# THERMODYNAMICS OF ASSOCIATED SOLUTIONS. PHASE EQUILIBRIA OF BINARY AND TERNARY WATER-ALCOHOL MIXTURES

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## ABSTRACT

The UNIQUAC associated solution theory is applied to the accurate reproduction of binary vapor-liquid equilibrium data for water-alcohol mixtures. It is able to predict ternary liquid-liquid and vapor-liquid equilibria for mixtures containing water and alcohols using binary parameters obtained from phase equilibrium data for the constituting binary mixtures.

## INTRODUCTION

Chemical models are well suited to the description of excess thermodynamic properties of associated solutions. Many investigators have studied vapor-liquid equilibria for binary alcohol-inert and -active component solutions with association models containing chemical and physical contribution terms to the excess Gibbs energy. Nath and Bender [1,2] for the first time successfully applied their association theory to binary aqueous mixtures and predicted isothermal vapor-liquid equilibria for ternary aqueous mixtures from binary parameters alone. However, Nath and Bender did not perform liquid-liquid equilibrium calculations for aqueous mixtures. The UNIQUAC associated solution theory [3,4] has been extended to associated solutions containing any number of alcohols to calculate vapor-liquid and liquid-liquid equilibria [5].

This paper presents an application of the UNIQUAC associated solution theory to phase equilibrium calculation for water-alcohol mixtures.

## **THEORY**

## *Binary mixtures containing two associating components*

The theory assumes three linear, polymer-forming reactions for self-association of components A and B and for solvation between the  $i$ -th polymer

$$
A_1 + A_i = A_{i+1}
$$
  
\n
$$
B_1 + B_i = B_{i+1}
$$
\n(1)

$$
\mathbf{A}_i + \mathbf{B}_j = \mathbf{A}_i \mathbf{B}_j \tag{3}
$$

The equilibrium constants for these reactions are given by

$$
K_{I} = (\Phi_{I_{i+1}} / \Phi_{I_{i}} \Phi_{I_{1}}) i / (i+1)
$$
\n(4)

for association of component  $I (I = A, B)$ , and

$$
K_{AB} = \left(\Phi_{A_iB_j}/\Phi_{A_i}\Phi_{B_j}\right)ij/(ir_A + jr_B)
$$
\n(5)

for solvation

The activity coefficient of component A is expressed as

$$
\ln \gamma_{A} = \ln (\Phi_{A_{1}} / \Phi_{A_{1}}^{0} x_{A}) + r_{A} (1 / V^{0} - 1 / V)
$$
  
– (Z/2) q<sub>A</sub> [ln( $\Phi_{A} / \theta_{A}$ ) + 1 –  $\Phi_{A} / \theta_{A}$ ]  
+ q<sub>A</sub>{ - ln( $\theta_{A}$  +  $\theta_{B} \tau_{BA}$ ) +  $\theta_{B} [\tau_{BA} / (\theta_{A} + \theta_{B} \tau_{BA}) - \tau_{AB} / (\theta_{B} + \theta_{A} \tau_{AB})]$  (6)

where the segment fraction,  $\Phi$ , the surface fraction,  $\theta$ , and adjustable binary parameter,  $\tau$ , are defined by

$$
\Phi_{\mathbf{A}} = r_{\mathbf{A}} x_{\mathbf{A}} / (r_{\mathbf{A}} x_{\mathbf{A}} + r_{\mathbf{B}} x_{\mathbf{B}}) \qquad \Phi_{\mathbf{B}} = 1 - \Phi_{\mathbf{A}} \tag{7}
$$

$$
\theta_{A} = q_{A} x_{A} / (q_{A} x_{A} + q_{B} x_{B}) \quad \theta_{B} = 1 - \theta_{A}
$$
\n(8)

$$
\tau_{BA} = \exp(-a_{BA}/T) \qquad \tau_{AB} = \exp(-a_{AB}/T) \qquad (9)
$$

The activity coefficient of component B is given by exchanging the subscripts A and B for B and A, respectively.

The nominal segment fractions of both components are expressed by the monomer segment fractions.

$$
\Phi_{A} = \left[ \Phi_{A_{1}} / (1 - K_{A} \Phi_{A_{1}})^{2} \right] \left[ 1 + r_{A} K_{AB} \theta_{B_{1}} / (1 - K_{B} \Phi_{B_{1}}) \right]
$$
(10)

$$
\Phi_{B} = \left[ \Phi_{B_{1}} / (1 - K_{B} \Phi_{B_{1}})^{2} \right] \left[ 1 + r_{B} K_{AB} \Phi_{A_{1}} / (1 - K_{A} \Phi_{A_{1}}) \right]
$$
(11)

and the true molar volume of the mixture is given by

$$
1/V = \Phi_{A_1}/r_A(1 - K_A \Phi_{A_1}) + K_{AB}\Phi_{A_1}\Phi_{B_1}/[(1 - K_A \Phi_{A_1})(1 - K_B \Phi_{B_1})]
$$
  
+  $\Phi_{B_1}/r_B(1 - K_B \Phi_{B_1})$  (12)

For the pure component I eqns.  $(10)$ – $(12)$  reduce to

$$
\Phi_{I_1}^0 = \left[ \left( 2K_I + 1 \right) - \left( 1 + 4K_I \right)^{1/2} \right] / 2K_I^2 \tag{13}
$$

and

$$
1/V_I^0 = (1 - K_B \Phi_{I_1}^0)/r_I
$$
 (14)

## **Ternary mixtures containing three associating components**

*In a* solution including three self-associating components, three general successive chemical reactions may proceed [5].

$$
I_1 + I_i = I_{i+1} \quad I = A, B, C \tag{15}
$$

$$
I_i + J_j = I_i J_j \t I = A, B; J = B, C \t (16)
$$

$$
\mathbf{A}_i + \mathbf{B}_j + \mathbf{C}_k = \mathbf{A}_i \mathbf{B}_j \mathbf{C}_k \tag{17}
$$

The equilibrium constants for these reactions additional to eqns. (4) and (5) are defined as

$$
K_{\rm AC} = \left(\Phi_{A,C}/\Phi_{A,\phi} - \mu_{C,\phi}\right) / (i r_{\rm A} + j r_{\rm B}) \tag{18}
$$

$$
K_{\rm BC} = \left(\Phi_{\rm B,C}/\Phi_{\rm B} \Phi_{\rm C_j}\right) i j / (i r_{\rm B} + j r_{\rm C}) \tag{19}
$$

$$
K_{ABC} = \left(\Phi_{A,B_jC_k}/\Phi_{A_j}\Phi_{B_j}\Phi_{C_k}\right)ijk/(ir_A + jr_B + kr_C)
$$
\n(20)

**The** monomer segment fractions of three components are related to the stoichiometric segment fraction of each component as shown for binary mixtures.

$$
\Phi_{A} = \left[ \Phi_{A_{1}} / (1 - K_{A} \Phi_{A_{1}}) \right] \left( 1 + r_{A} \left\{ K_{AB} \Phi_{B_{1}} / (1 - K_{B} \Phi_{B_{1}}) \right.\n+ K_{AC} \Phi_{C_{1}} / (1 - K_{C} \Phi_{C_{1}}) + K_{ABC} \Phi_{B_{1}} \Phi_{C_{1}} / [(1 - K_{B} \Phi_{B_{1}}) (1 - K_{C} \Phi_{C_{1}})] \right\}\n\right)
$$
\n(21)

$$
\Phi_{\rm B} = \left[ \Phi_{\rm B_{1}} / (1 - K_{\rm B} \Phi_{\rm B_{1}}) \right] \left( 1 + r_{\rm B} \left( K_{\rm AB} \Phi_{\rm A_{1}} / (1 - K_{\rm C} \Phi_{\rm C_{1}}) \right) + K_{\rm BC} \Phi_{\rm C_{1}} / (1 - K_{\rm C} \Phi_{\rm C_{1}}) + K_{\rm ABC} \Phi_{\rm A_{1}} \Phi_{\rm C_{1}} / [(1 - K_{\rm A} \Phi_{\rm A_{1}}) (1 - K_{\rm C} \Phi_{\rm C_{1}})] \right) \right)
$$
\n(22)

$$
\Phi_{\rm C} = \left[ \Phi_{\rm C_1} / (1 - K_{\rm C} \Phi_{\rm C_1}) \right] \left( 1 + r_{\rm C} \left\{ K_{\rm AC} \Phi_{\rm A_1} / (1 - K_{\rm A} \Phi_{\rm A_1}) \right.\n+ K_{\rm BC} \Phi_{\rm B_1} / (1 - K_{\rm B} \Phi_{\rm B_1}) + K_{\rm ABC} \Phi_{\rm A_1} \Phi_{\rm B_1} / \left[ (1 - K_{\rm A} \Phi_{\rm A_1}) (1 - K_{\rm B} \Phi_{\rm B_1}) \right] \right\} \right) \tag{23}
$$

The true molar volume of the solution is given by

$$
1/V = \Phi_{A_1}/r_A(1 - K_A\Phi_{A_1}) + K_{AB}\Phi_{A_1}\Phi_{B_1}/[(1 - K_A\Phi_{A_1})(1 - K_B\Phi_{B_1})]
$$
  
+  $\Phi_{B_1}/r_B(1 - K_B\Phi_{B_1}) + K_{BC}\Phi_{B_1}\Phi_{C_1}/[(1 - K_B\Phi_{B_1})(1 - K_C\Phi_{C_1})]$   
+  $\Phi_{C_1}/r_C(1 - K_C\Phi_{C_1}) + K_{AC}\Phi_{A_1}\Phi_{C_1}/[(1 - K_A\Phi_{A_1})(1 - K_C\Phi_{C_1})]$   
+  $K_{ABC}\Phi_{A_1}\Phi_{B_1}\Phi_{C_1}/[(1 - K_A\Phi_{A_1})(1 - K_B\Phi_{B_1})(1 - K_C\Phi_{C_1})]$  (24)

The activity coefficient of component  $I$  is

$$
\ln \gamma_I = \ln \left( \Phi_{I_1} / \Phi_{I_1}^0 x_I \right) + r_I \left( 1 / V^0 - 1 / V \right) - \left( 2 / 2 \right) q_I \left[ \ln \left( \Phi_I / \theta_I \right) + 1 - \Phi_I / \theta_I \right] + q_I \left[ 1 - \ln \left( \sum_J \theta_J \tau_{JI} \right) - \sum_J \left( \theta_J \tau_{IJ} / \sum_K \theta_K \tau_{KJ} \right) \right]
$$
(25)

where

$$
\theta_I = q_I x_I / \sum_J q_J x_J \tag{26}
$$

$$
\Phi_I = r_I x_I / \sum_J r_J x_J \tag{27}
$$

$$
\tau_{JI} = \exp(-a_{JI}/T) \tag{28}
$$

If component *I* does not self-associate,  $K_I = 0$  and  $\Phi_I^0 = 1$ .

## CALCULATED RESULTS AND DISCUSSION

The association constants of pure alcohols at 50°C were taken from Brandani [6]: for methanol, 173.9; for ethanol, 110.4; for l-propanol, 87.0; for l-butanol, 69.5. That for water was assumed as 500 at 50°C. The enthalpy of hydrogen-bond formation for all alcohols and water which self-associate through the O-H bond was selected as  $-23.2$  kJ mol<sup>-1</sup>, which is equivalent to the enthalpy of dilution of ethanol in *n*-hexane at  $25^{\circ}$ C [7].

The temperature dependence of the equilibrium constant is expresed by

$$
\frac{\partial \ln K}{\partial (1/T)} = -\frac{h}{R} \tag{29}
$$

Table 1 presents the solvation equilibrium constants and their temperature dependencies [5]. Any complex formation was not assumed in binary mixtures of hydrocarbons, tetrachloromethane and trichloromethane with water.

Binary vapor-liquid equilibrium data were reduced to obtain the energy parameters of the UNIQUAC associated solution theory using the relation

$$
\phi_I y_I P = \gamma_I x_I \phi_I^s P_I^s \exp\left\{ v_I^L (P - P_I^s) / RT \right\} \tag{30}
$$

where the fugacity coefficients,  $\phi_I$  and  $\phi_I^s$ , were calculated from

$$
\ln \phi_I = \left(2\sum_J y_J B_{IJ} - \sum_I \sum_J y_I y_J B_{IJ}\right) P/RT
$$
\n(31)

The second virial coefficients  $B_{IJ}$  were calculated by the Hayden-O'Connell method [8]. The pure molar liquid volumes  $v_t^L$  were estimated by the modified Rackett equation [9]. Pure component vapor pressures, *P;,* were obtained from the Antoine equation whose constants are available elsewhere [10,11]. Pure component structural parameters,  $r$  and  $q$ , were estimated by

TABLE 1

Solvation equilibrium constants and enthalpies of hydrogen bonding and of complex formation

A	B	$K_{AB}$ (50°C)	$-h_{AB}$ (kJ mol <sup>-1</sup> )
Methanol	Ethanol	2500	23.2
Methanol	Water	3500	23.2
Ethanol	Water	3000	23.2
1-Propanol	Water	700	23.2
1-Butanol	Water	400	23.2
Methanol	Benzene	4	8.3
Ethanol	Benzene	3	8.3
Ethanol	Trichloromethane	15	13
1-Propanol	Benzene	2.5	8.3
1-Propanol	Tetrachloromethane		5.5
1-Butanol	Benzene	2.5	8.3

the method of Vera et al. [12]. The computer program used for parameter estimation was similar to that based on the maximum likelihood principle as described by Prausnitz et al. [13]. The values of the standard deviations in the measured variables were equal to those given by Prausnitz et al. [13]: for pressure,  $\sigma_p = 1.0$  Torr; for temperature,  $\sigma_T = 0.05$  K; for liquid and vapor mole fractions,  $\sigma_x = 0.001$  and  $\sigma_y = 0.003$ .

Mutual solubility data were used to obtain energy parameters for a partially miscible mixture by solving eqn. (32) with a Newton-Raphson iterative technique.

$$
\left(\gamma_I x_I\right)^1 = \left(\gamma_I x_I\right)^{11} \tag{32}
$$

where the superscripts I and II indicate equilibrium liquid phases. The monomer segment fractions were obtained by solving related material balance equations with subroutine NOLBR built in a FACOM M-170F computer. Tables 2 and 3 give the binary calculated results as obtained from phase equilibrium data reduction and typical results are illustrated in Figs. 1 and 2. The calculated results in Fig. 2 show that the theory provided maxima in the plot of activity coefficient versus liquid mole fraction.

In ternary phase equilibrium calculations the ternary solvation constant, *K ABC,* was neglected. The predicted vapor-phase mole fractions and total pressures were compared with the experimental vapor-liquid equilibrium data of the water(1)-methanol(2)-ethanol(3) system at  $25^{\circ}$ C [14]. The average deviations on vapor-phase mole fraction and pressure were calculated by using the following equations

$$
\delta y_I = (100/N) \sum_i |\Delta y_I|_i \tag{33}
$$

$$
\delta P = (1/N) \sum_{i} |\Delta P_i| \tag{34}
$$



Root-mean square deviations as obtained from binary data reduction for water solutions Root-mean square deviations as obtained from binary data reduction for water solutions TABLE 2

 $A = M =$  mutual solubility. a MS = mutual solubility.



TABLE 3

 $A \times B$  mutual solubility. a MS = mutual solubility.

 $45$ 



Fig. 1. Vapor-liquid equilibria for (a) water-methanol and (b) water-ethanol at 25°C. Experimental  $(\bullet)$  [14]. Calculated  $(\_\_\_\_\)_$ .

$$
\delta P/P = (100/N) \sum_{i} |\Delta P_i|/P_i
$$
\n(35)

where  $\delta y_i = 2.3$ ,  $\delta y_2 = 3.6$ ,  $\delta y_3 = 3.8$ ,  $\delta P = 6.4$  Torr and  $\delta P/P = 9.2\%$ . Nath and Bender *[2]* presented the predictions of vapor-liquid equilibria for



Fig. 2. Vapor-liquid equilibria for (a) ethanol-n-heptane at  $30^{\circ}$ C and (b) 1propanol-cyclohexane at  $25^{\circ}$ C. Experimental: ( $\bullet$ ), ethanol-n-heptane [20]; lpropanol-cyclohexane [19]. Calculated (---------).

### TABLE 4



Ternary systems tested

ternary mixtures containing water and ethanol based on their association theory and the calculated results show exceedingly larger deviations than those obtained in this work. Table 4 lists eleven ternary, partially miscible systems chosen to test the ability of the theory: nine type-1 systems, i.e., including only one partially miscible binary; two type-II systems, i.e., containing two partially miscible binaries. Figure 3 compares the calculated values obtained from the theory with the experimental data [24,25]. The agreement for mixtures of normal alcohols with water is comparable to that obtained by commonly used models for the excess Gibbs energy.

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LIST OF SYMBOLS





 $\frac{1}{1}$ 

 $\frac{1}{2}$ 

 $\mathbf{I}$ 

 $\frac{1}{2}$ 

 $\bar{\rm l}$ 

 $\frac{1}{2}$ 

 $\frac{1}{1}$ 

 $\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array}$  $\ddot{\phantom{0}}$ 

48





49



*Greek letters* 



# *Subscripts*



# *Superscripts*



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