

DISTURBANCES IN WEIGHING — PART I

A SURVEY OF WORK PRESENTED AT THE PRECEDING VMT CONFERENCES

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ABSTRACT

A survey is given on disturbances which determine the accuracy of weighing in vacuum. The nature of the disturbances covers the range from fundamental to purely practical. Apart from the physical description of the disturbing phenomena, numerical estimates for beam-type balances are presented. Part I covers the following phenomena:

- Brownian motion
- Knudsen forces
- Cavity forces
- Unequal thermal expansion of balance arms
- Magnetostatic effects
- Radiation pressure
- Vibrations of the building
- Other effects

Additional effects will be discussed in Part II during the 21st Conference, 1985 at Dijon.

BROWNIAN MOTION

(refs. 1,3,4,9,19,47-49)

The phenomenon of Brownian Motion is of fundamental nature, its effect on weighing-accuracy is found through the application of the equipartition theorem of the balance as a system with one degree of motional freedom. From ref.47 we learn that for an automated balance the spurious mass effect  $\Delta m_B$  resulting from Brownian Motion satisfies:

$$\Delta m_B = \frac{2}{1g} \sqrt{\frac{J k T}{t_i t_D}} \quad (1)$$

where  $l$  is the armlength (m) of the balance beam,  
 $g$  is the acceleration due to gravity:  $10 \text{ m/s}^2$ ,  
 $J$  is the moment of inertia ( $\text{kg m}^2$ ) of the balance  
 $k$  is the Boltzmann-constant:  $1.4 \cdot 10^{-23} \text{ J/K}$ ,

$T$  is the temperature (K),  
 $t_i$  is the time (s) of integration and  
 $t_D$  is the characteristic damping time (s).

For an estimate of  $\Delta m_B$  we use:

$l = 0.1$  m,  $J = 2 \cdot 10^{-4}$  kg m<sup>2</sup>,  $T = 300$  K,  $t_i = 1$  s and  $t_D = 10$  s.

The result is:

$$\Delta m_B = 0.006 \text{ } \mu\text{g} \quad (2)$$

#### KNUDSEN FORCES

(refs. 1-6,9,11,15-21,30,31)

These forces act on the balance in the presence of temperature gradients at low pressures. In order to be able to give an estimate on Knudsen forces we will restrict ourselves here by introducing the following approximations:

- the hangdown tube is infinitely long,
- the radius of the sample  $R_i$  satisfies  $R_i \ll R_o$  where  $R_o$  is the radius of the hangdown tube (or furnace)
- at collisions the accommodation coefficient of molecules equals unity.

Two pressure ranges are to be distinguished:

- a) pressures at which the mean free path  $\lambda$  of the molecules satisfies  $\lambda > R_o$
- b) pressures at which  $\lambda < R_o$ .

Furthermore we distinguish forces acting on the sample on one hand and on the hangdown wire on the other.

a:  $\lambda > R_o$

Under the condition  $\lambda > R_o$  the Knudsen forces lead to a mass effect  $\Delta m_{Ka}$  which satisfies (ref.48)

$$\Delta m_{Ka} = 2 \frac{R_i R_o p}{g} \frac{\sqrt{T_t} - \sqrt{T_b}}{\sqrt{T_b}} \quad (3)$$

where  $p$  is the gas pressure (Pa)

$T_t$  is the temperature (K) at the top and

$T_b$  is the temperature (K) at the bottom.

We use  $R_o = 2 \cdot 10^{-2}$  m,  $p = 10^{-1}$  Pa and

$i$  for the force on the sample:  $R_i = 2 \cdot 10^{-3}$  m,  $T_t = 1140$  K and  $T_b = 1200$  K and we obtain:

$$\Delta m_{Ka} = 20 \text{ } \mu\text{g} \quad (4)$$

and for the force on the hangdown wire using  $R_i = 5 \cdot 10^{-5}$  m,

$T_t = 300$  K and  $T_b = 1200$  K we obtain:

$$\Delta m_{Ka} = 20 \text{ } \mu\text{g} \quad (5)$$

$$b: \lambda \ll R_0$$

Under the condition  $\lambda \ll R_0$  the Knudsen forces lead to a mass effect  $\Delta m_{Kb}$  which satisfies (refs.19,49):

$$\Delta m_{Kb} = \frac{\pi \lambda^2 p (T_t - T_b)}{2 T \ln(R_0/R_i) - 1} \quad (6)$$

where  $T = (T_t + T_b)/2$ .

Using  $p = 10^2$  Pa and  $\lambda = 10^{-4}$  m we obtain for the mass effect at the sample:

$$\Delta m_{Kb} = 10 \mu g \quad (7)$$

and for the mass effect on the hangdown wire:

$$\Delta m_{Kb} = 40 \mu g \quad (8)$$

So far we have calculated the effects resulting from the longitudinal Knudsen forces. Spurious mass changes due to the transverse Knudsen forces satisfy in case a:

$$\Delta m_{Ka} = \pi \frac{R_i^2 p}{g} \frac{\sqrt{T_b} - \sqrt{T_t}}{\sqrt{T_b}} \quad (9)$$

At the maximum of these transverse Knudsen forces the resulting mass effects become

$$\text{on the sample: } \Delta m_{Ka} = 300 \mu g \quad (10)$$

on the hangdown wire

$$\Delta m_{Ka} = 8 \mu g \quad (11)$$

#### CAVITY FORCES

(refs. 3,4,15,17,23,30)

These forces occur in case measurements are performed with a porous sample while a vertical temperature gradient prevails (ref.15). The origin of this effect is the same as that of Knudsen forces. In order to give an estimate of the accompanying spurious mass-effect  $\Delta m_c$  the assumption is made that pores are identical cylinders, radius  $R_p$ , perpendicular to the horizontal surface of the sample. The result is:

$$\Delta m_c = f \frac{\pi R_i^2 R_p}{4 g T} \frac{dT}{dz} \quad (12)$$

where  $f$  is the fraction of the sample surface covered with pores and  $dT/dz$  is the temperature gradient in the pore-material. We take  $f = 1/3$ ,  $R_p = 10^{-5}$  m and  $dT/dz = 500$  K/m. For the maximum effect which occurs at  $\lambda \approx R_p$  and so at  $p \approx 10^3$  Pa we find:

$$\Delta m_c \approx 2 \mu g \quad (13)$$

## UNEQUAL THERMAL EXPANSION OF BALANCE ARMS

(refs. 1,3,4,14,24,50)

This phenomenon can occur when there exists a temperature gradient along the balance beam resulting in an inequality of armlength and so in a spurious mass-effect  $\Delta m_a$ . From literature (refs.14,50) we learn:

$$\Delta m_a = \frac{3}{8} \frac{\alpha \sigma \epsilon_b}{\lambda_{\text{eff}}} \frac{l^2}{R_b} T^3 \Delta T \quad (14)$$

where  $\alpha$  is the linear expansion coefficient of the beam material ( $K^{-1}$ )

$\sigma$  is the Boltzmann constant:  $5.6 \cdot 10^{-8} \text{ J/m}^2 \text{K}^4$

$\epsilon_b$  is the emissivity of the beam surface

$\lambda_{\text{eff}}$  is the effective thermal conductivity (J/mKs) of the beam

$l$  is the armlength (m)

$R_b$  is the radius (m) of the beam

$m$  is the mass (kg) of the sample and the counterweight

$\Delta T$  is the temperature difference (K) between the surrounding of an arm and the middle of the beam.

The results of this effect are calculated by using  $l = 5 \cdot 10^{-2} \text{ m}$ ,  $T = 300 \text{ K}$ ,  $m = 2 \cdot 10^{-3} \text{ kg}$ ,  $R_b = 10^{-3} \text{ m}$  and  $\Delta T = 1 \text{ K}$ . Three different beam-compositions are considered: Aluminum, silica and a silica beam covered with a  $5 \cdot 10^{-6} \text{ m}$  thick layer of gold. Using the values of the physical properties from practical handbooks we find

$$\begin{aligned} \text{Al-beam:} & \quad \Delta m_a = 0.5 \text{ } \mu\text{g} \\ \text{Silica-beam:} & \quad \Delta m_a = 0.6 \text{ } \mu\text{g} \\ \text{Silica-Au-beam:} & \quad \Delta m_a = 0.03 \text{ } \mu\text{g} \end{aligned} \quad (15)$$

## MAGNETOSTATIC EFFECTS

(refs. 1,3,4,10)

Here we have to distinguish two influences on balance reading:

- a) the effect of a homogeneous magnetic field on a ferromagnetic impurity in the balance beam,
- b) the effect of an inhomogeneous magnetic field on a ferro-or paramagnetic sample.

a: Homogeneous field

This effect can be estimated by using (ref.3):

$$\Delta m_{\text{ma}} = \frac{\mu_o H M V}{g l} \quad (16)$$

where  $\mu_0$  is the magnetic permeability of vacuum:  $4\pi \cdot 10^{-7}$  Vs/Am  
 H is the external magnetic field (A/m)  
 M is the magnetization (A/m) of the ferromagnetic impurity and  
 V is the volume of the impurity ( $m^3$ )

As an example we use:  $H = 10$  A/m,  $M = 10^5$  A/m,  $V = 10^{-10}$   $m^3$  and  $l = 0.1$  m, so:

$$\Delta m_{ma} \approx 0.1 \text{ } \mu\text{g} \quad (17)$$

#### b: Inhomogeneous magnetic field

This effect can be estimated by using (ref.3):

$$\Delta m_{mb} = \frac{\mu_0 \chi V H \partial H / \partial z}{g} \quad (18)$$

where  $\chi$  is the magnetic susceptibility of the sample material  
 V is the volume of the sample and  
 $\partial H / \partial z$  is the vertical component of the magnetic field gradient.

As an example we use  $\chi = 10^{-4}$  (paramagnetism),  $V = 10^{-6}$   $m^3$ ,  
 $H = 10^3$  A/m and  $\partial H / \partial z = 10^4$  A/m<sup>2</sup> and obtain :

$$\Delta m_{mb} = 0.1 \text{ } \mu\text{g} \quad (19)$$

#### RADIATION PRESSURE

(refs. 3,4,33,35)

This phenomenon can result in spurious mass-effects which can be estimated by (ref.3):

$$\Delta m_r = \frac{P_z}{g c} \quad (20)$$

where  $P_z$  is the radiation power (W) absorbed by the sample from vertically directed radiation and  
 c is the velocity of light:  $3 \cdot 10^8$  cm/s

Taking as an example the effect of spark-image heating we can use  $P_z = 3$  W and find:

$$\Delta m_r \approx 1 \text{ } \mu\text{g} \quad (21)$$

#### BUILDING VIBRATIONS

(refs. 3,4,51-54)

The effect of building vibrations on balance performances has been discussed in literature (ref.51). Two types of vibrations are distinguished viz.

- a) translational vibrations and
- b) rotational vibrations.

a: Translational vibrations

This type of vibration effects mass determination when the displacements take place in a direction parallel to the beam. The resulting spurious mass effect  $\Delta m_{va}$  satisfies (ref.51):

$$\Delta m_{va} = 2\pi \frac{m h X f}{g l t_i} \quad (22)$$

where  $m$  is the mass (kg) of the balance

$h$  is the (vertical) distance (m) between the axis of rotation of the beam and the center of mass of the balance

$l$  is the armlength (m)

$t_i$  is the time (s) of integration

$X$  is the amplitude (m) of the vibration and

$f$  is the frequency of the vibration

As typical values we take  $m = 0.02$  kg,  $h = 10^{-4}$  m,  $l = 0.1$  m and  $t_i = 10$  s. For  $X$  and  $f$  we choose literature data (ref.52-54)  $X = 1.5 \cdot 10^{-5}$  m at 20 Hz and find:

$$\Delta m_{va} \approx 0.3 \text{ } \mu\text{g} \quad (23)$$

b: Rotational vibrations

The effect of building rotation is only significant in case the axes are directed parallel to the rotation axis of the balance. Here the resulting mass effect reads:

$$\Delta m_{vb} = 2\pi \frac{J A f}{g l t_i} \quad (24)$$

where  $J$  is the moment of inertia ( $\text{kgm}^2$ ) of the balance and

$A$  is the amplitude of the angular building vibrations

We use  $J = 10^{-5}$   $\text{kgm}^2$  and from literature  $A = 3 \cdot 10^{-7}$  at  $f = 1$  Hz.

The result is:

$$\Delta m_{vb} = 0.002 \text{ } \mu\text{g} \quad (25)$$

In practice the effect of such building vibrations are coped with by a suitable damping system so that building vibrations are not directly transferred to the balance case.

## OTHER EFFECTS

Further disturbances may arise from imperfect, soiled or damaged fulcrums or suspension bearings (refs. 1,3,4,55), from fingerprints, corrosion and adsorption at balance parts or counterweights (refs. 4,13,46), buoyancy (refs. 1-5,7,8,22,25,45),

convective currents in the balance case (refs. 1-5,8,15,17,18, 27-30,32,34), temperature deviation of the sample (refs. 4,13, 25,26,45), electrostatic fields (refs. 3,4), or from noise caused by network and control circuits (refs. 4,55). The resulting errors will be estimated in Part II, which will be presented at the next VMT Conference.

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