symmetry number of 12 [5–7]. According to the above this compound should have an entropy at absolute zero of an astonishing $20.65\,J\,K^{-1}\,mol^{-1}$.

This prediction can be easily checked by comparing the entropy of the standard state by two independent methods. Firstly it can be obtained by integration of heat capacity data from low temperatures and secondly by calculation from theory assuming the ideal gas state. Thus for the gas at 298.15 K the former method yields a value of about $269.5\,J\,K^{-1}\,mol^{-1}$ while the latter gives about $269.2\,J\,K^{-1}\,mol^{-1}$ [22]. Within experimental error it is evident that benzene has no residual entropy at absolute zero at all.

As symmetry cannot directly increase the entropy of the crystalline phase, given the arguments and evidence raised above, then it must be asked whether it can reduce the entropy of the melt instead. However it would appear that this would require a significant degree of free rotation in the liquid phase. But the general consensus seems to be that the overall motion of liquid molecules is basically oscillational in nature [23,24] at least near T_{mp}, though not without entirely excluding short free translational and rotational excursions. Indeed the proposed mechanism for entropy–enthalpy dependence [11] is based on a vibrational representation.

While accepting a small contribution from free rotation, could there be another factor in reducing ΔS_f? Some studies of aromatic symmetric organic molecules, e.g. benzofuran, fluorene and pentachloro-nitrobenzene, have found that they

show second order (glass) transitions in the solid phase [25,26]. These are not of course listed as s–s transitions [12,13] but as they involve an upwards shift in the heat capacity curve the entropy of the solid at T_{mp} is higher than it might be thus reducing ΔS_f.

Unfortunately there does not seem to be sufficient work published to gauge whether glass transitions are largely a feature of symmetric molecules or not and also whether the prevalence reflects the magnitude of σ in some way. Certainly many globular (and symmetric) molecules are well known for exhibiting transitions of various types in the solid and therefore low values of ΔS_f.

It may be that both non-'first' order transitions (solid) and free rotation (liquid) play a role in the 'symmetry effect'.

4. Conclusion

Because of the overall dependence of ΔS_f on ΔH_f, the effect of symmetry on the entropy is masked but offsetting this dependence reveals that increased symmetry lowers ΔS_f from its expected value for most compounds studied. This depression is sufficient in most cases to raise the melting point of the more symmetric isomers over their less symmetric counterparts.

Acknowledgement

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.tca.2006.10.023.

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