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A NOISE FILTER FOR CALORIMETER SIGNALS

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ABSTRACT

This article describes a general method for filtering noise from thermograms which uses the previously determined autocorrelation function of the noise produced by the calorimeter apparatus together with an estimate of the second derivative of the signal being processed. The method is applied to a standard Calvet microcalorimeter, the autocorrelation function of whose noise is shown, and the variable number of points required by the filter in this case is plotted against a convenient function of the second derivative of the signal. Comparison of deconvolved calorimeter output signals with and without previous filtering shows the filter to function as desired.

INTRODUCTION

Recent articles [1,2] have reported the application of deconvolution techniques to thermograms for separating the true thermal reaction signal from distortion introduced by the calorimeter system. Such techniques are particularly sensitive to noise in the input signal. The present article describes a digital filter designed to eliminate this noise.

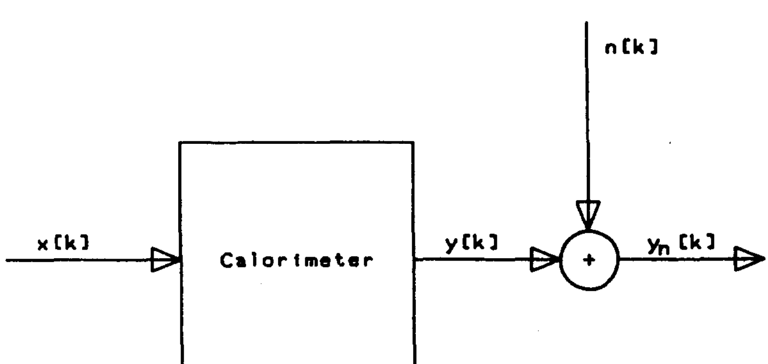

Fig. 1. Block diagram of the calorimeter system. The signal $y_n(k)$ is the measured output signal contaminated with additive noise $n(k)$.

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We assume that the noise, $n(k)$, is additive, stationary, ergodic and of mean value zero. The noisy signal $y_n(k) = u_n(kT)$ sampled from the calorimeter output $u_n(t)$ at some suitable frequency $1/T$ is, therefore, given by

$$y_n(k) = y(k) + n(k) \tag{1}$$

where $y(k)$ is the desired noiseless signal (Fig. 1), and since

$$E[n(k)] = 0, \forall k \tag{2}$$

then

$$E[y_n(k)] = y(k), \forall k \tag{3}$$

The variance σ_n^2 of $y_n(k)$ is, accordingly, given by

$$\sigma_n^2 = E\{[y_n(k) - y(k)]^2\} = E\{[n(k)]^2\} = R_{nn}(0) \tag{4}$$

where R_{nn} is the autocorrelation function of the noise, defined by

$$R_{nn}(k) = E[n(l)n(l+k)] \tag{5}$$

Since the noise is assumed to be stationary and ergodic, $R_{nn}(k)$ is given [3,4]

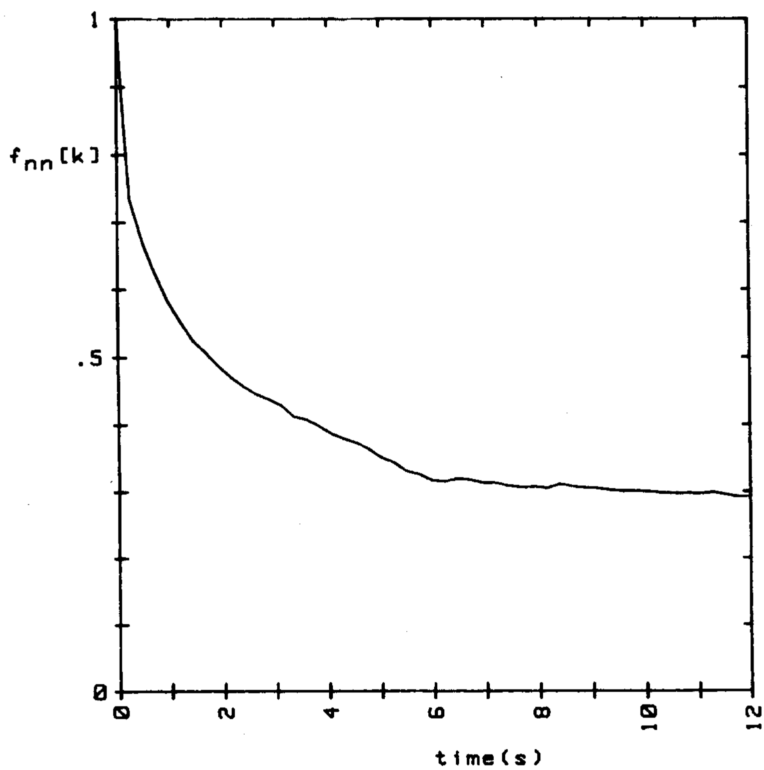

Fig. 2. Normalized mean autocorrelation function of the noise produced by a Calvet microcalorimeter.

by

$$R_{nn}(k) = \lim_{N \to \infty} \frac{1}{N} \sum_{l=1}^{N} [n(l)n(l+k)] \tag{6}$$

of course, if σ_n^2 is comparable to $y^2(k)$, the noise completely obliterates the signal to be measured.

THE FILTER

To estimate $y(k)$ we use the smoothing function

$$\hat{y}(k) = \frac{1}{2P+1} \sum_{l=k-P}^{k+P} y_n(l) \tag{7}$$

The number of points, P, over which this average is taken should be such as to minimize the mean squared error $\sigma_e^2(k)$ given by

$$\sigma_e^2(k) = E\{[\hat{y}(k) - y(k)]^2\} = E\langle\{\hat{y}(k) - E[\hat{y}(k)]\}^2\rangle + \{E[\hat{y}(k)] - y(k)\}^2 \tag{8}$$

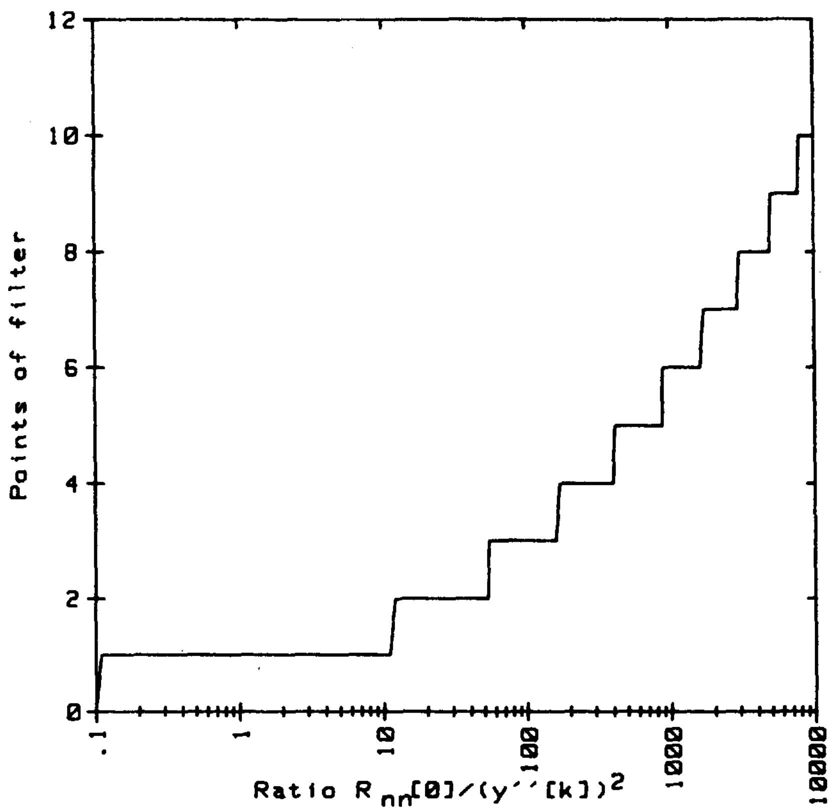

Fig. 3. Number of points required for the filter plotted against $R_{nn}(0)/[y''(k)]^2$

The second of the two terms on the right-hand side is the square of the mean error $b(k) = E[\hat{y}(k) - y(k)]$, which may be approximated (see Appendix) by

$$b(k) = \tfrac{1}{6}P(P+1)y''(k) \tag{9}$$

The first term is the variance of $\hat{y}(k)$, σ^2, given by

$$\sigma^2 = E\langle\{\hat{y}(k) - E[\hat{y}(k)]\}^2\rangle = \frac{1}{(2P+1)^2}E\left\{\left[\sum_{l=k-P}^{k+P} n(l)\right]^2\right\} \tag{10}$$

or, in terms of the autocorrelation function of $n(k)$, by

$$\sigma^2 = \frac{1}{2P+1}R_{nn}(0) + \frac{2}{(2P+1)^2}\sum_{j=1}^{2P} jR_{nn}(2P+1-j) \tag{11}$$

Dividing both sides of eqn. (8) by $R_{nn}(0)$ so as to introduce the normalized autocorrelation function $f_{nn}(k) = R_{nn}(k)/R_{nn}(0)$ in the term corresponding to σ^2, we find that the number of points, P, should be such as to minimize the function

$$E = \tfrac{1}{36}P^2(P+1)^2\frac{[y''(k)]^2}{R_{nn}(0)} + \frac{1}{2P+1}\left[1 + \frac{2}{2P+1}\sum_{j=1}^{2P} jf_{nn}(2P+1-j)\right] \tag{12}$$

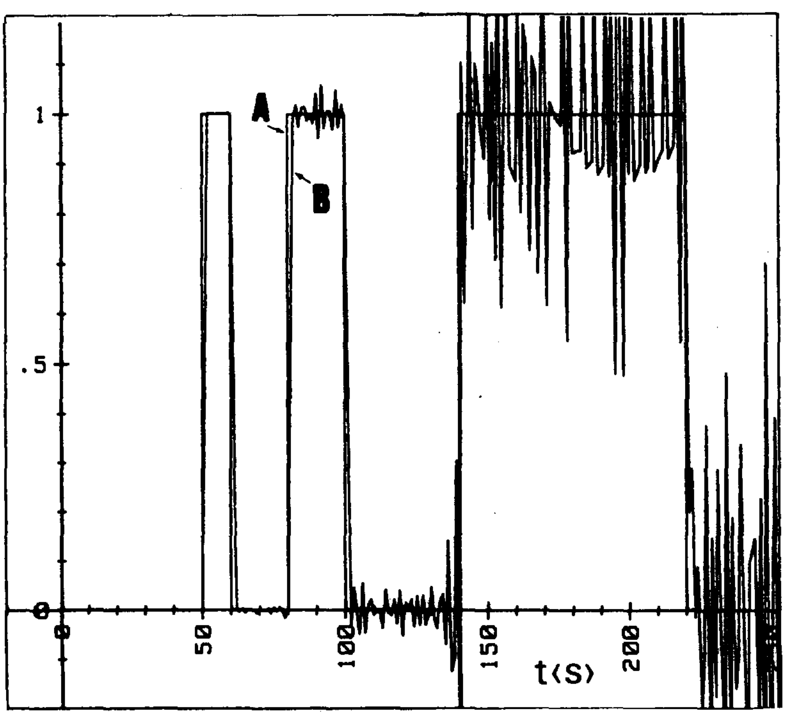

Fig. 4. True heat signal (A) together with the deconvolved signal (B) obtained from the measured signal before filtering.

In practical applications with a given calorimeter, the computer program implementing the filter would have access to a pre-calculated table giving the value of P as $y''(k)$ varies. For a given point k, P would be looked up in this table after estimating $y''(k)$ over a short time interval.

EXPERIMENTAL RESULTS AND CONCLUSIONS

The filter described above was calculated for a standard Calvet conduction microcalorimeter with a cell volume of 10 cm^3. A mean autocorrelation function for the calorimeter noise was estimated by averaging the results of applying eqn. (6) to each of a number of calorimeter output signals recorded for different constant inputs (Fig. 2). On fitting the function

$$f_{nn}(t) = A_0 + A_1 t + (1 - A_0)\ e^{-A_2 t} \tag{13}$$

to the initial 13 s of this empirical autocorrelation function, the values $A_0 = 0.44498$, $A_1 = -0.01527$ and $A_2 = 1.4205$ were obtained. In Fig. 3 the value of P which minimizes E in eqn. (12) is plotted against $R_{nn}(0)/[y''(k)]^2$.

To test the method, a known heat signal was generated electrically within the calorimeter and the measured output signal deconvolved both with and

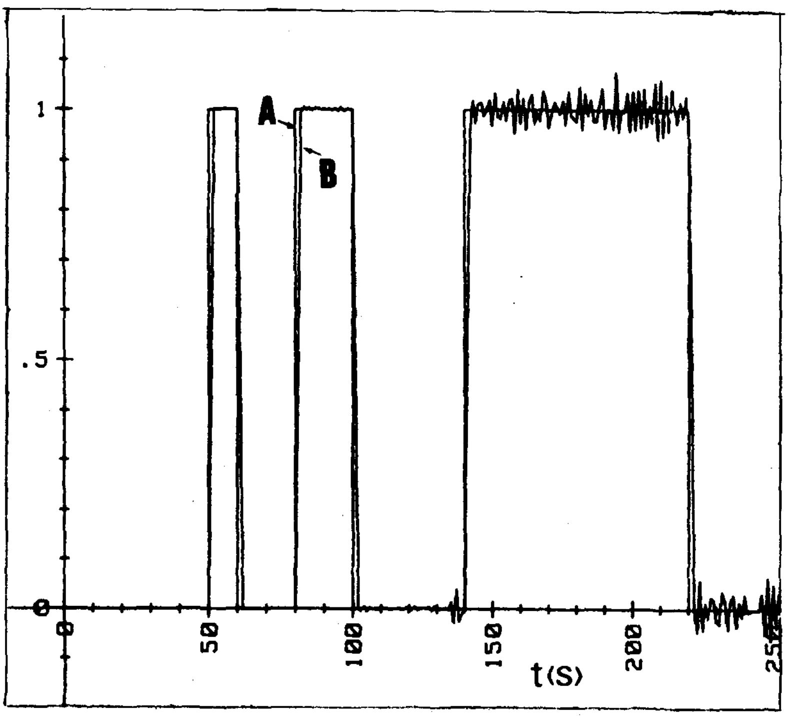

Fig. 5. True heat signal (A) together with the deconvolved signal (B) obtained after filtering the measured signal by the method described in this article.

without previous filtering (Figs. 5 and 4, respectively). The second derivative of the signal was estimated at each point by fitting a third-degree polynomial to the measured output signal at that point. Deconvolution was carried out using the Z-transform [1]. It is clear from Figs. 4 and 5 that the use of the filter results in a significant improvement in the reconstruction of the heat signal.

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APPENDIX

The Taylor expansion of y about $y(k)$ is

$$y(l) = y(k) + (l-k)y'(k) + \tfrac{1}{2}(l-k)^2 y''(k) + \tfrac{1}{6}(l-k)^3 y'''(k) + \ldots \quad \text{(A1)}$$

If derivatives of order four or higher are ignored (calorimeter signals normally vary quite slowly), then

$$E\{\hat{y}(k)\} = \frac{1}{2P+1} \sum_{l=k-P}^{k+P} y(l)$$

$$= \frac{1}{2P+1}\left[(2P+1)y(k) + y'(k)\left[\sum_{l=k-P}^{n+P} (l-k)^2\right] + \tfrac{1}{2}y''(k) \sum_{l=k-P}^{k+P} (l-k)^2 + \tfrac{1}{6}y'''(k) \sum_{l=k-P}^{k+P} (l-k)^3\right] \quad \text{(A2)}$$

Since

$$\sum_{l=k-P}^{k+P} (l-k) = \sum_{l=k-P}^{k+P} (l-k)^3 = 0$$

and

$$\sum_{l=k-P}^{k+P} (l-k)^2 = \tfrac{1}{3}P(P+1)(2P+1) \quad \text{(A3)}$$

then

$$b = E[\hat{y}(k)] - y(k) = \tfrac{1}{6}P(P+1)y''(k) \quad \text{(A4)}$$