# MOLECULAR INTERACTIONS IN TERNARY MIXTURES OF NON-ELECTROLYTES: MOLAR EXCESS VOLUMES

**R.K. NIGAM \* and SADHNA AGGARWAL** 

Department of Chemistry, Maharshi Dayanand University, Rohtak 124 001 (India) (Received 10 January 1985)

#### ABSTRACT

Molar excess volumes,  $V_{ijk}^{E}$ , of 1,2-dichloroethane (i)+pyridine (j)+ $\alpha$ -picoline (k), + *n*-heptane (j)+pyridine (k), +  $\alpha$ -picoline (k), *n*-heptane (i)+pyridine (j)+ $\alpha$ -picoline (k), aniline (i)+pyridine (j)+ $\alpha$ -picoline (k) and +  $\gamma$ -picoline (k) mixtures have been measured dilatometrically at 298.15 K. The data have been examined in terms of the Sanchez and Lacombe theory and the graph-theoretical approach and they were found to be described better by the latter. Self- and cross-volume interaction coefficients,  $V_{jk}$ ,  $V_{jjk}$  and  $V_{jkk}$ , etc., have also been determined and the values utilized to study molecular interactions between the *j* th and *k* th in the presence of the *i*th molecular species in these i + j + k mixtures.

#### **INTRODUCTION**

In a binary i + j mixture, i-i and j-j contacts in the pure components iand j are replaced by i-j contacts in the mixture. If interactions in a ternary i+j+k mixture are assumed to be closely dependent on the interactions in the constituent i+j, j+k and i+k mixtures, it should be possible to evaluate the thermodynamic excess functions for ternary mixtures of nonelectrolytes when the corresponding functions for the binary i+j, j+k and i+k mixtures are known.

Singh and Sharma [1] recently employed a graph-theoretical approach that utilises connectivity parameters of the third degree,  ${}^{3}\xi_{i}$ , which successfully describes the molar excess volumes,  $V^{\rm E}$ , of ternary mixtures of non-heterocyclic non-electrolytes. Lacombe and Sanchez [2,3] have also developed a theory of fluid mixtures which suggests that the thermodynamic properties of ternary mixtures of non-electrolytes are determinable from the corresponding properties of the constituent binary mixtures. This prompted us to study the molar excess volumes,  $V^{\rm E}_{ijk}$ , of some ternary mixtures of non-electrolytes further and to analyse data in terms of (i) the Lacombe and Sanchez theory, and (ii) the graph-theoretical approach of fluid mixtures.

<sup>\*</sup> Author for correspondence.

#### EXPERIMENTAL

1,2-Dichloroethane, pyridine,  $\alpha$ - and  $\gamma$ -picolines, aniline and *n*-heptane (all BDH, Analar grade) were purified by standard procedures [4]. The purity was checked by measuring their densities at 293.15 ± 0.01 K, which agreed well with the corresponding literature values [4].

Molar excess volumes,  $V_{ijk}^{E}$ , were determined in a dilatometer similar to that used by Brown and Smith (1962) [4a], the difference being that there were now three limbs for the three components. The temperature of the water bath was controlled to within less than  $\pm 0.01$  K by means of a toluene regulator, and the change in the level of the liquid in the dilatometer capillary was measured by a cathetometer that could be read to  $\pm 0.001$  cm. Further, to ensure the complete mixing of the components in the capillary, the dilatometer was placed in a cold bath, so that there was a minimum amount of liquid in the capillary, and then returned to the experimental water bath. This process was repeated two or three times. The uncertainty in the measured  $V_{ijk}^{E}$  values is ~ 0.55% at most.

#### RESULTS

The  $V_{ijk}^{E}$  data for the various ternary (i + j + k) mixtures as a function of composition at 298.15 K are recorded in Table 1 and shown graphically in Figs. 1–6. These data were expressed as [1]

$$V_{ijk}^{E} = x_{i}x_{j} \left[ \sum_{n=0}^{2} A_{ij}^{n} (x_{i} - x_{j})^{n} \right] + x_{j}x_{k} \left[ \sum_{n=0}^{2} A_{jk}^{n} (x_{j} - x_{k})^{n} \right] \\ + x_{i}x_{k} \left[ \sum_{n=0}^{2} A_{ik}^{n} (x_{i} - x_{k})^{n} \right] + x_{i}x_{j}x_{k} \left[ \sum_{n=0}^{2} A_{ijk}^{n} (x_{j} - x_{k})^{n} x_{i}^{n} \right]$$
(1)

Where  $x_i$  and  $x_j$  are the mole fractions of the *i*th and *j*th components in the i+j+k mixture, and  $A_{ij}^n$  (n = 0-2), etc., are parameters characteristic of the binary (i+j), etc., mixtures. The  $A_{ijk}^n$  (n = 0-2) parameters in eqn. (1) were evaluated by fitting

$$\left\{ V_{ijk}^{\rm E} - x_i x_j \left[ \sum_{n=0}^{2} (x_i - x_j)^n A_{ij}^n \right] - x_j x_k \left[ \sum_{n=0}^{2} (x_j - x_k) A_{jk}^n \right] - x_i x_k \left[ \sum_{n=0}^{2} (x_i - x_k)^n A_{ik}^n \right] \right\} / x_i x_j x_k$$

data to

$$\left[\sum_{n=0}^{2} A_{ijk}^{n} (x_j - x_k)^n x_i^n\right]$$

## TABLE 1

Comparison of the measured  $V_{ijk}^{E}$  values for the various (i + j + k) ternary mixtures at 298.15 and 308.15 K with their corresponding  $V_{ijk}^{E}$  values as evaluated from the graph theory and Sanchez and Lacombe theory

| <i>x</i> <sub><i>i</i></sub> | $x_j \qquad V_{ijk}^{\mathbf{E}} \; (\mathrm{cm}^3 \; \mathrm{mol}^{-1})$ |  |                          |                      |  |  |
|------------------------------|---|--|--------------------------|----------------------|--|--|
|                              |   | Graph  | Expt.                    | Sanchez and Lacombe  |  |  |
| 1,2-Dichlo                   | roethane $(i)$ +  | pyridine $(j) + \alpha$ -pico                    | line $(k)$ at 29         | 8.15 K               |  |  |
| $\alpha_{ij} = -7.2$         | 00 (-2.259) c   | $m^3 mol^{-1}; \ \alpha_{jk} = -2.$              | 6088 (-0.434             | 4) $cm^3 mol^{-1}$ ; |  |  |
| $\alpha_{ik} = -1.8$         | 517 (~11.524  | ) cm <sup>3</sup> mol <sup><math>-1</math></sup> |                          |                      |  |  |
| 0.0299                       | 0.0660  | 0.006 (0.035)                                    | 0.010                    | -0.239               |  |  |
| 0.0441                       | 0.0891  | 0.027 (0.047)                                    | 0.013                    | -0.314               |  |  |
| 0.0591                       | 0.0947  | 0.034 (0.052)                                    | 0.019                    | -0.421               |  |  |
| 0.0714                       | 0.0596  | 0.038 (0.038)                                    | 0.019                    | -0.339               |  |  |
| 0.0784                       | 0.3378  | 0.047 (0.141)                                    | 0.040                    | -0.084               |  |  |
| 0.1778                       | 0.0297  | 0.083 (0.037)                                    | 0.052                    | -0.463               |  |  |
| 0.2022                       | 0.1170  | 0.093 (0.083)                                    | 0.067                    | -0.341               |  |  |
| 0.2080                       | 0.1018  | 0.095 (0.077)                                    | 0.068                    | -0.348               |  |  |
| 0.2095                       | 0.7310  | 0.055 (0.243)                                    | 0.137                    | 0.512                |  |  |
| 0.2184                       | 0.3827  | 0.089 (0.194)                                    | 0.098                    | 0.046                |  |  |
| 0.3494                       | 0.4800  | 0.096 (0.259)                                    | 0.178                    | 0.281                |  |  |
| 0.4470                       | 0.3502  | 0.117 (0.230)                                    | 0.185                    | 0.185                |  |  |
| 0.4678                       | 0.3706  | 0.112 (0.241)                                    | 0.200                    | 0.237                |  |  |
| 0.5771                       | 0.2417  | 0.127 (0.187)                                    | 0.183                    | 0.167                |  |  |
| 0.7520                       | 0.1502  | 0.102 (0.132)                                    | 0.151                    | 0.272                |  |  |
| 1,2-Dichlo                   | roethane $(i)$ +  | <i>n</i> -heptane $(j)$ +pyric                   | line (k) at 29           | 8.15 K               |  |  |
| $\alpha_{ij} = -94.$         | 4958 cm <sup>3</sup> mol  | <sup>-1</sup> ; $\alpha_{ik} = -25.69$ (-        | 1.327) cm <sup>3</sup> m | $ol^{-1};$           |  |  |
| $\alpha_{ik} = -7.2$         | 200 ( <del>-</del> 2.259) c   | $cm^3 mol^{-1}$                                  |                          |                      |  |  |
| 0.0369                       | 0.8619  | 0.122 (0.948)                                    | 0.239                    | -0.478               |  |  |
| 0.0568                       | 0.8951  | 0.186 (1.354)                                    | 0.235                    | -0.515               |  |  |
| 0.0621                       | 0.8551  | 0.219 (1.835)                                    | 0.268                    | -0.443               |  |  |
| 0.1034                       | 0.8720  | 0.285 (1.346)                                    | 0.296                    | -0.410               |  |  |
| 0.1081                       | 0.0595  | 0.313 (1.389)                                    | 0.104                    | 0.503                |  |  |
| 0.1192                       | 0.6755  | 0.277 (1.763)                                    | 0.391                    | -0.191               |  |  |
| 0.1462                       | 0.7521  | 0.211 (0.968)                                    | 0.419                    | -0.236               |  |  |
| 0.1618                       | 0.4608  | 1.007 (3.387)                                    | 0.380                    | -0.030               |  |  |
| 0.1910                       | 0.1736  | 0.501 (2.785)                                    | 0.215                    | 0.326                |  |  |
| 0.2050                       | 0.1004  | 0.517 (2.456)                                    | 0.181                    | 0.408                |  |  |
| 0.3221                       | 0.3615  | 0.548 (2.236)                                    | 0.356                    | 0.214                |  |  |
| 0.3701                       | 0.2008  | 0.673 (2.461)                                    | 0.344                    | 0.303                |  |  |
| 0.4065                       | 0.4258  | 0.454 (1.387)                                    | 0.542                    | 0.253                |  |  |
| 0.4524                       | 0.1626  | 0.723 (2.228)                                    | 0.336                    | 0.341                |  |  |
| 0.6550                       | 0.2009  | 0.498 (1.072)                                    | 0.412                    | 0.445                |  |  |
| 0.7793                       | 0.0991  | 0.450 (0.842)                                    | 0.260                    | 0.453                |  |  |
| 0.8821                       | 0.0695  | 0.232 (0.364)                                    | 0.167                    | 0.471                |  |  |
| 1,2-Dichlo                   | roethane $(i)$ +  | <i>n</i> -heptane $(j)$ + $\alpha$ -pi           | coline $(k)$ at 2        | 298.15 K             |  |  |
| $\alpha_{ij} = -94.$         | .4958 cm <sup>3</sup> mol   | $\alpha_{jk} = -5.933 (-$                        | 5.078) cm <sup>3</sup> m | ol <sup>-1</sup> ;   |  |  |
| $\alpha_{ik} = -1.8$         | 3517 (-11.524   | ) cm <sup>3</sup> mol <sup><math>-1</math></sup> |                          |                      |  |  |
| 0.0474                       | 0.8514  | 0.389 (0.201)                                    | 0.216                    | 0.129                |  |  |
| 0.0666                       | 0.9121  | 0.322 (0.181)                                    | 0.195                    | 0.358                |  |  |

| $\overline{x_i}$      | <i>x<sub>j</sub></i>                                 | $V_{ijk}^{E} (\mathrm{cm}^3 \mathrm{mol}^{-1})$ |                |  |  |  |
|-----------------------|--|---|----------------|--|--|--|
|                       |  | Graph   | Expt.          | Sanchez and Lacombe                    |  |  |
| 0.0752                | 0.0492   | 0.420 (0.254)                                   | 0.016          | -0.313                                 |  |  |
| 0.0828                | 0.8310   | 0.411 (0.270)                                   | 0.273          | 0.252                                  |  |  |
| 0.1048                | 0.7829   | 0.422 (0.336)                                   | 0.320          | 0.231                                  |  |  |
| 0.1329                | 0.848  | 0.691 (0.389)                                   | 0.033          | -0.297                                 |  |  |
| 0.1852                | 0.6128   | 0.703 (0.503)                                   | 0.393          | 0.147                                  |  |  |
| 0.1864                | 0.5269   | 0.852 (0.617)                                   | 0.354          | 0.012                                  |  |  |
| 0.2814                | 0.1694   | 1.149 (0.535)                                   | 0.104          | -0.154                                 |  |  |
| 0.2881                | 0.5428   | 0.726 (0.449)                                   | 0.454          | 0.381                                  |  |  |
| 0.3343                | 0.3175   | 1.122 (0.575)                                   | 0.316          | 0.084                                  |  |  |
| 0.3676                | 0.4001   | 0.958 (0.501)                                   | 0.425          | 0.326                                  |  |  |
| 0.5574                | 0.0800   | 1.1417 (0.345)                                  | 0.121          | 0.205                                  |  |  |
| 0.6628                | 0.2024   | 0.846 (0.281)                                   | 0.360          | 0.734                                  |  |  |
| 0.8057                | 0.0705   | 0.839 (0.176)                                   | 0.177          | 0.693                                  |  |  |
| n-Heptane             | e(i)+pyridine  | $(j) + \alpha$ -picoline $(k)$                  | at 298.15 K    |  |  |  |
| $\alpha_{ij} = -25$   | .69 (-1.327) c                                       | $m^3 mol^{-1}; \ \alpha_{jk} = -2$              | 2.6088 (-0.434 | 4) cm <sup>3</sup> mol <sup>-1</sup> ; |  |  |
| $\alpha_{ik} = -5.5$  | 933 ( <i>-</i> 5.078) c                              | $m^3 mol^{-1}$                                  |                |  |  |  |
| 0.0370                | 0.6448   | 0.064 (0.090)                                   | -0.001         | 0.455                                  |  |  |
| 0.0412                | 0.0974   | 0.045 (0.33)                                    | -0.002         | -0.290                                 |  |  |
| 0.0575                | 0.1062   | 0.057 (0.39)                                    | 0.001          | -0.294                                 |  |  |
| 0.0652                | 0.0354   | 0.052 (0.024)                                   | 0.002          | -0.404                                 |  |  |
| 0.0858                | 0.0932   | 0.073 (0.043)                                   | 0.007          | -0.413                                 |  |  |
| 0.0882                | 0.0584   | 0.070 (0.035)                                   | 0.008          | -0.364                                 |  |  |
| 0.0990                | 0.3218   | 0.095 (0.094)                                   | -0.005         | -0.046                                 |  |  |
| 0.1114                | 0.8050   | 0.039 (0.131)                                   | -0.006         | -0.653                                 |  |  |
| 0.1318                | 0.1047   | 0.101 (0.057)                                   | 0.021          | -0.389                                 |  |  |
| 0.1932                | 0.1970   | 0.133 (0.093)                                   | 0.046          | -0.312                                 |  |  |
| 0.2754                | 0.5507   | 0.103 (0.194)                                   | -0.160         | 0.268                                  |  |  |
| 0.4984                | 0.2523   | 0.217 (0.074)                                   | 0.181          | -0.582                                 |  |  |
| 0.5592                | 0.0776   | 0.212 (0.084)                                   | 0.215          | -0.481                                 |  |  |
| 0.7208                | 0.0558   | 0.178 (0.065)                                   | 0.237          | -0.351                                 |  |  |
| 0.8484                | 0.0662   | 0.096 (0.056)                                   | 0.181          | -0.101                                 |  |  |
| Aniline ( <i>i</i>    | )+pyridine (j)                                       | $+ \alpha$ -picoline (k) at                     | 298.15 K       |  |  |  |
| $\alpha_{ij} = (7.50$ | )5) cm <sup>3</sup> mol <sup><math>-1</math></sup> ; | $\alpha_{jk} = -2.6088 (-0)$                    | ).434) cm³ mol | l <sup>-1</sup> ;                      |  |  |
| $\alpha_{ik} = 62.7$  | 353 (4709.2) cr                                      | $n^3 \text{ mol}^{-1}$                          |                |  |  |  |
| 0.0565                | 0.0431   | (-6.454)  | -0.141         | 0.081                                  |  |  |
| 0.0651                | 0.0730   | (-10.667)                                       | -0.161         | 0.192                                  |  |  |
| 0.0983                | 0.6585   | (-42.065)                                       | -0.243         | 0.364                                  |  |  |
| 0.1052                | 0.0614   | (-9.036)  | -0.241         | 0.411                                  |  |  |
| 0.1121                | 0.8213   | (-28.126)                                       | -0.211         | 0.533                                  |  |  |
| 0.1167                | 0.0841   | (-12.137)                                       | -0.271         | 0.601                                  |  |  |
| 0.1569                | 0.1136   | (-16.184)                                       | -0.345         | -0.442                                 |  |  |
| 0.2212                | 0.2836   | (-33.273)                                       | -0.479         | -0.820                                 |  |  |
| 0.2272                | 0.1646   | (-21.936)                                       | -0.442         | -0.471                                 |  |  |
| 0.2433                | 0.1640   | (-22.079)                                       | -0.462         | -0.506                                 |  |  |
| 0.3042                | 0.1744   | (-22.606)                                       | -0.509         | 0.566                                  |  |  |
| 0.3192                | 0.1450   | (-20.478)                                       | -0.500         | 0.615                                  |  |  |

TABLE 1 (continued)

| x <sub>i</sub>          | x <sub>j</sub>                       | $V_{ijk}^{\mathbf{E}}$ (cm <sup>3</sup> mol <sup>-1</sup> ) |                            |                     |  |
|-------------------------|--------------------------------------|---|----------------------------|---------------------|--|
|                         |                                      | Graph   | Expt.                      | Sanchez and Lacombe |  |
| 0.4339                  | 0.2500                               | (-31.916)   | -0.569                     | -0.558              |  |
| 0.4350                  | 0.1811                               | (-23.498)   | -0.548                     | -0.647              |  |
| 0.4929                  | 0.3817                               | (-38.416)   | -0.420                     | -0.394              |  |
| 0.7851                  | 0.1289                               | (-16.718)   | -0.216                     | -0.621              |  |
| Aniline (i)             | + pyridine ( j                       | $)+\gamma$ -picoline (k) a                                  | t 298.15 K                 |                     |  |
| $\alpha_{ii} = (7.505)$ | 5) cm <sup>3</sup> mol <sup>-1</sup> | $\alpha_{ik} = -6.2114(-$                                   | 0.858) cm <sup>3</sup> mol | -1,                 |  |
| $\alpha_{ik} = 36.02$   | 62 (280.42) cr                       | $n^3 mol^{-1}$  |                            |                     |  |
| 0.0254                  | 0.0764                               | (-0.809)  | -0.042                     | 0.184               |  |
| 0.0498                  | 0.0271                               | (-0.304)  | -0.139                     | 0.211               |  |
| 0.0635                  | 0.0914                               | (-0.954)  | -0.111                     | 0.082               |  |
| 0.0800                  | 0.0488                               | (-0.534)  | -0.141                     | 0.189               |  |
| 0.1260                  | 0.7680                               | (-2.320)  | -0.191                     | 0.242               |  |
| 0.1471                  | 0.1408                               | (-1.374)  | -0.223                     | 0.144               |  |
| 0.1532                  | 0.3158                               | (-2.581)  | -0.216                     | 0.144               |  |
| 0.2337                  | 0.1790                               | (-1.671)  | -0.283                     | - 0.079             |  |
| 0.2357                  | 0.5025                               | (-2.967)  | -0.283                     | -0.058              |  |
| 0.2374                  | 0.3002                               | (-2.431)  | -0.290                     | -0.073              |  |
| 0.2470                  | 0.5097                               | (-2.966)  | -0.290                     | -0.086              |  |
| 0.4534                  | 0.2883                               | (-2.187)  | -0.318                     | -0.486              |  |
| 0.5648                  | 0.1486                               | (-1.290)  | -0.274                     | - 0.638             |  |
| 0.7470                  | 0.0819                               | (-0.735)  | -0.152                     | - 0.795             |  |

TABLE 1 (continued)



Fig. 1. Molar excess volumes,  $V^{E}$ , of 1,2-dichloroethane (*i*)+pyridine (*j*)+ $\alpha$ -picoline (*k*) at 298.15 K.



Fig. 2. Molar excess volumes,  $V^{E}$ , of 1,2-dichloroethane (i) + *n*-heptane (j) + pyridine (k) at 298.15 K.



Fig. 3. Molar excess volumes,  $V^{\rm E}$ , of 1,2-dichloroethane (i) + n-heptane  $(j) + \alpha$ -picoline (k) at 298.15 K.



Fig. 4. Molar excess volumes,  $V^{E}$ , of *n*-heptane (*i*)+pyridine (*j*)+ $\alpha$ -picoline (*k*) at 298.15 K.



Fig. 5. Molar excess volumes,  $V^{E}$ , of aniline (i)+pyridine (j)+ $\alpha$ -picoline (k) at 298.15 K.



Fig. 6. Molar excess volumes,  $V^{\rm E}$ , of aniline (i)+pyridine (j)+ $\gamma$ -picoline (k) at 298.15 K.

by the method of least squares, and are recorded in Table 2 together with standard deviations,  $\sigma(V_{ijk}^{E})$ , defined as

$$\left[\sum \left(V_{ijk(\exp.)}^{\rm E} - V_{ijk(\text{calc.from eqn.1})}^{\rm E}\right)^2 / (m-p)\right]^{1/2}$$
(2)

# TABLE 2

 $V_{ijk}^0$ ,  $V_{ijk}^1$  and  $V_{ijk}^2$  parameters (cm<sup>3</sup> mol<sup>-1</sup>) of eqn. (3) along with the standard deviation,  $\sigma(V_{ijk}^E)$  (cm<sup>3</sup> mol<sup>-1</sup>) of the molar excess volume for the (i + j + k) mixtures at 298.15 K

| System  | Temp.<br>(K) | $V^0_{ijk}$ | $V_{ijk}^1$    | $V_{ijk}^2$  | $\sigma(V_{ijk}^{\rm E})$ |
|---|--------------|-------------|----------------|--------------|---------------------------|
| 1,2-Dichloroethane (i)<br>+ pyridine (j)                        |              |             |                |              |                           |
| $+ \alpha$ -picoline (k)  | 298.15       | - 1.0643    | 1.6922         | 25.0235      | 0.001                     |
| 1,2-Dichloroethane $(i)$<br>+ <i>n</i> -heptane $(j)$           |              |             |                |              |                           |
| + pyridine $(k)$  | 298.15       | -2.800      | -6.200         | -93.400      | 0.002                     |
| 1,2-Dichloroethane $(i)$<br>+ <i>n</i> -heptane $(j)$           |              |             |                |              |                           |
| + $\alpha$ -picoline $(k)$                                      | 298.15       | - 4.090     | 0.100          | -158.600     | 0.002                     |
| <i>n</i> -Heptane ( <i>i</i> )<br>+ pyridine ( <i>j</i> )       | 200.15       | 1 (20)      | <b>61 11 (</b> | <b>222</b> 0 | 0.002                     |
| + $\alpha$ -picoline (k)  | 298.15       | -4.620      | -51.114        | -232.0       | 0.002                     |
| Aniline $(i)$<br>+ pyridine $(j)$<br>+ $\alpha$ -picoline $(k)$ | 298.15       | - 5.700     | -2.800         | 145 000      | 0.003                     |
| Aniline $(i)$<br>+ pyridine $(j)$                               | -, 5.10      | 2.700       | 2.000          | 1.0.000      | 0.005                     |
| $+\gamma$ -picoline (k)   | 298.15       | - 1.9925    | 4.7533         | 14.552       | 0.003                     |

where *m* is the number of data points and *p* the number of adjustable parameters in eqn. (1). The parameters  $A_{ij}^n$ ,  $A_{jk}^n$ ,  $A_{ik}^n$ , etc., for the i + j, j + k and k + i binary mixtures were taken from the literature [4,5].

#### DISCUSSION

We are unaware of any previous  $V_{ijk}^{E}$  data with which to compare the present results for i + j + k mixtures. The  $V_{ijk}^{E}$  data for the ternary mixtures were first analysed [1] using the Lacombe and Sanchez [2,3] theory of classical fluids and their mixtures.

However, it was observed that the right-hand side (RHS) of the equation of state (eqn. 34a in ref. 1) varied from 0.0124 to 0.0441 for the various binary mixtures. Since, therefore, the i + j, j + k and i + k mixtures do not satisfy the equation of state for binary mixtures, the resulting i + j + kternary mixture would also not satisfy the equation of state. Thus,  $V_{ijk}^{\rm E}$ values evaluated in the manner originally proposed by Lacombe and Sanchez [3] would not compare well with the corresponding experimental values. The equation of state for a ternary mixture has been shown [1] to be given by

$$\bar{\rho}_{ijk}^{2} + p_{ijk} + (RT/\epsilon_{ijk}^{*}) \left[ \ln(1 - \bar{\rho}_{ijk}) + (1 - r_{ijk}^{-1}) \bar{\rho}_{ijk} \right]$$

$$= 1/3 \left( \sum \text{RHS of the equation of state for } i + j, \ j + k \text{ and } i + k \text{ mixtures} \right)$$
(3)

where  $\bar{\rho}_{ijk}$  and  $\epsilon^*_{ijk}$  have the same significance as explained earlier [1]. Once this equation of state for a ternary mixture has been established,  $\bar{\rho}_{ijk}$  and, hence,  $V^{\rm E}_{ijk}$  for the mixture at any composition and temperature can be evaluated readily.  $V^{\rm E}_{ijk}$  values thus obtained for the various ternary mixtures are recorded in Table 1, where they are also compared with the corresponding experimental values.

Examination of Table 1 reveals that  $V_{ijk}^{E}$  values calculated at 298.15 K are of the right order of magnitude, but that the quantitative agreement is not very impressive. In some cases even the sign is not predicted correctly. A possible reason for this failure may be the assumption that the right-hand side of eqn. (3) for the i + j, j + k and i + k binary mixtures comprising the i + j + k mixture is constant over the entire range of  $x_i$ ,  $x_j$  and  $x_k$  at the given temperature. Nevertheless, the theory makes a significant theoretical attempt to evaluate  $V_{ijk}^{E}$  data for ternary mixtures of non-electrolytes from a knowledge of their binary interaction coefficients.

The  $V_{ijk}^{E}$  data for the various ternary mixtures were then analysed [1] in terms of the graph-theoretical approach [6].  $V_{ijk}^{E}$  according to this approach is given by

$$V_{ijk}^{\rm E} = \alpha_{ijk} \left( 1 / \sum x_i^3 \xi_i - \sum x_i / {}^3 \xi_i \right) \tag{4}$$

where  $\alpha_{ijk}$  is a constant characteristic of the i+j+k mixture and is assumed to be independent of composition, and  $x_i$ , etc., are the mole fractions of the *i*th, etc., components in the mixture. The term  ${}^{3}\xi_i$  is a connectivity parameter of the third degree for the *i*th component, and is defined [7] by

$${}^{3}\xi_{i} = \sum_{l < m} \sum_{m < n} \sum_{n < o} \left(\delta_{l} \delta_{m} \delta_{n} \delta_{o}\right)^{-1/2}$$
(5)

where  $\delta_l$ ,  $\delta_m$ ,  $\delta_n$ ,  $\delta_o$ , etc., denote the degrees of the *l*th, *m*th, *n*th and *o*th, etc., vertices of the graph for the *i*th molecule [8] and were evaluated in the manner described earlier [6].

Evaluation of  $V_{ijk}^{E}$  for a ternary mixture thus requires a knowledge of  $\alpha_{ijk}$  for the mixture. The latter was shown [1] to be given by

$$\alpha_{ijk} = \frac{1}{5} (\alpha_{ij} + \alpha_{jk} + \alpha_{ik})$$
(6)

so that eqn. (4) reduces to

$$V_{ijk}^{\mathsf{E}} = 0.2(\alpha_{ij} + \alpha_{jk} + \alpha_{ik}) \left( 1 / \sum x_i^3 \xi_i - \sum x_i / {}^3 \xi_i \right)$$
(7)

Thus, from a knowledge of the binary interaction coefficients  $\alpha_{ij}$ ,  $\alpha_{jk}$  and  $\alpha_{ik}$  for the i + j, j + k and i + k binary mixtures,  $V_{ijk}^{E}$  for a ternary i + j + k mixture can be evaluated readily if  ${}^{3}\xi_{i}$  parameters for the constituent molecules are known.

Since aniline, pyridine and  $\alpha$ - and  $\gamma$ -picolines undergo reactions that are characteristic of the aromatic ring (disregarding the basic nature of these compounds for the present analysis), and as their molar volumes are almost the same (~10% lower) as those of benzene and toluene, the  ${}^{3}\xi_{i}$  parameters for pyridine and picolines were taken to be the same as those for benzene and toluene, respectively [6]. However, such an approach can be highly restrictive, and Singh [9] has recently advocated the use of  $\delta^{v}$  (valence  $\delta$ , which explicitly reflects the valency of the atoms forming the bonds) rather than  $\delta$  to evaluate  ${}^{3}\xi_{i}$  in calculating  $H^{E}$  and  $V^{E}$  values for binary mixtures of non-electrolytes. In the present analysis we thus evaluated  ${}^{3}\xi_{i}$  parameters for the constituent molecules of the ternary mixtures using both these approaches and then utilised both sets of results to calculate  $V^{E}_{ijk}$  data for the mixtures. The  $V^{E}_{ijk}$  values thus obtained are recorded in Table 1 ( $V^{E}_{ijk}$  values obtained from eqn. (4) using  ${}^{3}\xi$  parameters evaluated from  $\delta^{v}$  are enclosed in parentheses), where they are also compared with the corresponding experimental values.

Examination of Table 1 reveals that the  $V_{ijk}^{E}$  ( $\delta$  or  $\delta^{v}$ ) values calculated from eqn. (4) for all the present ternary mixtures correctly predict the sign of experimental  $V_{ijk}^{E}$  values; the quantitative agreement (for all except aniline (*i*) + pyridine (*j*) +  $\alpha$ -picoline (*k*) and + $\gamma$ -picoline (*k*) mixtures), however, is dictated by  $x_i$ ,  $x_j$  and  $x_k$ . This suggests that, depending upon  $x_i$ ,  $x_j$ ,  $x_k$ , either the C–C skeleton or the valency of individual atoms in the molecular graph of the constituent molecules dictate the nature of packing of *i*, *j* and *k* components in the (i + j + k) mixtures. For aniline (i) + pyridine (j) +  $\alpha$ -picoline (k) and + $\gamma$ -picoline (k) mixtures, the calculated  $V_{ijk}^{E}$  values are about 7 times more than the corresponding experimental values. This suggests that the addition of pyridine and  $\alpha$ - or  $\gamma$ -picoline to aniline causes stronger interaction between them than that envisaged by the theory.

 $V_{ijk}^{\rm E}(T,x_i,x_j)$  data were next analysed in terms of the Mayer-McMillan formalism [10] to gain information about the effect of *i* on the *j*-*j* and *k*-*k*, etc., interactions. According to this formalism [11], an excess function, say,  $V^{\rm E*}$ , of a solution containing *n* solute species can be expressed as

$$V^{E*} = V - V_{sol} - \sum_{i=1}^{n} V_i^0 m_i$$
(8)

$$=\sum_{i=1}^{n}\sum_{i=1}^{n}V_{ij}m_{i}m_{j}+\sum_{i=1}^{n}\sum_{i=1}^{n}\sum_{i=1}^{n}V_{ijk}m_{i}m_{j}m_{k}+\dots$$
(9)

where V and  $V^{E*}$  are the volume and excess volume, respectively, of a solution containing 1 kg of solvent and  $m_1, \ldots, m_n$  moles of each solute species;  $V_{sol}$  is the standard volume of 1 kg of the solvent; and  $V_i^0$  is the partial molal volume of each solute. For a ternary solution, eqn. (9) reduces to

$$V^{E*}(m_j, m_k) = V_{jj}m_j^2 + 2V_{jk}m_jm_k + V_{kk}m_k^2 + V_{jjj}m_j^3 + 3V_{jjk}m_j^2m_k + V_{jkk}m_jm_k^2 + V_{kkk}m_k^3$$
(10)

and for two binary (i + p), (p = j or k) solutions, eqn. (9) yields

$$V^{\mathsf{E}}(m_{j}) = V_{jj}m_{j}^{2} + V_{jjj}m_{j}^{3}$$
(11)

$$V^{\rm E}*(m_k) = V_{kk}m_k^2 + V_{kkk}m_k^3$$
(12)

In order to calculate the cross-volume coefficients  $V_{jk}$ ,  $V_{jkk}$  and  $V_{jjk}$  from eqn. (10), we used an auxiliary function,  $\Delta V^{E*}$ , defined [12] as

$$\Delta V^{E*} = V^{E*}(m_j, m_k) - V^{E*}(m_j) - V^{E*}(m_k)$$
(13)

which, in view of eqns. (11) and (12), yields

$$\Delta V^{\text{E}*}(m_j, m_k)^{-1} = 2V_{jk} + 3V_{jjk}m_j + 3V_{jkk}m_k + \dots$$
(14)

Thus, if  $V^{E*}(m_j, m_k)$  data for the (i + j + k) ternary mixture, along with the  $V^{E*}(m_j)$  and  $V^{E*}(m_k)$  data for (i + j) and (i + k) binary mixtures are known, it should be possible to evaluate the cross-volume coefficients. Ideally,  $V^{E*}(m_j, m_k)$ ,  $V^{E*}(m_j)$  or  $V^{E*}(m_k)$  data for dilute solutions should be utilized to evaluate these cross-volume coefficients. This led us to use  $V^{E*}(m_j, m_k)$  data for a ternary mixture in the range  $0.025 \le x_j$ ,  $x_k \le 0.11$  (corresponding to  $0.2 \le m_j$ ,  $m_k$ ) with the corresponding  $V^{E*}(m_p)$  (p = j or k) data of the (i + p) binary mixtures in the range  $0.2 \le m_p$  to calculate  $V_{jk}$ ,  $V_{jjk}$  and  $V_{jkk}$  from eqn. (14) by the method of least squares. For this purpose,  $V_{ijk}^{E}(x_j, x_k)$  data (cm<sup>3</sup> mol<sup>-1</sup>) of the (i + j + k) mixture were converted to  $V^{E*}(m_j, m_k)$  data (cm<sup>3</sup> mol<sup>-1</sup>; mol kg<sup>-1</sup> of i) by means of  $V^{E*}(m_j, m_k)$  cm<sup>3</sup> mol<sup>-1</sup> (mol kg<sup>-1</sup> of i)  $= \left[ V^{E}(x_j, x_k) \operatorname{cm}^3 \operatorname{mol}^{-1} \operatorname{of} (i + j + k) \operatorname{mixture} \right] \left[ 1000/x_i \operatorname{mol} \mathrm{kg}^{-1} \operatorname{of} i \right]$ (15)

while the  $m_p$  (p = j or k) values were obtained from

$$m_p = x_p \left[ \frac{1000}{x_i} \cdot (\text{mol kg}^{-1} \text{ of } i) \right]$$
 (16)

The corresponding  $V^{E*}(m_p)$  (p = j or k) values (cm<sup>3</sup> mol<sup>-1</sup>; mol kg<sup>-1</sup> of i) in the (i + p) binary mixtures were read from  $V^{E}(m_p)$  vs.  $m_p$  plots, and for this purpose  $V^{E}(x_p)$  data (cm<sup>3</sup> mol<sup>-1</sup>) of the (i + p) mixture were first converted to  $V^{E}(m_p)$  data by an expression analogous to eqn. (15). Such  $V_{jk}$ ,  $V_{ijk}$  and  $V_{ikk}$  data at 298.15 K are recorded in Table 3.

Examination of Table 3 reveals: (1) the  $V_{jk}$  interaction coefficients at 298.15 K of pyridine (j) with 1,2-dichloroethane (k), *n*-heptane (k) and aniline (k) in the presence of  $\alpha$ -picoline vary in the order 1,2-dichloroethane < n-heptane < aniline; (2) the  $V_{jk}$  interaction coefficients at 298.15 K of 1,2-dichloroethane (j) with pyridine (k) and  $\alpha$ -picoline (k) in the presence of *n*-heptane vary in the order  $\alpha$ -picoline < pyridine; and the  $V_{jk}$  interaction coefficients at 298.15 K of 1,2-dichloroethane (j) with pyridine (k) and  $\alpha$ -picoline (k) in the presence of *n*-heptane vary in the order  $\alpha$ -picoline < pyridine; and the  $V_{jk}$  interaction coefficients at 298.15 K of aniline (k) with pyridine (j) in the presence of

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Self and cross-volume interaction coefficients (cm<sup>3</sup> mol<sup>-3</sup>; mol kg<sup>-1</sup>) at 298.15 K

| $V_{jj}$ | V <sub>kk</sub>   | $V_{jk}$   | $V_{jjk}$   | V <sub>jkk</sub>   |
|----------|---|--|---|--|
|          |   |  |   |  |
|          |   |  |   |  |
| 0.079    | 0.700   | -0.1284  | 0.0919  | 0.0190   |
|          |   |  |   |  |
|          |   |  |   |  |
| 6.95     | 4.5   | 0.8247   | -0.0202   | -0.6592  |
|          |   |  |   |  |
|          |   |  |   |  |
| 6.95     | 3.31  | -0.0955  | -0.0635   | -0.0498  |
|          |   |  |   |  |
| 0.079    | -0.07   | -0.0002  | -0.0073   | 0.0026   |
|          |   |  |   |  |
| 0.079    | -6.45   | 1.8894   | 2.0458  | -3.1540  |
|          |   |  |   |  |
| 0.623    | - 6.95  | -1.4006  | 0.7771  | 0.1933   |
|          | V <sub>jj</sub><br>0.079<br>6.95<br>6.95<br>0.079<br>0.079<br>0.079 | $V_{jj}$ $V_{kk}$ 0.079         0.700           6.95         4.5           6.95         3.31           0.079         -0.07           0.079         -6.45           0.623         -6.95 | $V_{jj}$ $V_{kk}$ $V_{jk}$ 0.079         0.700 $-0.1284$ 6.95         4.5         0.8247           6.95         3.31 $-0.0955$ 0.079 $-0.07$ $-0.0002$ 0.079 $-6.45$ 1.8894           0.623 $-6.95$ $-1.4006$ | $V_{jj}$ $V_{kk}$ $V_{jk}$ $V_{jjk}$ 0.079         0.700 $-0.1284$ 0.0919           6.95         4.5         0.8247 $-0.0202$ 6.95         3.31 $-0.0955$ $-0.0635$ 0.079 $-0.07$ $-0.0002$ $-0.0073$ 0.079 $-6.45$ $1.8894$ $2.0458$ 0.623 $-6.95$ $-1.4006$ $0.7771$ |

 $\alpha$ -picoline (i) or  $\gamma$ -picoline (i) vary in the order  $\alpha$ -picoline >  $\gamma$ -picoline.

The  $V_{jk}$  data of the pyridine with 1,2-dichloroethane, *n*-heptane and aniline in the presence of  $\alpha$ -picoline then suggests that, compared to aniline or *n*-heptane, 1,2-dichloroethane comes closer to pyridine. This is understandable, since NMR studies on 1,2-dichloroethane +  $\alpha$ -picoline mixtures have already suggested that 1,2-dichloroethane and  $\alpha$ -picoline interact in the following manner



The addition of pyridine to 1,2-dichloroethane in  $\alpha$ -picoline would then cause pyridine to approach the electron-deficient end of  $\alpha$ -picoline in the above molecular complex with its own electron-deficient end, i.e.



The addition of *n*-heptane to pyridine in  $\alpha$ -picoline as a solvent would not allow the components to come closer while the addition of pyridine to aniline in  $\alpha$ -picoline may bring about additional rupture of the self association in aniline. The net effect would thus be that  $V_{jk}$  for aniline or 1,2-dichloroethane in pyridine in the presence of  $\alpha$ -picoline would vary in the order *n*-heptane < aniline. The fact that the  $V_{jk}$  data of 1,2-dichloroethane with pyridine or with  $\alpha$ -picoline in the presence of *n*-heptane vary in the order  $\alpha$ -picoline < pyridine lends further support to the possible interaction of 1,2-dichloroethane with pyridine in the presence of  $\alpha$ -picoline.

Again, NMR studies have suggested that aniline interacts with pyridine in

the following manner



The introduction of  $\alpha$ -picoline or  $\gamma$ -picoline into aniline + pyridine would then cause minimum disturbance in this scheme of molecular interaction when  $\gamma$ -picoline, rather than  $\alpha$ -picoline, is added. The fact that  $V_{jk}$  values of aniline with pyridine in the presence of  $\alpha$ -picoline or  $\gamma$ -picoline vary in the order  $\gamma$ -picoline <  $\alpha$ -picoline further supports this conjecture.

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