

Dispersive kinetic models for isothermal solid-state conversions and their application to the thermal decomposition of oxacillin

Peter J. Skrdla^{a,*}, Rebecca T. Robertson^b

^a 640 Maple Street, Westfield, NJ 07090, USA

^b Novartis Pharmaceuticals, 1 Health Plaza, East Hanover, NJ 07936, USA

Received 16 October 2006; received in revised form 2 November 2006; accepted 6 November 2006

Available online 16 November 2006

Abstract

The authors recently published works in which the use of two novel equations for modeling the dispersive kinetics observed in various solid-state conversions are described. These equations are based on the assumptions of a ‘Maxwell–Boltzmann (M–B)-like’ distribution of activation energies and a first-order rate law. In the present work, it is shown that it may be possible to expand the approach to include mechanisms other than first-order, i.e. some of those commonly encountered in the field of thermal analysis, thus obtaining ‘dispersive versions’ of these kinetic models. The application of these dispersive kinetic models to the slightly sigmoidal, isothermal conversion–time ($x-t$) data of Rodante and co-workers for the degradation of the antibiotic, oxacillin, is described. This is done in an effort to test the limitations of the proposed dispersive models in describing kinetic data which is not clearly sigmoidal (i.e. as shown in previous works). Finally, it is demonstrated that, using graphical analysis, the typically sigmoidal $x-t$ plots of first-order dispersive processes are the direct result of (asymmetric) activation energy distributions that are either ‘ \cap -shaped’ (for heterogeneous conversions) or ‘U-shaped’ (for homogeneous conversions) in appearance, i.e. when the activation energy is plotted as a function of conversion. This finding lends support to the founding hypothesis of the authors’ approach for modeling dispersive kinetic processes: the existence of ‘M–B-like’ distributions of activation energies.

© 2006 Elsevier B.V. All rights reserved.

Keywords: Oxacillin; Dispersive kinetic model; Activation energy distribution; Thermal decomposition

1. Introduction

While several common kinetic models exist in the literature for solid-state applications [1–4], their general application is often limited only to certain, simple phase transformations. These models are summarized in the first few columns of Table 1. For more complex transformations, generally those which exhibit multiple activation energies over the course of the conversion (as observed using ‘model-free’ isoconversional kinetic techniques [5–7]), these kinetic models, used either individually or in combination, often fail to adequately describe the data; a specific example will be discussed later in this work. Additionally, the traditional kinetic models are often plagued by factors including poor fits to experimental data, the inability to accurately estimate the start time of conversion, the empirical

nature of the (sometimes many) fit parameters, etc. Alternatively, the authors have recently discussed the development of two novel, simple, ‘dispersive kinetic models’ which were shown to fit/describe well the experimental data for various solid-state conversions, including a solvent-mediated polymorphic transformation and a thermal decomposition, using only two fit parameters (each of which has physical units) [8].

Dispersive kinetics are observed in chemical systems exhibiting ‘renewing environments’, i.e. typically those in which molecular dynamics impact the measured rate of conversion [10–12]. The authors believe that many solid-state conversions may fall into this category (e.g. nucleation/de-nucleation and nucleation-and-growth, rate-limited processes, in which kinetic energy quantization may be important in affecting the conversion rate e.g. [9]). Dispersive kinetics are often explained by the existence of a distribution of activation energies. This distribution of activation energies, in turn, relates a time-dependent rate constant for the conversion, which can impact the observed kinetic behavior [8–12].

* Corresponding author. Tel.: +1 908 232 0572.

E-mail address: skrdla@earthlink.net (P.J. Skrdla).

Table 1
Compilation of common kinetic models used in condensed phase kinetics and novel dispersive variants for similar applications

Classical model	Differential form, $f(x)$	Integral form, $g(x)$	Dispersive model using $-\beta$, $n=2$	Dispersive model using $+\beta$, $n=0$
Avrami–Erofe'ev (A2)	$2(1-x)[- \ln(1-x)]^{1/2}$	$[- \ln(1-x)]^{1/2}$	n/a	n/a
Avrami–Erofe'ev (A3)	$3(1-x)[- \ln(1-x)]^{2/3}$	$[- \ln(1-x)]^{1/3}$	n/a	n/a
Avrami–Erofe'ev (A4)	$4(1-x)[- \ln(1-x)]^{3/4}$	$[- \ln(1-x)]^{1/4}$	n/a	n/a
Power Law (P2)	$2x^{1/2}$	$x^{1/2}$	n/a	n/a
Power Law (P3)	$3x^{2/3}$	$x^{1/3}$	n/a	n/a
Power Law (P4)	$4x^{3/4}$	$x^{1/4}$	n/a	n/a
Contracting area (R2)	$2(1-x)^{1/2}$	$1-(1-x)^{1/2}$	n/a	n/a
Contracting volume (R3)	$3(1-x)^{2/3}$	$1-(1-x)^{1/3}$	n/a	n/a
1D diffusion (D1)	$1/2x$	x^2	n/a	n/a
2D diffusion (D2)	$[- \ln(1-x)]^{-1}$	$(1-x)\ln(1-x)+x$	n/a	n/a
3D diffusion: Jander equation (D3)	$\{3(1-x)^{2/3}\} \{2[1-(1-x)^{1/3}]\}$	$[1-(1-x)^{1/3}]^2$	n/a	n/a
Ginstling–Brounshtein (D4)	$3\{2[(1-x)^{-1/3}-1]\}$	$1-(2x/3)-(1-x)^{2/3}$	n/a	n/a
Zero-order (F0)	1	x	$x = [-\alpha t/2\beta][\exp(-\beta t^2) - 1]$	$x = [\alpha/2\beta t][\exp(\beta t^2) - 1]$
First-order (F1)	$1-x$	$-\ln(1-x)$	$x = 1 - \exp\{[\alpha t/2\beta][\exp(-\beta t^2) - 1]\}$	$x = 1 - \exp\{-\alpha/2\beta t[\exp(\beta t^2) - 1]\}$
Second-order (F2)	$(1-x)^2$	$(1-x)^{-1} - 1$	$x = 1 - \{[-\alpha t/2\beta][\exp(-\beta t^2) - 1] + 1\}^{-1}$	$x = 1 - \{[\alpha/2\beta t][\exp(\beta t^2) - 1] + 1\}^{-1}$
Third-order (F3)	$(1-x)^3$	$1/2[(1-x)^{-2} - 1]$	$x = 1 - \{[-\alpha t/\beta][\exp(-\beta t^2) - 1] + 1\}^{-1/2}$	$x = 1 - \{[\alpha/\beta t][\exp(\beta t^2) - 1] + 1\}^{-1/2}$

Dispersive kinetic models are provided only for the F-series (see text for details). n/a, not applicable.

The authors assume that the distributions of activation energies for various dispersive processes take the general functional form/shape of the Maxwell–Boltzmann (M–B) kinetic energy distribution due to the role of molecular dynamics in affecting the observed reaction kinetics; the distribution is ‘concave-down’ for heterogeneous systems (i.e. those producing acceleratory, sigmoid conversion versus time, $x-t$, curves—which are characterized by slower rates earlier in the conversion than towards the end) and ‘concave-up’ (i.e. inverted) for homogeneous systems (i.e. those producing deceleratory, sigmoid $x-t$ curves—which are characterized by faster rates earlier in the conversion, post-induction period, than towards the end) [8,9]. [An example of a dispersive homogeneous solid-state conversion is the thermal decomposition of silver permanganate e.g. [8]; it is a process which yields products from a single, homogeneous reagent. Heterogeneous solid-state conversions, on the other hand, typically involve polymorphic transformations and crystallizations, i.e. systems which involve the formation a ‘product’ condensed phase from a homogeneous/dissolved ‘reagent’ phase.] In either case, these activation energy distributions ultimately define (mathematically) a time-dependent rate ‘constant’, which the authors believe can be approximated by a specific functional form, as will be discussed later. When the time-dependent rate constant is coupled with an assumed first-order mechanism for the conversion, the dispersive kinetic model equations of the authors’ previous works are obtained (i.e. one for treating homogeneous conversions and one for use in heterogeneous applications [8,9]).

In this work, it is demonstrated that it may be possible to convert some of the traditional kinetic models shown in Table 1 into corresponding ‘dispersive kinetic models’, using the approach described above. Additionally, the application of these various dispersive models to the isothermal decomposition of the antibiotic, oxacillin, originally investigated by Rodante et al. [14–16], is shown. This particular conversion is of interest to the authors for three main reasons: (1) the data is ‘complex’ and it cannot be satisfactorily fit/interpreted using most of the traditional kinetic models, even in combination, (2) the $x-t$ data is ‘less sigmoidal’ in appearance than any other data the authors have attempted to model with their dispersive kinetic equations previously (thus, it may provide a good test of the potential limitations of the proposed dispersive kinetic models) and (3) the solid-state thermal decomposition of oxacillin, unlike other compounds the authors have previously investigated [8,9], appears to proceed via a heterogeneous mechanism rather than a homogeneous one (from visually examining the shape of the $x-t$ trends). Finally, by utilizing some recent results [9], the authors attempt to show, via graphical analysis, that the dispersive model equation for first-order heterogeneous processes (i.e. the model which is ultimately used to treat the oxacillin data) predicts a ‘M–B-like’ (i.e. asymmetric, ‘∩-shaped’) distribution of activation energies, as a function of the extent of conversion. This is an important goal because such a finding would lend support to the fundamental assumption (i.e. in the derivation of the various dispersive kinetic model equations presented in this work) of the existence of such activation energy distributions in dispersive conversions.

2. Results and discussion

2.1. Dispersive model equations for condensed phase kinetics

The differential form of the standard kinetic models in Table 1 is defined by the function, $f(x)$, where:

$$f(x) = \left(\frac{1}{k}\right) \left(\frac{dx}{dt}\right) \quad (1)$$

and dx/dt is the rate of conversion, x the degree of conversion (i.e. the amount of product formed at time t) and k is the rate constant for the process (as per the traditional Arrhenius definition). Note that the definitions of ‘ x ’ and ‘ $1 - x$ ’, for the product and reagent fractions, respectively, are reversed in the authors’ previous works [8,9,13]. The integrated forms of these reaction models, denoted $g(x)$, are also provided in the table. For these variants, $g(x)$ is defined as below:

$$g(x) = kt \quad (2)$$

The first six models (the A-series) are typical nucleation models, the P-series are geometrical contracting models, the D-series represent diffusion models and the last four (F-series) are common reaction order models [3]. Focusing on the integral forms of these models (i.e. mainly the F-series, as will be discussed later) for the purposes of this work, the equations can be converted to corresponding ‘dispersive models’, using the general expression:

$$g(x) = t^n \int k(t) dt \quad (3)$$

which introduces a time-dependent rate constant, $k(t)$, and a reaction dimension, n , to the conversion (note: the integration limits are 0 and t). Typically, $n=0$ for heterogeneous conversions where the activated state is dissolved (e.g. solvent-mediated polymorphic transformations, see Ref. [8]), but $n=2$ for homogeneous solid-state conversions, such as the thermal decomposition of AgMnO_4 [8], where the rate-limiting mechanism may involve two-dimensional de-nucleation from flat crystal surfaces e.g. [14]. Considering only isothermal conditions, we have shown elsewhere that for dispersive processes the rate constant may be approximated using a Gaussian function:

$$k(t) = \alpha \exp(\pm\beta t^2) \quad (4)$$

based on the initial assumption of a M–B distribution of activation energies [8]. In Eq. (4), α and β are assumed to be independent constants; the sign of β indicates whether the reaction is generally acceleratory (+) or deceleratory (–) in nature (the former conversions typically include heterogeneous phase changes while the latter are homogeneous solid-state reactions) [8]. Note that both α and β , which translate directly from Eq. (4) to the proposed dispersive kinetic models in Table 1 (discussed more below), each have units in the time domain. Thus, they have more physical significance than the typical empirical ‘fit parameters’ of traditional model equations [8,9]. [Essentially, these parameters are akin to traditional rate constants:

in previous works, it was discussed that α is a kind of first-order, time-independent rate constant while β describes the time-dependence of the overall rate constant, $k(t)$, for the conversion (i.e. as typically observed in sigmoidal, dispersive kinetic $x-t$ trends). Given that one rate constant is normally sufficient to describe a simple, first-order exponential (F1), it may be realistic to expect that the simplest kinetic model that can describe sigmoidal $x-t$ trends should contain at least two such parameters.] Using Eq. (4) in Eq. (3) and integrating, one obtains:

$$g(x) = \left\{ \left[\frac{\alpha\pi^{1/2}t^2}{2\beta^{1/2}} \right] [\text{erf}(\beta^{1/2}t)] \right\} \quad \text{for } -\beta \text{ and } n = 2 \quad (5)$$

$$g(x) = \left\{ \left[\frac{\alpha\pi^{1/2}}{2\beta^{1/2}} \right] [\text{erfi}(\beta^{1/2}t)] \right\} \quad \text{for } +\beta \text{ and } n = 0 \quad (6)$$

Unfortunately, these results are not entirely satisfying for kinetic modeling applications since the error function (denoted ‘erf’; $\text{erf}(\beta^{1/2}t) \equiv 2\pi^{-1/2}\beta^{1/2} \int \exp(-\beta t^2) dt$, where the integration limits are 0 and t) requires numerical solution and such capability may not be readily available in some laboratories; furthermore, Eq. (6) uses an imaginary error function, denoted as ‘erfi’. Fortunately, using an approximation for the error function integral [13], one can obtain the following expressions (which the authors believe are valid down to low integer values of t , the units of which can be selected quite arbitrarily in most kinetic modeling [9]):

$$g(x) = \left[\frac{-\alpha t}{2\beta} \right] [\exp(-\beta t^2) - 1] \quad \text{when } -\beta \text{ and } n = 2 \quad (7)$$

$$g(x) = \left[\frac{\alpha}{2\beta t} \right] [\exp(\beta t^2) - 1] \quad \text{when } +\beta \text{ and } n = 0 \quad (8)$$

Thus, for the various integral models, $g(x)$, listed in Table 1 which describe different conversion mechanisms, one can theoretically obtain corresponding dispersive kinetic variations of these models by using either Eq. (7) or Eq. (8), depending on whether the conversion is homogeneous/deceleratory or heterogeneous/acceleratory in nature (respectively). It is highlighted here, however, that it may not be meaningful to obtain ‘dispersive’ versions of the so-called Avrami–Erofe’ev or Power Law equations, for example, using the authors’ procedure, since these equations have already been shown to be obtainable directly from the dispersive kinetics approach of Plonka [10–12]. Additionally, since the ‘geometric’ and ‘diffusion’ models in Table 1 are already based on molecular motion (but that which is not quantized as per the authors’ dispersive approach), it may hold true that the only physically meaningful dispersive kinetic variations are those of the F-series models. Thus, only these dispersive models are shown in the table. Note that, to-date, the authors have only utilized the first-order reaction model, labeled F1 in Table 1, with either Eq. (7) or (8), for applications ranging from solid-state phase changes [8,9] to femtosecond gas phase reaction kinetics [13].

2.2. Isothermal decomposition of oxacillin

Rodante et al. recently described both the simple and complex thermal decompositions of some pharmaceutical compounds; most interesting to the authors are the decompositions of the antibiotics these workers investigated as these systems exhibit the most complex kinetic behavior [15–17]. For example, Rodante and co-workers found that the isothermal ‘conversion versus reduced time’ plots for oxacillin yielded consistent activation energies and Arrhenius frequency factors for the majority of the kinetic models in Table 1, despite the relatively poor data fits [17]. However, as a direct result of the poor data fits, the various kinetic treatments were not able to provide reliable parameters that could be used to estimate the half-life of decomposition. In Ref. [15], it is stated that “. . . kinetic calculations are not able to determine the decomposition mechanisms for the studied compounds. The failure of the model-fitting method to define multi-step decomposition processes . . .” was found to be a key limitation in that work. [As an aside, Rodante and co-workers also suggest that non-isothermal approaches can better reveal the true complexity of the data because a wider temperature range can be more readily studied, however, this is mainly an experimental limitation which will not be pursued in the present work.] In Ref. [16], Rodante et al. attempted to use a ‘superimposed series’ (i.e. a linear combination in time) of models to better describe the complex kinetic trends. However, this approach was only able to support the R3 and F3 models for what these workers believed to be the second and fourth steps of the decomposition, respectively; the other two proposed steps in the decomposition could not be adequately described by any of the other models.

The data from Ref [15], i.e. for the isothermal decomposition of oxacillin at different temperatures as characterized by TG (see Fig. 1), was modeled using the various dispersive kinetic equations discussed earlier (see Table 1). It was found that the

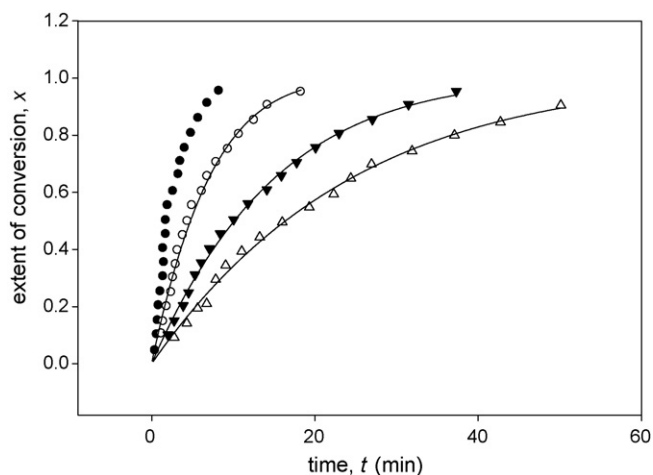


Fig. 1. Kinetic data for the isothermal conversion of oxacillin at various temperatures, taken from Ref. [15]. The data fits were performed using Eq. (9) in the text for the 225 °C (Δ), 230 °C (\blacktriangledown) and 240 °C (\circ) data; the 250 °C data (\bullet) was not fit (see text for details). The regression fits are as follows: $R^2 = 0.995$, $\alpha^0 = (6.2 \pm 4.6) \times 10^2 \text{ min}$, $\beta = (6.7 \pm 4.9) \times 10^{-5} \text{ min}^{-2}$ (Δ); $R^2 = 0.997$, $\alpha^0 = (3.2 \pm 1.4) \times 10^2 \text{ min}$, $\beta = (2.1 \pm 0.9) \times 10^{-4} \text{ min}^{-2}$ (\blacktriangledown); $R^2 = 0.990$, $\alpha^0 = (1.3 \pm 0.9) \times 10^2 \text{ min}$, $\beta = (1.1 \pm 0.7) \times 10^{-3} \text{ min}^{-2}$ (\circ). The errors represent one standard deviation.

first-order dispersive model for heterogeneous phase transformations, derived using model F1 in Eq. (8) – shown below as Eq. (9) – was the simplest dispersive model that was able to describe the data collected at 225, 230 and 240 °C, with the best precision:

$$x = 1 - \exp \left\{ \left[\frac{-\alpha^0}{t} \right] [\exp(\beta t^2) - 1] \right\}, \quad \alpha^0 \equiv \frac{\alpha}{2\beta} \quad (9)$$

The data fits using Eq. (9) were generally found to be better than those of the non-dispersive kinetic models (by comparison with the plots in Ref. [15]); of note, while the traditional F1 model appeared to provide reasonably good fits to the curves in Fig. 1 (not shown), it was found to be unable to describe some of the slightly sigmoidal $x-t$ curvature observed in the data. Unfortunately, the data collected at 250 °C was not able to be fit satisfactorily with Eq. (9), likely due, at least in part, to the limited number of data points collected early in the conversion which poorly defines the curvature of this plot. It is also important to point out that the first-order, dispersive kinetic model for homogeneous phase transformations (i.e. which can be derived by using model F1 in Eq. (7); see Table 1) was not as successful as Eq. (9) in fitting the data in Fig. 1.

In previous work [9], it was shown that a plot of $\ln(\alpha/2)$ versus $1/T$ can yield an estimate for the ‘global activation energy’ (i.e. the potential energy component of the activation energy), E_a^0 , and the Arrhenius frequency factor, A . For heterogeneous conversions, such as the thermal decomposition of oxacillin, the change in the activation energy, $E_a(t)$, as a function of the conversion time, t , may be related by the equation [9]:

$$E_a(t) = E_a^0 - RT\beta t^2 \quad (10)$$

where R is the gas constant and the other parameters are defined as before (note that β , like any traditional rate constant, is a function of T although it is not shown explicitly in the above equation; see Refs. [8,9] for a physical interpretation of the ‘fit parameters’ α and β , as they are currently understood by the authors). While E_a^0 represents the (time-independent) *potential energy* component of the activation energy, the term $RT\beta t^2$ describes the (time/conversion-dependent) *kinetic energy* component of the activation energy. Thus, analogously to the Arrhenius equation, one may write [9]:

$$\ln \left(\frac{\alpha}{2} \right) = \ln \left(\frac{A}{2} \right) - \frac{E_a^0}{RT} \quad (11)$$

From Eq. (11), the slope of a plot of $\ln(\alpha/2)$ versus $1/T$ is given by $-E_a^0/R$ and the vertical axis intercept yields a value for $\ln(A/2)$. From Fig. 2, the global activation energy was estimated at $173 \pm 9 \text{ kJ/mol}$ and the frequency factor obtained was $\sim 10^{22} \text{ s}^{-1}$. While the global activation energy is consistent with the activation energies presented in Ref. [15], the frequency factor is about six orders of magnitude higher than those reported in the reference. At present, the authors are not sure of the reason for this difference in the magnitude of A , other than the fact that it is related to the difference between the rate parameters (α and β) extracted using the proposed dispersive kinetic treatment of the data and the traditional rate constants (k) determined

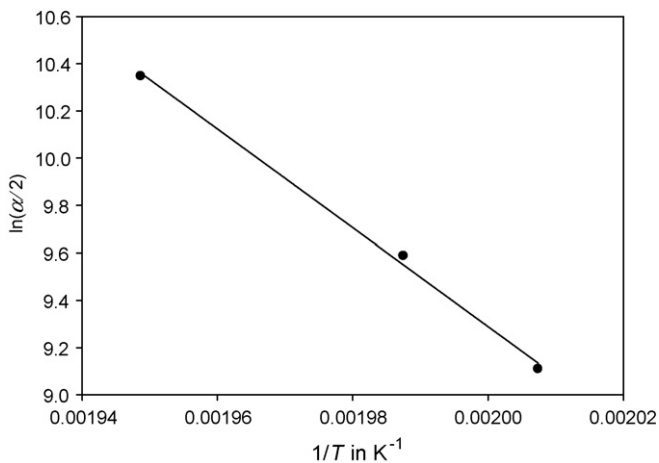


Fig. 2. Plot of $\ln(\alpha/2)$ vs. $1/T$ for the solid-state decomposition of oxacillin using the regression fit data presented in the caption of Fig. 1, for temperatures in the range 225–240 °C. From the linear regression fit of the plot, $R^2 = 0.997$, vertical axis intercept = $(5.11 \pm 0.22) \times 10^1$ and slope = $-(2.09 \pm 0.11) \times 10^4$ K. The errors represent one standard deviation.

by Rodante and co-workers. Note that after 40 min of reaction time (i.e. at ~80% conversion), at 225 °C, Eq. (10) predicts that the activation energy for the decomposition decreases by ~0.5 kJ/mol. This slight decrease in the activation energy barrier with increasing conversion/time produces the acceleratory trend in the weakly sigmoidal x - t plot of this heterogeneous conversion. Due to the fact that the variation in the activation energy is quite small (relative to the magnitude of E_a^0), the plot is only slightly sigmoidal; the converse is also true. Note that if one neglects any dispersion in $E_a(t)$, E_a becomes single-valued (as per the Arrhenius kinetic description) and the dispersive kinetic model, Eq. (9), reduces to the F1 model (which, as stated earlier, also does a reasonable job at fitting the data).

While most solid-state decompositions that the authors have modeled to-date have been homogeneous in nature, from the modeling efforts presented here it is believed that the thermal decomposition of oxacillin (i.e. the rate-determining step, r.d.s., of this process) is a heterogeneous process. This idea may be supported by the exothermic decomposition peak (onset at 231 °C, observed by DSC) which is accompanied by a 59% mass loss (by TG) due to the formation of volatile product(s) [15]. Essentially, the authors believe that, for this decomposition, the r.d.s. involves the nucleation-and-growth of product species from ‘dimensionless’ (i.e. ‘dissolved’) starting materials. Similarly, from a survey of the literature, the thermal decomposition of TNT also appears to be a heterogeneous dispersive kinetic process (producing acceleratory x - t sigmoids); this may be due to the nucleation-and-growth of reaction centers in the liquid portion of the confined sample [18].

2.3. Eq. (9) predicts an asymmetric, concave-down distribution of activation energies for heterogeneous condensed phase conversions

Workers in the field of thermal analysis, such as those using isoconversional kinetic methods e.g. [5–7] to investigate various

solid-state processes, are often interested in how the activation energy changes with the extent of conversion, x , rather than with time, t . That is because isoconversional methods typically yield values of E_a as a function of x , not t . While the idea that the activation energy may change during the course of conversion has been a topic of much controversy in the literature in the recent past e.g. [19], in the authors’ view this idea is supported by both experimental data (as will be discussed below) and by the dispersive kinetics literature [10–12]. Recently, Vyazovkin has clarified that the observation of multiple activation energies during isoconversional analyses are not necessarily due to experimental errors or errors associated with the data treatment(s), but, rather, they may be the result of multiple conversion mechanisms for a given phase transformation [1]. The authors would like to expand on this idea to include conversions that predominantly occur via a single mechanism, i.e. for elementary conversions (i.e. $R \rightarrow P$) or those that react with a single r.d.s. over the course of the conversion, but which are dispersive in nature.

From Eq. (9), one can see that x is a non-linear function of t ; thus, an analytical solution for $t(x)$ is non-trivial. However, one can use the ‘exact solution’ for x , i.e. Eq. (6) (assuming $g(x) = -\ln(1-x)$), directly to yield:

$$t = \beta^{-1/2} \operatorname{erfi}^{-1}[-2\pi^{-1/2}\alpha^{-1}\beta^{1/2} \ln(1-x)] \quad (12)$$

where ‘erfi⁻¹’ denotes an inverse imaginary error function. Using Eq. (12) in Eq. (10), one finds that:

$$E_a(x) = E_a^0 - RT\{\operatorname{erfi}^{-1}[-2\pi^{-1/2}\alpha^{-1}\beta^{1/2} \ln(1-x)]\}^2 \quad (13)$$

for the conversion-dependence of the activation energy for a first-order, heterogeneous, dispersive process. By plotting Eq. (13), using various values of α and β (as well as E_a^0 and T) – not shown here – one may observe the general shape of the E_a versus x plots, as predicted by the equation. More simply, however, one may use graphical arguments to show that $E_a(x)$ can be expected to exhibit the general characteristics of a M–B kinetic energy distribution; i.e. an asymmetric, concave-down curve. As mentioned in the Introduction, this is an important goal because the authors’ fundamental assumption in the original development of Eq. (9) was a M–B-like distribution of activation energies [8]. Thus, the graphical analysis may lend support to this assumption and provide a level of ‘self-consistency’ [9] for the derivations of the dispersive kinetic model equations presented herein.

In Fig. 3, typical reagent ($1-x$) and product (x) fractions, modeled as a function of t using Eq. (9), are depicted schematically. For heterogeneous conversions, these curves generally take on the shape of asymmetric, acceleratory sigmoids e.g. [8,9,13]. Considering only the x fraction in Fig. 3 (a), one may approximate the shape of the corresponding t versus x plot in (b) by using the function $t = \operatorname{arctanh}(x)$, as shown in (c); note that the $\operatorname{tanh}(x)$ function was first proposed by Prout and Tompkins for modeling solid-state kinetics [20]. However, as the $t = \operatorname{arctanh}(x)$ function has an inflection point at (0, 0), whereas the schematic x versus t curve has an inflection point at some location (t_i, x_i) in the positive quadrant for both variables, one may choose to (arbitrarily) translate the data curves in (a) to make the inflection point coincide with the origin. Doing so, one finds that it

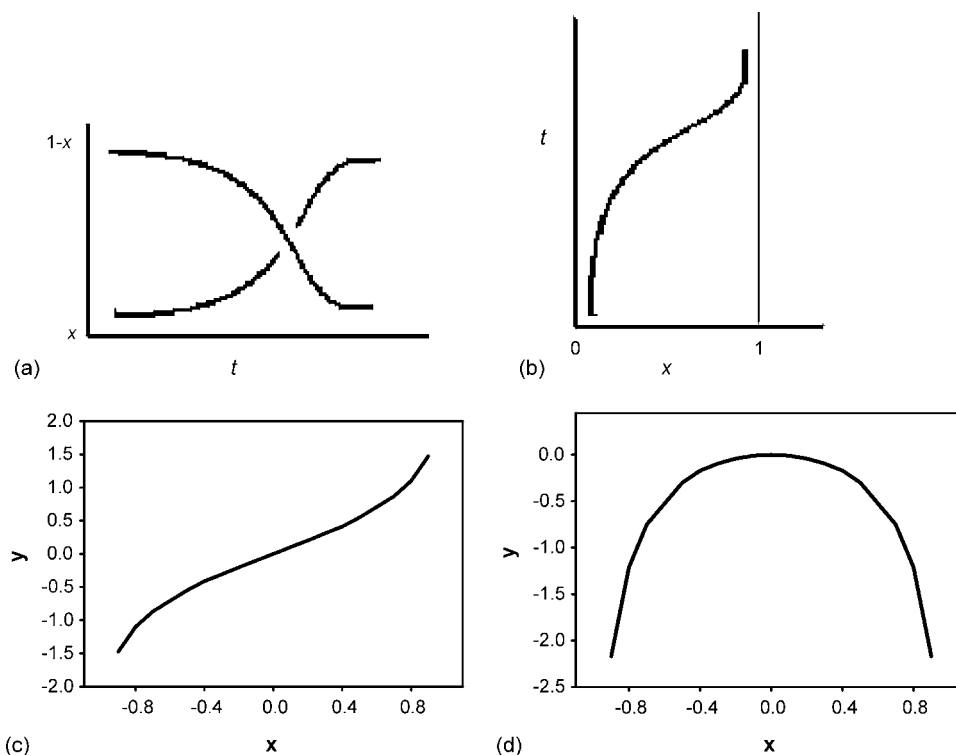


Fig. 3. Schematic depiction of: (a) a general heterogeneous solid-state conversion; both $1-x(t)$ and $x(t)$ trends are shown, (b) the predicted shape of the $t(x)$ function for the system in (a); the line $x=1$ is added simply to guide the reader, (c) a plot of $y = [\text{arctanh}(x)]$, on an x, y -coordinate; an approximation for the shape of the curve in (b), and (d) a plot of $y = -[\text{arctanh}(x)]^2$, on an x, y -coordinate, as an approximation to the shape of $E_a(x)$ for the system described in (a).

is possible to consider all $x \in (-1 < x < 1)$; it also holds that as $t \rightarrow \infty$, $x \rightarrow 1$ and as $t \rightarrow 0$, $x \rightarrow -1$. Using $t = \text{arctanh}(x)$ (or, more precisely, $t = [\text{arctanh}(x - x_i) + t_i]$) in the second term on the r.h.s. of Eq. (10), i.e. for the kinetic energy portion of $E_a(t)$, one discovers the source of the non-linear behavior in $E_a(x)$; this is demonstrated graphically in Fig. 3(d) by the plot of the function, $y = -[\text{arctanh}(x)]^2$, on an x, y -coordinate. Note that this plot is symmetrical due to the fact that the symmetrical $\text{arctanh}(x)$ function was used to approximate the asymmetrical data in Fig. 3(a). Thus, it is not difficult to envision that an asymmetric $t(x)$ function would correspondingly yield an asymmetric, ‘concave-down’ $E_a(x)$ function, much like a M–B distribution. The authors point out, however, that the $E_a(x)$ function may not be precisely described by Eq. (12), since even this so-called ‘exact solution’ contains some fundamental assumptions, in particular, those which are introduced in defining the time-dependent rate constant in Eq. (4); see Ref. [8]. It may be of interest to highlight the fact that Eq. (4) is ultimately responsible for allowing the dispersive kinetic models discussed in this work (and in previous works) to fit asymmetric $x-t$ sigmoids; this can be seen by considering a general Gaussian $k(t)$ versus t plot, centered at the origin (not shown), and only ‘real’ values of $t \geq 0$: at both low and high values of t , the degree of curvature of the plot is very different, thus introducing the desired asymmetry to the kinetic model functions.

The authors believe that this graphical demonstration supports their standing hypothesis regarding the existence of a ‘M–B-like’ distribution of activation energies, in the case of heterogeneous, dispersive conversions (i.e. those typically

producing acceleratory $x-t$ sigmoids). Such concave-down, asymmetric $E_a(x)$ distributions have been observed experimentally e.g. [4,18]. Similarly, for homogeneous, dispersive conversions (i.e. those producing deceleratory $x-t$ sigmoids), where one may apply the relation [9]: $E_a(t) = E_a^0 + RT\beta t^2$, the $E_a(x)$ function can be expected to be an asymmetric, concave-up distribution, consistent with the experimental observations in some of the thermal analysis literature e.g. [21,22].

3. Conclusions

Semi-empirical, ‘dispersive kinetic’ versions of some of the traditional solid-state models may be adapted using the novel approach described in this work. These dispersive models are thought to be applicable to cases in which quantized molecular kinetic energies impact the r.d.s. of the conversion (e.g. in the solid-state, processes that are rate-limited by nucleation/denucleation). Thus, they may potentially be of broad applicability; this idea is supported both by the present work and by previous papers by the authors. The dispersion in the activation energy, which is common to dispersive kinetic treatments and is fundamentally caused by molecular motion, may help to explain the variation in the activation energy with conversion that has been described in some recent works aimed at studying various solid-state conversions using isoconversional kinetic approaches.

The data for the thermal decomposition of oxacillin was found to be able to be fit by the heterogeneous, first-order, dispersive kinetic model proposed by the authors, despite producing only slightly (acceleratory) sigmoidal $x-t$ trends. Despite

this atypical (i.e. F1-like) appearance of the data curves (i.e. relative to other dispersive processes that the authors have modeled), isothermal experiments performed at multiple temperatures were able to provide reasonable estimates of both the potential (time-independent) and kinetic (time-dependent) energy portions of the activation energy for this conversion, indicating that the model is robust. The dispersion (i.e. change) in the activation energy as a function of the extent of conversion/time was found to be small, supporting the weakly sigmoidal appearance of the observed $x-t$ trends.

Using a schematic $x-t$ curve for a general heterogeneous, first-order dispersive process, coupled with the elementary mathematical ‘fit function’, $t = \text{arctanh}(x)$, it was possible to provide graphical evidence to support the founding assumption of the authors’ dispersive kinetic models: the sigmoidal $x-t$ trends exhibited by typical dispersive processes are the direct result of the existence of an asymmetric distribution of activation energies, $E_a(x)$, which may take the functional form of the M–B distribution.

References

- [1] S. Vyazovkin, *J. Therm. Anal. Calorim.* 83 (2006) 45.
- [2] M.E. Brown, *Thermochim. Acta* 300 (1997) 93.
- [3] A. Khawam, D.R. Flanagan, *J. Pharm. Sci.* 95 (2006) 472.
- [4] J. Malek, T. Mitsuhashi, J.M. Criado, *J. Mater. Res.* 16 (2001) 1862.
- [5] S. Vyazovkin, *New J. Chem.* 24 (2000) 913.
- [6] S. Vyazovkin, C.A. Wight, *Thermochim. Acta* 340/341 (1999) 53.
- [7] S. Vyazovkin, *J. Comput. Chem.* 18 (1997) 393.
- [8] P.J. Skrdla, R.T. Robertson, *J. Phys. Chem. B* 109 (2005) 10611.
- [9] P.J. Skrdla, *J. Phys. Chem. A* 110 (2006) 11494.
- [10] J. Bednarek, A. Plonka, B. Pacewska, J. Pysiak, *Thermochim. Acta* 282/283 (1996) 51.
- [11] A. Plonka, *Annu. Rep. Prog. Chem. Sect. C* 97 (2001) 91.
- [12] A. Plonka, *Kinet. Catal.* 37 (1996) 661.
- [13] P.J. Skrdla, *Chem. Phys. Lett.* 419 (2006) 130.
- [14] I.V. Markov, *Crystal Growth for Beginners*, World Scientific, Singapore, 2003.
- [15] F. Rodante, S. Vecchio, M. Tomassetti, *Thermochim. Acta* 394 (2002) 7.
- [16] F. Rodante, S. Vecchio, V. Slovak, *Int. J. Chem. Kinet.* 35 (2003) 611.
- [17] F. Rodante, S. Vecchio, M. Tomassetti, *J. Pharm. Biomed. Anal.* 29 (2002) 1031.
- [18] G.T. Long, B.A. Brems, C.A. Wight, *Thermochim. Acta* 388 (2002) 175.
- [19] A.K. Galwey, *Thermochim. Acta* 413 (2004) 139.
- [20] E.G. Prout, F.C. Tompkins, *Trans. Faraday Soc.* 40 (1944) 488.
- [21] Z. Dong, J.S. Salsbury, D. Zhou, E.J. Munson, S.A. Schroeder, I. Prakash, S. Vyazovkin, C.A. Wight, D.J.W. Grant, *J. Pharm. Sci.* 91 (2002) 1423.
- [22] D.L. Zhou, E.A. Schmitt, G.G.Z. Zhang, D. Law, C.A. Wight, S. Vyazovkin, D.J.W. Grant, *J. Pharm. Sci.* 92 (2003) 1367.